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INSIGHT

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INSIGHT

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INSIGHT

PREFACE

Rational self-consciousness is a peak above the clouds. Intelligent and reasonable, responsible and free, scientific and metaphysical, it stands above romantic spontaneity and the psychological depths, historical determinism and social engineering, the disconcerted existential subject and the undeciphered symbols of the artist and the modernist.

Yet if man can scale the summit of his inner being, also he can fail to advert to the possibility of the ascent or, again, he can begin the climb only to lose his way. If then he knows himself as in fact he is, he can know no more than that he has been cast into the world to be afflicted with questions he does not answer and with aspirations he does not fulfil. For it is the paradox of man that what he is by nature is so much less than what he can become: and it is the tragedy of man that the truth, which portrays him as actually he is, can descend like an iron curtain to frustrate what he would and might be.

Facts, it is said, are stubborn things. But there is a sense in which, I believe, it is true to say that the facts about man can be outflanked. For a change in man, a development of potentialities that are no less real because, like all potentialities they are latent, not only is itself a fact but also can be a permanent source of new facts that cumulatively alter the complexion of the old.

So it is that the present work is a program rather than an argument. It begins not by assuming premisses but by presuming readers. It advances not by deducing conclusions from the truths of a religious faith or from the principles of a philosophy but by issuing to readers an invitation, ever more precise and more detailed, to apprehend, to appropriate, to envisage in all its consequences, the inner focus of their own intelligence and reasonableness. That focus, it will be claimed, is insight. But to apprehend the focus is to gain insight into insight, to pierce the outer verbal and conceptual exhibitions of mathematics, of science, and of common sense, and to penetrate to the inner dynamism of intelligent inquiry and critical reflection. To appropriate the focus is both to know and to know what it is to know one's own intelligence, one's own reasonableness, one's own essential and restrictedly effective freedom. To envisage the focus in the full range of its implications is to discover for oneself what is meant by being, by objectivity, by meta-

physics, by ethics, by God, and by evil.

Frankly, even as a program, even as a sketch that offers only to indicate the detailed map that is needed, the present work may be reproached for excessive ambition. But if I may borrow a phrase from Ortega y Gasset, one has to strive to mount to the level of one's time. The twentieth century has been described as the end of the Renaissance. Some four centuries ago there was projected a new world: new nations had arisen in new political constellations; a new art was matched with the promise of a new science; and new philosophies were disseminated through a new education. That new world has been realized, but the ideas that fostered its genesis have been discredited by its maturity. What was so new has become so old. To have been educated is no longer a matter of speaking Latin and writing Greek. Modern art would puzzle Raffaello, as modern technology would astound da Vinci. The new nations are not in Europe, and the issues of modern politics seem transcribed from the pages of Utopia. Einstein has revised momentously the thought of Galileo, and Heisenberg has contended that good Laplace, like Homer, nods. The novel outlook that is transforming the natural sciences cannot but affect profoundly the methods that were transferred with so sedulous a fidelity from the natural to the human sciences. Not even Renaissance ridicule of the Middle Ages has been able to prevent a rebirth of interest

in logic. Not even the Enlightenment's insistence on the autonomy of man has been able to prevent the recurrence of theological themes under the guise of existentialist philosophy.

So it is that a new world has been bequeathed us and yet we, the heirs of the Renaissance, have been denied its spirit of bold confidence, of venturous assurance. For we know too much in too many fields, we have witnessed too much suffering in too many unexpected quarters, to purchase confidence by an easy exuberence of feeling or to accept words of assurance without answers to our questions. Nor was the basic question missed, when the late Prof. Ernst Cassirer, towards the end of a long and highly productive career, endeavored to communicate within a brief compass some of the main conclusions of his vast erudition and ever penetrating thought. Just what is man? Answers, he remarked, have been worked out by theologians and scientists, by politicians and sociologists, by biologists and psychologists, by ethnologists and economists. But not only do the many answers not agree, not only is there lacking some generally accepted principle that would select one and reject the others, but even within specialized fields there seems to be no method that can confront basic issues without succumbing to individual temperament and personal evaluations.

In the midst of this widespread disorientation, man's problem of self-knowledge ceases to be simply the

individual concern inculcated by the ancient sage. It takes on the dimensions of a social crisis. It can be read as the historical issue of the twentieth century. If in that balance human intelligence and reasonableness, human responsibility and freedom, are to prevail, then they must be summoned from the dim and confused realm of latent factors and they must burst forth in the full power of self-awareness and self-possession.

If such is the urgency of personal appropriation of rational self-consciousness, the difficulty of achievement should not discourage attempts at making a beginning. If the extent and the complexity of modern knowledge preclude the possibility in our time both of the *uomo universale* of the Renaissance and of the medieval writer of a *Summa*, at least the collaboration of many contains a promise of success, where the unaided individual would have to despair.

Still a collaboration has its conditions. It supposes a common vision of a common goal. It supposes at least a tentative idea that would unify and coordinate separate efforts in different fields. It supposes a central nucleus that somehow could retain its identity yet undergo all the modifications and enrichments that could be poured into its capacious frame from specialized investigations.

It is with the conditions, preliminary to an effective collaboration, that the present work is concerned.

For in the measure that potential collaborators move towards a personal appropriation of their rational self-consciousness, in the same measure they will begin to attain the needed common vision of the common goal. In the measure that they discover in themselves the structure of developing intelligence, in the same measure they will share a tentative idea that can unify and coordinate separate efforts in different fields. In the measure that they reach the invariants of intellectual development, in the same measure they will possess a central nucleus that retains its identity through all the possible developments of human intelligence.

Prof. Cassirer has told us that, from the viewpoint of a phenomenology of human culture, the explanatory definition of man is <u>animal symbolicum</u> rather than <u>animal rationale</u>. But in the measure that men appropriate their rational self-consciousness, not only do they re-establish the <u>animal rationale</u> but also they break through the phenomenological veil. For, as will be argued, they can reach a universal viewpoint from which individual temperament can be discounted, personal evaluations can be criticized, and the many and disparate reports on man, emanating from experts in various fields, can be welded into a single view.

But if I believe that man's self-awareness and self-possession can add a further, overarching component to Prof. Cassirer's portrayal of man, it is not to be over-

looked that a possibility is claimed and not an achievement. I could not convey my meaning without venturing into many fields, into mathematics and physics, into the subtleties of common sense and depth psychology, into the processes of history, the intricacies of interpretation, the dialectic of the philosophies, and the possibility of transcendent knowledge. I would not wish anyone to entertain the fanciful nonsense that I can speak with authority or even competence in so many fields. I do not expect many experts to recognize their science in the formulations that suit my purpose. Yet, perhaps, I may hope that there will be some that share my preoccupations and interests, that will divine what I am endeavoring to say and will proceed to say it more adequately, that will grasp how my ignorance and oversights can be remedied without completely invalidating the fundamental structures that make possible a common vision of a common goal. Finally, if in any measure that hope is realized, the relative isolation of my efforts will have ended and the preliminary conditions will begin to be fulfilled for the collaboration I would merely initiate.

It is customary to conclude a preface with an acknowledgement of one's indebtedness. Naturally I am inclined to think in the first place of the teachers and writers who have left their impress upon me in the course of the twenty-seven years since first I was initiated into epistemological issues. But so long a gestation contains too many half-lights, too many detours, for me to indicate

in a brief yet intelligible fashion my proximate sources. So it is that I must be content to restrict my expression of gratitude to immediate benefactors: to the staff of L'Immaculée Conception, Montreal, where the underlying studies on Gratia Operans[1] and Verbum[2] were undertaken; to the staff of the Jesuit Seminary, Toronto, where I enjoyed the freedom to write the present work; to the Rev. Joseph Wulftange, Joseph Clark, Norris Clarke, Frederick Crowe, Frederick Copleston, and Andrew Godin, who were generously read the typescript and gave me the benefit of reactions and criticisms from specialists in different fields; to the Rev. Patrick Plunkett who labored (to my shame, rather vainly) to reduce the solecisms of my style; and to the Rev. Eric Gallagher O'Connor who was ever ready to allow me to draw upon his knowledge of mathematics and of science.

June 1949 to September 1953.

1) St. Thomas' Thought on Gratia Operans, Theological Studies [Woodstock, Md.] II(1941), 289-324; III(1942), 69-88, 375-402, 533-578.

2) The Concept of Verbum in the Writings of St. Thomas Aquinas, Ibid. VII(1946), 349-392; VIII(1947), 35-79, 404-444; X(1949), 3-40, 359-393.

a sequence of lower contexts for the purpose of reaching an upper context; and the basic upper context is to be pre-logical, not in the sense made current by M. Lévy-Bruhl, but in the sense that developing intelligence and reasonableness are prior to intelligently grasped and reasonably affirmed utterances. Still it may not be amiss to indicate a single instance in which the genetic order of developing insights differs from the logical order of defining thought. Thus, logically it is illegitimate to speak, for example, of the equality of the spokes of a cart-wheel without explaining that the spokes will be said to be equal if the same number is reached in measuring each of them. In turn, this statement calls for a further statement in which the meaning of the word, measuring, is explained; and that explanation calls for an account of units of measurement, of their standardization, of the numbers employed in measuring, and of the isomorphism of mathematical and physical relations. On the other hand, genetically it seems clear enough that Euclidean geometry existed for some centuries before there occurred any effective advertence to its metrical suppositions. More generally, it seems true that prior to every correct logical formalization there is a sufficiently univocal communication of insights, that this prior communication grounds not only non-technical discourse but also the possibility of discussing the adequacy or inadequacy of any formalization, and that from a pedagogical viewpoint the correct procedure is to begin by communicating the insights.

To turn from logical to metaphysical considera-

INSIGHT

CHAPTER I

ELEMENTS

In the midst of that vast and profound stirring of human minds, which we name the Renaissance, Descartes was convinced that too many people felt it beneath them to direct their efforts to apparently trifling problems. Again and again, in his Regulae ad directionem ingenii, he reverts to this theme. Intellectual mastery of mathematics, of the departments of science, of philosophy, is the fruit of a slow and steady accumulation of little insights. Great problems are solved by being broken down into little problems. The strokes of genius are but the outcome of a continuous habit of inquiry that grasps clearly and distinctly all that is involved in the simple things that anyone can understand.

I thought it well to begin by recalling this conviction of a famous mathematician and philosopher, for our first task will be to attain familiarity with what is meant by insight, and the only way to achieve this end is, it seems, to attend very closely to a series of instances all of which are rather remarkable for their banality.

(35)

1 A Dramatic Instance

1. Our first illustrative instance of insight will be the story of Archimedes rushing naked from the baths of Syracuse with the cryptic cry, "Eureka"! King Hiero, it seems, had had a votive crown fashioned by a smith of rare skill and doubtful honesty. He wished to know whether or not baser metals had been added to the gold. Archimedes was set the problem and in the bath had hit upon the solution. Weigh the crown in water! Implicit in this directive were the principles of displacement and of specific gravity.

With those principles of hydrostatics we are not directly concerned. For our objective is an insight into insight. Archimedes had his insight by thinking about the crown; we shall have ours by thinking about Archimedes. What we have to grasp is that insight 1) comes as a release to the tension of inquiry, 2) comes suddenly and unexpectedly, 3) is a function not of outer circumstances but inner conditions, 4) pivots between the concrete and the abstract, and 5) passes into the habitual texture of one's mind.*

First, then, insight comes as a release to the tension of inquiry. This feature is dramatized in the story by Archimedes' peculiarly uninhibited exultation. But the point I would make does not lie in this outburst of delight but in the antecedent desire and effort that it betrays. For if the typical scientist's satisfaction in success is more sedate, his earnestness in inquiry can still exceed that of Archimedes. Deep within us all, emergent when the noise of other appetites is stilled, there is a drive to

* A profusion of instances of insight is offered by E. D. Hutchinson in three articles originally published in Psychiatry and reprinted in A Study of Interpersonal Relations (edited by P. Mullaly, New York, 1949).

know, to understand, to see why, to discover the reason, to find the cause, to explain. Just what is wanted, has many names. In what precisely it consists, is a matter of dispute. But the fact of inquiry is beyond all doubt. It can absorb a man. It can keep him for hours, day after day, year after year, in the narrow prison of his study or his laboratory. It can send him on dangerous voyages of exploration. It can withdraw him from other interests, other pursuits, other pleasures,other achievements. It can fill his waking thoughts, hide from him the world of ordinary affairs, invade the very fabric of his dreams. It can demand endless sacrifices that are made without regret though there is only the hope, never a certain promise, of success. What better symbol could one find for this obscure, exigent, impetuous drive, than a man, naked, running, excitedly crying, "I've got it".

Secondly, insight comes suddenly and unexpectedly. It did not occur when Archimedes was in the mood and posture that a sculptor would select to portray "The Thinker". It came in a flash, on a trivial occasion, in a moment of relaxation. Once more there is dramatized a universal aspect of insight. For it is reached, in the last analysis, not by learning rules, not by following precepts, not by studying any methodology. Discovery is a new beginning. It is the origin of new rules that supplement or even supplant the old. Genius is creative. It is genius precisely because it disregards established routines, because it originates the novelties that will be the routines of the future. Were there rules for discovery, then discoveries

would be mere conclusions. Were there precepts for genius, then men of genius would be hacks. Indeed, what is true of discovery, also holds for the transmission of discoveries by teaching. For a teacher cannot undertake to make a pupil understand. All he can do is present the sensible elements in the issue in a suggestive order and with a proper distribution of emphasis. It is up to the pupils themselves to reach understanding, and they do so in varying measures of ease and rapidity. Some get the point before the teacher can finish his exposition. Others just manage to keep pace with him. Others see the light only when they go over the matter by themselves. Some finally never catch on at all; for a while they follow the classes but, sooner or later, they drop by the way.

Thirdly, insight is a function, not of outer circumstances, but of inner conditions. Many frequented the baths of Syracuse without coming to grasp the principles of hydrostatics. But who bathed there without feeling the water, or without finding it hot or cold or tepid? There is, then, a strange difference between insight and sensation. Unless one is deaf, one cannot avoid hearing. Unless one is blind, one has only to open one's eyes to see. The occurrence and the content of sensation stand in some immediate correlation with outer circumstance. But with insight, internal conditions are paramount. Thus, insight depends upon native endowment and so, with fair accuracy, one can say that insight is the act that occurs frequently in the intelligent and rarely in the stupid. Again, insight depends upon a habitual orientation, upon a perpetual alertness

ever asking the little question,"Why?". Finally, insight depends on the accurate presentation of definite problems. Had Hiero not put his problem to Archimedes, had Archimedes not thought earnestly, perhaps desperately, upon it, the baths of Syracuse would have been no more famous than any others.

Fourthly, insight pivots between the concrete and the abstract. Archimedes' problem was concrete. He had to settle whether a particular crown was made of pure gold. Archimedes' solution was concrete. It was to weigh the crown in water. Yet if we ask what was the point to that procedure we have to have recourse to the abstract formulations of the principles of displacement and of specific gravity. Without that point, weighing the crown in water would be mere eccentricity. Once the point is grasped, King Hiero and his golden crown become minor historical details of no scientific importance. Once more the story dramatizes a universal aspect of insight. For if insights arise from concrete problems, if they reveal their value in concrete applications, none the less they possess a significance greater than their origins and a relevance wider than their original applications. Because insights arise with reference to the concrete ~~teachers use diagrams~~, mathematicians invent symbols, teachers need black-boards, pupils have to perform experiments for themselves, doctors have to see their patients, trouble-shooters have to travel to the spot, people with a mechanical bent take things apart to see how they work. But because the significance and relevance of insight goes beyond any concrete problem or application, men formulate ab-

stract sciences with their numbers and symbols, their technical terms and formulae, their definitions, postulates, and deductions. Thus, by its very nature, insight is the mediator, the hinge, the pivot. It is insight into the concrete world of sense and imagination. Yet what is known by insight, what insight adds to sensible and imagined presentations, finds its adequate expression only in the abstract and recondite formulations of the sciences.

Fifthly, insight passes into the habitual texture of one's mind. Before Archimedes could solve his problem, he needed an instant of inspiration. But he needed no further inspiration when he went to offer the king his solution Once one has understood, one has crossed a divide. What a moment ago was an insoluble problem, now becomes incredibly simple and obvious. Moreover, it tends to remain simple and obvious. However laborious the first occurrence of an insight may be, subsequent repetitions occur almost at will. This, too, is a universal characteristic of insight and, indeed, it constitutes the possibility of learning. For we can learn inasmuch as we can add insight to insight, inasmuch as the new does not extrude the old but complements and combines with it. Inversely, inasmuch as the subject to be learnt involves the acquisition of a whole series of insights, the process of learning is marked by an initial period of darkness in which one gropes about insecurely, in which one cannot see where one is going, in which one cannot grasp what all the fuss is about; and only gradually, as one begins to catch on, does the initial darkness yield to a subsequent period of increasing light, confidence,

interest, absorption. Then, the infinitesimal calculus or theoretical physics or the issues of philosophy cease to be the mysterious and foggy realms they had seemed. Imperceptibly we shift from the helpless infancy of the beginner to the modest self-confidence of the advanced student. Eventually we become capable of taking over the teacher's role and complaining of the remarkable obtuseness of pupils that fail to see what, of course, is perfectly simple and obvious to those that understand.

2. Definition. As every school#boy knows, a circle is a locus of coplanar points equidistant from a center. What every school#boy does not know is the difference between repeating that definition, as a parrot might, and uttering it intelligently. So, with a sidelong bow to Descartes' insistence on the importance of understanding very simple things, let us inquire into the genesis of the definition of the circle.

2.1 The Clue. Imagine a cart-wheel with its bulky hub, its stout spokes, its solid rim.

Ask a question. Why is it round?

Limit the question. What is wanted is the immanent ground of the roundness of the wheel. Hence a correct answer will not introduce new data such as carts, carting, transportation, wheelwrights, or their tools. It will refer simply to the wheel.

Consider a suggestion. The wheel is round because its spokes are equal. Clearly, that will not do. The spokes could be equal yet sink unequally into the hub and rim. Again, the rim could be flat between successive spokes.

Still, we have a clue. Let the hub decrease to a point: let the rim and spokes thin out into lines: then, if there were an infinity of spokes and all were exactly equal, the rim would have to be perfectly round; inversely, were any of the spokes unequal, the rim could not avoid bumps or dents. Hence, we can say that the wheel necessarily is round, inasmuch as the distance from the center of the hub to the outside of the rim is always the same.

A number of observations are now in order. The foregoing brings us close enough to the definition of the circle. But our purpose is to attain insight, not into the circle, but into the act illustrated by insight into the circle.

The first observation, then, is that points and lines cannot be imagined. One can imagine an extremely small dot. But no matter how small a dot may be, still it has magnitude. To reach a point, all magnitude must vanish, and with all magnitude there vanishes the dot as well. One can imagine an extremely fine thread. But no matter how fine a thread may be, still it has breadth and depth as well as length. Remove from the image all breadth and depth, and there vanishes all length as well.

2.2 The Concepts The second observation is that points and lines are concepts.

Just as imagination is the playground of our desires and our fears, so conception is the play ground of our intelligence. Just as imagination can create objects never seen or heard or felt, so too conception can create

objects that cannot even be imagined. How? By supposing. The imagined dot has magnitude as well as position, but the geometer says, Let us suppose it has only position. The imagined line has breadth as well as length, but the geometer says, Let us suppose it has only length.

Still, there is method in this madness. Our images and especially our dreams seem very random affairs, yet psychologists offer to explain them. Similarly, the suppositions underlying concepts may appear very fanciful, yet they too can be explained. Why did we require the hub to decrease to a point and the spokes and rim to mere lines? Because we had a clue - the equality of the spokes - and we were pushing it for all it was worth. As long as the hub had any magnitude, the spokes could sink into it unequally. As long as the spokes had any thickness, the wheel could be flat at their ends. So we supposed a point without magnitude, and lines without thickness to obtain a curve that would be perfectly, necessarily round.

Note, then, two properties of concepts. In the first place, they are constituted by the mere activity of supposing, thinking, considering, formulating, defining. They may or may not be more than that. But if they are more, then they are not merely concepts. And if they are no more than supposed or considered or thought about, still that is enough to constitute them as concepts. In the second place, concepts do not occur at random: they emerge in thinking, supposing, considering, defining, formulating: and that many named activity occurs, not at random, but in conjunction

with an act of insight.

2.3 The Image The third observation is that the image is necessary for the insight.

Points and lines cannot be imagined. But neither can necessity or impossibility be imagined. Yet in approaching the definition of the circle, there occurred some apprehension of necessity and of impossibility. As we remarked, if all the radii are equal, the curve must be perfectly round; and if any radii are unequal, the curve cannot avoid bumps or dents.

Further, the necessity in question was not necessity in general but a necessity of roundness resulting from these equal radii. Similarly, the impossibility in question was not impossibility in the abstract but an impossibility of roundness resulting from these unequal radii. Eliminate the image of the center, the radii, the curve, and by the same stroke there vanishes all grasp of necessary or of impossible roundness.

But it is that grasp that constitutes the insight. It is the occurrence of that grasp that makes the difference between repeating the definition of a circle, as a parrot might, and uttering it intelligently, uttering it with the ability to make up a new definition for oneself.

It follows that the image is necessary for the insight. Inversely, it follows that the insight is the act of catching on to a connection between imagined equal radii and, on the other hand, a curve that is bound to look perfectly round.

The Question

2.4 The fourth observation adverts to the question. There is the question as expressed in words, Why is the wheel round?

Behind the words there may be conceptual acts of meaning, such as "wheel", "round", etc.

Behind these concepts there may be insights in which one grasps how to use such words as "wheel", "round", etc.

But what we are trying to get at, is something different. Where does the "Why?" come from? What does it reveal or represent? Already we had occasion to speak of the psychological tension that had its release in the joy of discovery. It is that tension, that drive, that desire to understand, that constitutes the primordial "Why?". Name it what you please, alertness of mind, intellectual curiosity, the spirit of inquiry, active intelligence, the drive to know. Under any name, it remains the same and is, I trust, very familiar to you.

This primordial drive, then, is the pure question. It is prior to any insights, any concepts, any words, for insights, concepts, words, have to do with answers; and before we look for answers, we want them; such wanting is the pure question.

On the other hand, though the pure question is prior to insights, concepts, and words, it presupposes experiences and images. Just as insight is into the concretely given or imagined, so the pure question is about the concretely given or imagined. It is the wonder, which Aristotle

claimed to be the beginning of all science and philosophy. But no one just wonders. We wonder about something.

2.5 1 Genesis A fifth observation distinguishes moments in the genesis of a definition.

When an animal has nothing to do, it goes to sleep. When a man has nothing to do, he may ask questions. The first moment is an awakening to one's intelligence. It is release from the dominance of biological drive and from the routines of everyday living. It is the effective emergence of wonder, of the desire to understand.

The second moment is the hint, the suggestion, the clue. Insight has begun. We have got hold of something. There is a chance that we are on the right track. Let's see.

The third moment is the process. Imagination has been released from other cares. It is free to cooperate with intellectual effort, and its cooperation consists in endeavoring to run parallel to intelligent suppositions while at the same time, restraining supposition within some limits of approximation to the imaginable field.

The fourth moment is achievement. By their cooperation, by successive adjustments, question and insight, image and concept, present a solid front. The answer is a patterned set of concepts. The image strains to approximate to the concepts. The concepts, by added conceptual determinations, can express their differences from the merely approximate image. The pivot between images and concepts is the insight. And setting the standard which insight, images, and concepts must meet is the question, the desire to know, that could have kept the process in motion

by further queries, had its requirements not been satisfied.

9.6 Nominal and Explanatory Definition A sixth observation distinguishes different kinds of definition. As Euclid defined a straight line as a line lying evenly between its extremes, so he might have defined a circle as a perfectly round plane curve. As the former definition, so also the latter would serve to determine unequivocally the proper use of the names, straight line, circle. But, in fact, Euclid's definition of the circle does more than reveal the proper use of the name, circle. It includes the affirmation that in any circle, all radii are exactly equal: and were that affirmation not included in the definition, then it would have had to be added as a postulate.

To view the same matter from another angle, Euclid did postulate that all right angles are equal. Let us name the sum of two adjacent right angles a straight angle. Then, if all right angles are equal, necessarily all straight angles will be equal. Inversely, if all straight angles are equal, all right angles must be equal. Now if straight lines are really straight, if they never bend in any direction, must not all straight angles be equal? Could not the postulate of the equality of straight angles be included in the definition of the straight line, as the postulate of the equality of radii is included in the definition of the circle?

At any rate, there is a difference between nominal and explanatory definitions. Nominal definitions merely tell us about the correct usage of names. Explanatory

definitions also include something further that, were it not included in the definition, would have to be added as a postulate.

What constitutes the difference? It is not that explanatory definitions suppose an insight while nominal definitions do not. For a language is an enormously complicated tool with an almost endless variety of parts that admit a far greater number of significant combinations. If insight is needed to see how other tools are to be used properly and effectively, insight is similarly needed to use a language properly and effectively.

Still, this yields, I think, the answer to our question. Both nominal and explanatory definitions suppose insights. But a nominal definition supposes no more than an insight into the proper use of language. An explanatory definition, on the other hand, supposes a further insight into the objects to which language refers. The name, circle, is defined as a perfectly round plane curve, as the name, straight line, is defined as a line lying evenly between its extremes. But when one goes on to affirm that all radii in a circle are equal or that all right angles are equal, one no longer is talking merely of names. One is making assertions about the objects which names denote.

2.7 Primitive Terms A seventh observation adds a note on the old puzzle of primitive terms.

Every definition presupposes other terms. If these can be defined, their definitions will presuppose still other terms. But one cannot regress to infinity. Hence, either definition is based on undefined terms or

else terms are defined in a circle so that each virtually defines itself.

Fortunately, we are under no necessity of accepting the argument's supposition. Definitions do not occur in a private vacuum of their own. They emerge in solidarity with experiences, images, questions, and insights. It is true enough that every definition involves several terms, but it is also true that no insight can be expressed by a single term, and it is not true that every insight presupposes previous insights.

Let us say, then, that for every basic insight there is a circle of terms and relations, such that the terms fix the relations, the relations fix the terms, and the insight fixes both. If one grasps the necessary and sufficient conditions for the perfect roundness of this imagined plane curve, then one grasps not only the circle but also the point, the line, the circumference, the radii, the plane, and equality. All the concepts tumble out togethe because all are needed to express adequately a single insigh All are coherent, for coherence basically means that all hang together from a single insight.

Again, there can be a set of basic insights. Such is the set underlying Euclidean geometry. Because the set of insights is coherent, they generate a set of coherent definitions. Because different objects of definition are composed of similar elements, such terms as point, line, surface, angle, keep recurring in distinct definitions. Thus, Euclid begins his exposition from a set of images, a set of insights, and a set of definitions; some of his defin tions are merely nominal; some are explanatory; some are

derived, partly from nominally and partly from explanatorily defined terms.

2.8 Implicit Definition. A final observation introduces the notion of implicit definition.

D. Hilbert has worked out Foundations of Geometry that satisfy contemporary logicians. One of his important devices is known as implicit definition. Thus, the meaning of both point and straight line is fixed by the relation that two and only two points determine a straight line.

In terms of the foregoing analysis, one may say that implicit definition consists in explanatory definition without nominal definition. It consists in explanatory definition, for the relation that two points determine a straight line is a postulational element such as the equality of all radii in a circle. It omits nominal definition, for one cannot restrict ~~the meaning of~~ Hilbert's point to the Euclidean meaning of position without magnitude. An ordered pair of numbers satisfies Hilbert's implicit definition of a point, for two such pairs determine a straight line. Similarly, a first degree equation satisfies Hilbert's implicit definition of a straight line, for such an equation is determined by two ordered pairs of numbers.

The significance of implicit definition is its complete generality. The omission of nominal definitions is the omission of a restriction to the objects which, in the first instance, one happens to be thinking about. The exclusive use of explanatory or postulational elements concentrates attention upon the set of relationships in which

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the whole of scientific significance is contained.

3.0, Higher Viewpoints The next significant step to be taken in working out the nature of insight is to analyze development. Single insights occur either in isolation or in related fields. In the latter case, they combine, cluster, coalesce, into the mastery of a subject: they ground sets of definitions, postulates, deductions: they admit applications to enormous ranges of instances. But the matter does not end there. Still further insights arise. The shortcomings of the previous position become recognized. New definitions and postulates are devised. A new and larger field of deductions is set up. Broader and more accurate applications become possible. Such a complex shift in the whole structure of insights, definitions, postulates, and deductions, and applications may be referred to very briefly as the emergence of a higher viewpoint. Our question is, Just what happens?

Taking our clue from Descartes' insistence on understanding simple things, we select as our pilot instance the transition from arithmetic to elementary algebra. Moreover, to guard against possible misinterpretations, let us say that by arithmetic is meant a subject studied in grade school and that by elementary algebra is meant a subject studied in high school.

3.1, Positive Integers. A first step is to offer some definition of the positive integers, 1, 2, 3, 4, ...

Let us suppose an indefinite multitude of instances of "one". They may be anything anyone pleases, from sheep to instances of the act of counting or ordering.

Further, let us suppose as too familiar to be defined, the notions of "one", "plus", and "equals".

Then, there is an infinite series of definitions for the infinite series of positive integers, and it may be indicated symbolically by the following:

1 + 1 = 2
2 + 1 = 3
3 + 1 = 4
etc., etc., etc.....

This symbolic indication may be interpreted in any of a variety of manners. It means one plus one equals two, or two is one more than one, or the second is the next after the first, or even the relations between classes of groups each with one, or two, or three, etc., members. As the acute reader will see, the one important element in the above series of definitions, is the etc., etc., etc.,... Without it, the positive integers cannot be defined; for they are an indefinitely great multitude; and it is only in so far as some such gesture as etc., etc., etc., is really significant, that an infinite series of definitions can occur. What, then, does the etc., etc., mean? It means that an insight should have occurred. If one has had the relevant insight, if one has caught on, if one sees how the defining can go on indefinitely, no more need be said. If one has not caught on, then the poor teacher has to labor in his apostolate of the obvious. For in defining the positive integers there is no alternative to insight.

Incidentally, it may not be amiss to recall what already has been remarked, namely, that a single insight is expressed in many concepts. In the present instance, a

single insight grounds an infinity of concepts.

Addition Tables.

3.21 A second step will consist in making somewhat more precise the familiar notion of equality. Let us say that when equals are added to equals, the results are equal; that one is equal to one; and that therefore, an infinite series of addition tables can constructed.

The table for adding 2 is constructed by adding one to each side of the equations that define the positive integers. Thus,

From the table	$2 + 1 = 3$
Adding 1	$2 + 1 + 1 = 3 + 1$
Hence, from the table	$2 + 2 = 4$

In like manner the whole table for adding 2 can be constructed. From this table, once it is constructed, there can be constructed a table for adding 3. From that table it will be possible to construct a table for adding 4. etc., etc., etc., which again means that an insight should have occurred.

Thus, from the definitions of the positive integers and the postulate about adding equals to equals, there follows an indefinitely great deductive expansion.

The Homogeneous Expansion

3.3 A third step will be to venture into a homogeneous expansion. The familiar notion of addition is to be complemented by such further notions as multiplication, powers, subtraction, division, and roots. This development, however, is to be homogeneous and by that is meant that no change is to be involved in the notions already employed.

Thus, multiplication is to mean adding a number to itself so many times, so that five by three will

mean the addition of three five's. Similarly, powers are to mean that a number is multiplied by itself so many times, so that five to the third will mean five multiplied by five with the result multiplied again by five. On the other hand, subtraction, division, and roots will mean the inverse operations that bring one back to the starting-point.

By a few insights, that need not be indicated, it will be seen that tables for multiplication and for powers can be constructed from the addition tables. Similarly, tables for subtraction, division, and roots can be constructed from the tables for addition, multiplication and powers.

The homogeneous expansion constitutes a vast extension of the initial deductive expansion. It consists in introducing new operations. Its characteristic is that the new operations involve no modification of the old.

The Need of a Higher Viewpoint

3.4 A fourth step will be the discovery of the need of a higher viewpoint. This arises when the inverse operations are allowed full generality, when they are not restricted to bringing one back to one's starting-point. Then, subtraction reveals the possibility of negative numbers, division reveals the possibility of fractions, roots reveal the possibility of surds. Further, there arise questions about the meaning of operations. What is multiplication when one multiplies negative numbers or fractions or surds? What is subtraction when one subtracts a negative number? etc., etc., etc.) Indeed, even the meaning of "one" and of "equals" becomes confused, for there are recurring

decimals and it can be shown that point nine recurring is equal to one.

Let	X	=	$0.\overline{9}$
then	10X	=	$9.\overline{9}$
hence	9X	=	9
and so	X	=	1

Formulation of the Higher Viewpoint

3.5, A fifth step will be to formulate a higher viewpoint. Distinguish 1) rules, 2) operations, and 3) numbers. Let numbers be defined implicitly by operations, so that the result of any operation will be a number and any number can be the result of an operation.

Let operations be defined implicitly by rules, so that what is done in accord with rules is an operation.

The trick will be to obtain the rules that fix the operations which fix the numbers.

The emergence of the higher viewpoint is the performance of this trick. It consists in an insight that 1) arises upon the operations performed according to the old rules and 2) is expressed in the formulation of the new rules.

Let me explain. From the image of a cart-wheel we proceeded by insight to the definition of the circle. But, while the cart-wheel was imagined, the circle consists of $\overset{a}{\underset{\wedge}{\text{point}}}$ and $\overset{a}{\underset{\wedge}{\text{line}}}$ neither of which can be imagined. Between the cart-wheel and the circle there is an approximation but only an approximation. Now, the transition from arithmetic to elementary algebra is the same sort of thing. For an image of the cart-wheel one substitutes the image of

what may be named "doing arithmetic"; it is a large, dynamic, virtual image that includes writing down, adding, multiplying, subtracting, and dividing numbers in accord with the precepts of the homogeneous expansion. Not all of this image will be present at once, but any part of it can be present and, when one is on the alert, any part that happens to be relevant will pop into view. In this large and virtual image, then, there is to be grasped a new set of rules governing operations. The new rules will not be exactly the same as the old rules. They will be more symmetrical. They will be more exact. They will be more general. In brief, they will differ from the old much as the highly exact and symmetrical circle differs from the cart-wheel.

What are the new rules? In high school the rules for fractions were generalized; rules for signs were introduced; rules for equations and for indices were worked out. Their effect was to redefine the notions of addition, multiplication, powers, subtraction, division, and roots; and the effect of the redefinitions of the operations was that numbers were generated, not merely by addition, but by any of the operations.

Successive Higher Viewpoints

3.61 The reader familiar with group theory will be aware that the definition of operations by rules on numbers or, more generally, symbols by operations is a procedure that penetrates deeply into the nature of mathematics. But there is a further aspect to the matter, and it has to do with the gradual development by which one advances through intermediate stages from elementary to higher mathematics.

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The logical analyst can leap from the positive integers to group theory, but one cannot learn mathematics in that simple fashion. On the contrary, one has to perform, over and over, the same type of transition as occurs in advancing from arithmetic to elementary algebra.

At each stage of the process there exists a set of rules that govern operations which result in numbers. To each stage there corresponds a symbolic image of doing arithmetic, doing algebra, doing calculus. In each successive image there is the potentiality of grasping by insight a higher set of rules that will govern the operations and by them elicit the numbers or symbols of the next stage. Only in so far as a man makes his slow progress up that [illegible] escalator does he become a technically competent mathematician. Without it, he may acquire a rough idea of what mathematics is about: but he will never be a master, perfectly aware of the precise meaning and the exact implications of every symbol and operation.

The Significance of Symbolism.

3.7 The analysis also reveals the importance of an apt symbolism.

There is no doubt that, though symbols are signs chosen by convention, still some choices are highly fruitful while others are not. It is easy enough to take the square root of 1764. It is another matter to take the square root of MDCCLXIV. The development of the calculus is easily designated in using Leibniz' symbol dy/dx, for the differential coefficient; Newton's symbol, on the other hand, can be used only in a few cases and, what is worse, it does not

suggest the theorems that can be established.

Why is this so? It is because mathematical operations are not merely the logical expansion of conceptual premises. Image and question, insight and concepts, all combine. The function of the symbolism is to supply the relevant image, and the symbolism is apt inasmuch as its immanent patterns as well as the dynamic patterns of its manipulation run parallel to the rules and operations that have been grasped by insight and formulated in concepts.

The benefits of this parallelism are manifold. In the first place, the symbolism itself takes over a notable part of the solution of problems, for the symbols, complemented by habits that have become automatic, dictate what has to be done. Thus, a mathematician will work at a problem up to a point and then announce that the rest is mere routine. In the second place, the symbolism constitutes a heuristic technique; the mathematician is not content to seek his unknowns; he names them; he assigns them symbols; he writes down in equations all their properties; he knows how many equations he will need; and when he has reached that number, he can say that ~~the rest of the problem is just routine~~ now the solution is automatic. In the third place, the symbolism offers clues, hints, suggestions. Just as the definition of the circle was approached from the clue of the equality of the spokes, so generally insights do not come to us in their full stature; we begin from little hints, from suspicions, from possibilities; we try them out: if they lead nowhere, we drop them; if they promise success, we push them for all they are worth. But this can be done only if we chance upon the hints, the

clues, the possibilities and the effect of the apt symbolism is to remove, if not entirely eliminate, this element of chance. Here, of course, the classical example is analytic geometry. To solve a problem by Euclidean methods, one has to stumble upon the correct construction. To solve a problem analytically, one has only to manipulate the symbols.

In the fourth place, there is the highly significant notion of invariances. An apt symbolism will endow the pattern of a mathematical expression with the totality of its meaning. Whether or not one uses the Latin, Greek, or Hebrew alphabet, is a matter of no importance. The mathematical meaning of an expression resides in the distinction between constants and variables and in the signs or collocations that dictate operations of combining, multiplying, summing, differentiating, integrating, and so forth. It follows that, as long as the symbolic pattern of a mathematical expression is unchanged, its mathematical meaning is unchanged. Further, it follows that if a symbolic pattern is unchanged by any substitutions of a determinate group, then the mathematical meaning of the pattern is independent of the meaning of the substitutions.

In the fifth place, as has already been mentioned, the symbolism appropriate to any stage of mathematical development, provides the image in which may be grasped by insight the rules for the next stage.

4.7 Inverse Insight

So far we have been asking questions that can be answered, now can one tell whether a crown is made

of pure gold without melting it down? What accounts for a wheel being round? What is arithmetic and how does one go on to algebra? In each case, there is an appropriate image or set of images that, under the stress of inquiry, results in an insight that expresses itself in some formulation called the answer.

So attention has to be directed to a quite different case. There is the question. There is the answer. But the answer consists in showing the question to be misconceived, and it is grounded in an insight that grasps why the question, as conceived, cannot be answered.

4.1 Surds How big is the square root of two? Clearly, it is greater than one, for the square ~~root~~ of one is one; and it is less than two, for the square of two is four. It would seem, then, that it is some improper fraction lying between one and two.

Now an improper fraction is the quotient of some positive integer divided by some other, smaller positive integer. Moreover, it is always possible to reduce such a fraction to its lowest terms by removing all common factors. Let us suppose then, that:

$$\sqrt{2} \quad = \quad M/N$$

where M and N are positive integers with no common factors. Multiplying across by N and squaring, one obtains:

$$2N^2 \quad = \quad M^2$$

It follows that M must be an even number and so twice, say, P.

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Substituting and dividing by two, one obtains:

$$N^2 = 2P^2$$

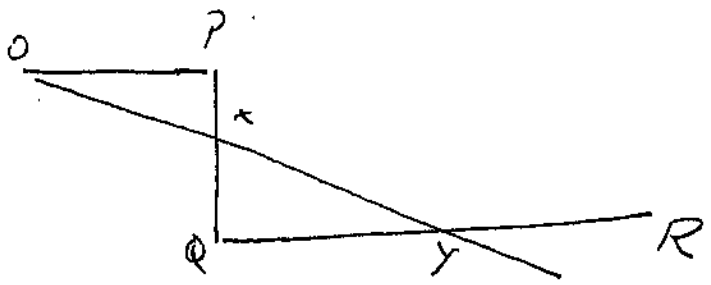
so that N also must be an even number, which contradicts the assumption that all common factors were eliminated. It follows that there is no "rational" fraction, M/N, that is equal to the square root of two. Moreover, since any recurring decimal can be reduced to such a fraction, there is no recurring decimal equal to the square root of two. However, one can apply to 2 the ordinary method for taking the square root, and so it remains that the square root of two will be an infinite, non-recurring decimal. Finally, the foregoing argument can be generalized and applied to any surd. Thus, if

$$3N^2 = M^2$$

then 3 must be a factor of M, so that M can be replaced by 3P, whence, it will follow that 3 must be a factor of N.

4.2 Non-countable Infinity

Again, to raise another, similar question. How many points are there in a straight line one inch long? Clearly, the number must be very large, for a point is position without magnitude. But, at least, one would be inclined to say that there are twice as many points in a straight line two inches long as in a straight line one inch long. Still, that would be erroneous, as appears from the following construction. Let the straight line, PQ, be perpendicular to the straight lines, OP and QR. Let QR be twice as long as PQ. And let OXY be a straight line cutting PQ in X and QR in Y.



Then, from the construction, it is clear that for every point, Y, in QR, there is a corresponding, distinct point, X, in PQ. Indeed, this remains true if one produces QR to infinity in the direction of R. No matter where Y is taken on QR produced, there is always a corresponding and distinct point X in PQ. Hence, there are as many points in an inch of straight line as there are in two inches, or in a foot, or in a mile, or in as many light-years as you please.

However, we have not met the question. We have said there are as many as ..... We have not said how many. Accordingly, let us distinguish between the counted, the countable, and the non-countable. A set is counted when one says it contains N members, where N is some positive integer. A set is countable when it can be arranged in some determinate order that contains all its members once each and only once; for then there can be established a one-to-one correspondence between the members of the set and the positive integers. Finally, a set is non-countable when it is not possible to establish a one-to-one correspondence between its members and the positive integers.

It is to be noted that by "countable" is not meant the possibility of finishing the counting. Thus, an infinite series, such as

1/2, 1/4, 1/8, 1/16, ...

is countable, for all its members lie in a determinate order and so can be placed in a one-to-one correspondence with the positive integers. Again, an infinite series of infinite series of elements is countable, for all its elements can be regarded as lying within a single determinate order. Thus, the reciprocals of the nth powers of the prime numbers form an infinite series of infinite series. Their elements can be arranged in a column of rows, thus:

1/2	1/4	1/8	1/16	.....
1/3	1/9	1/27	1/81	.....
1/5	1/25	1/125	1/625	.....
1/7	1/49	1/343	1/2401	.....

and any column of rows can be counted in the following manner:

1	2	5	10	17
4	3	6	11	etc....
9	8	7	12	etc....
16	15	14	13	etc....

Thus, any infinite series of infinite series can be assigned the order of a single infinite series. It follows that an infinite series of infinite series of infinite series can be arranged in a column of rows and so can be assigned the order of a single infinite series. The theorem can be repeated indefinitely. Thus, consider the rational, proper fractions:

1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5, 1/6, ...

From this infinite series there can be derived an infinite series of infinite series, for one can take, first, the

square root of the lot, then, the cube root, then square the cube roots, then take the fourth root, then cube the fourth roots, etc...Now, as has been shown, this infinite series of infinite series can be arranged in a single series. Once this is done, one can use these new terms as powers to be applied to the rational proper fractions to derive a new infinite series of infinite series. This can be arranged in a single series, applied as powers to the rational proper fractions, yield a new infinite series of infinite series, etc., etc.

From the foregoing it is clear that any infinite set is countable, provided it is possible to assign some order to its members. It is also clear that a non-countable infinite set must contain such a multitude of members in such a manner that ordering them is impossible. Such is the case with the points in a straight line. Thus, in the line, QR, it is impossible to pick any point, q', that is nearest to Q: for however short Qq' may be, it contains as many points as there are in a line as long as you please. Nor is there any use trying to proceed by dividing the line. For if this could be done in an orderly fashion, then one would be appealing to an ordered series of all the numbers greater than zero and less than unity. But the range of numbers is a non-countable infinite set, for it cannot be arranged in a single order. Suppose there were some single column containing all the infinite decimals. Then consider the diagonal. It is always possible to construct another infinite decimal that differs from the first infinite decimal by the digit in the first place, from the second by the digit in the second

place, from the nth by the digit in the nth place. Therefore, the initial assumption is false. The column did not contain all the infinite decimals. There is, then, no single series that contains all the infinite decimals and so the infinite decimals are a non-countable infinite set.

Well, how many points are there in a straight line an inch long? There is no answer. They form a non-countable infinite set. They do so, because they cannot be placed in a single order and so cannot be correlated in a one-to-one correspondence with the positive integers. However, they can be placed in a one-to-one correspondence with other non-countable infinite sets. Thus, there are as many points in an inch as in a mile or in a light-year or in as many light-years as you please. But that does not mean that there is some determinate number of points in an inch or in a mile. Much less does it mean that some smaller number is equal to a greater number. There just is no numbering, no counting. And there is no numbering or counting because there is no possibility of effecting an order, a system, an arrangement.

4.3 Function and Limit One might think that this exclusion of number and of order blocked the mathematician. In fact, it gives him a new lease of life. What is the mathematician's continuous function? In the elementary case, it is a one-to-one correspondence between non-countable infinite sets. Moreover, since such a correspondence can be set up between an inch and a foot, or an inch and a mile, or an inch and a

light-year, or any intermediate or still odder pair, since, visually, length is independent of the number of points, the mathematician can develop the infinitesimal calculus. But he does so, not by finding some order in the non-countable infinite set, but by developing a technique of getting around it. This technique is named proceeding to the limit.

Thus, consider the continuous function, $y = x^2$. ~~It is a function if, for every value of x, there is one corresponding value of y. It is continuous if, if the values of x are considered infinitely close, then every distinct value of x there is a distinct value of y.~~

(once x = 1 2)

Or, as x increases, y must increase very rapidly, for it equals the square of x. Hence, visually, as one moves from point to point along x, one must move very rapidly from point to point along y. Moreover, the farther one advances along x, the greater must be one's strides along y. Still, there are no points omitted along x and there are no points omitted along y.

What, then, is the ratio of the increment of y to the increment of x ? Clearly, if x increases by some slight amount, h, y will increase by

$$(x + h)^2 - x^2 = 2xh + h^2$$

Hence the ratio of the corresponding increment of y to the increment, h, of x will be $(2x + h)$. The smaller the increment, h, the nearer is the ratio to 2x. In the limit, it is exactly 2x. Then, if the limit of the ratio of the increment in y to the increment in x is denoted by the symbol,

(66)

dy/dx, then besides the initial function, $y = x^2$, we have also the derivative function, $dy/dx = 2x$.

Now what is this business of proceeding to the limit? There is said to be a limit, P, to a non-determined quantity, ζ, if the difference of ζ from P can be made smaller than any number one cares to assign. Thus, above, by making the increment, h, smaller and smaller, one can make the difference of $(2x + h)$ from $2x$ as small as we please. Still, this is only the conceptual formulation of the procedure of taking a limit. What is the underlying insight? What is the image that the insight presupposes?

Clearly enough the image will differ in different cases. Similarly, the insight will be reached in different manners. But the peculiarity of the insight is that it grasps, not that something is to some point, but that something is beside the point. No matter how small h is, there is a non countable infinite set of values between $2x$ and $(2x + h)$. They are non-countable because they defy arrangement, order, system. They exhibit a non-systematic aspect of continuous variables and continuous functions. But what one is trying to do in mathematics, is to reach the systematic. If that is all one wants, one can disregard the non-systematic. One can leap over the non-countable infinity because it is without order if one's aim is to reach just what admits order. Again, the ratio of the increment of y to the increment of x is any of a non-countable infinite set of values. But the limit of that ratio is unique. It can be determined systematically. It pertains to system.

Abstraction

4.4 1 When one comes to think of it, we have been doing this sort of thing all along. The principles of displacement and of specific gravity would not enable Archimedes to determine that there was nothing but pure gold in the crown; they would enable him to say merely that there was extremely little el e. Again, the definition of the circle paid no attention to the size, the weight, the strength, the origin, the materials, the purpose of the cart-wheel: on the contrary, it went off to a realm of the non-imaginable where points have position without magnitude and lines have length without thickness. Finally, the transition from arithmetic to algebra did not consist in paying closer attention to the things one might count by the positive integers; it consisted in deserting the good, common sense notion of adding and in developing a new notion that gave a meaning to adding negative numbers, multiplying fractions, and doing other things that have no *prima facie* meaning.

It is time, then, for us to reflect on certain general aspects of the process from image through insight to conceptions, and so we had best begin a new section.

The Empirical Residue

5.1 The assemblies and conceptions resulting from insight are only and abstract. They abstract from the irrelevant, the insignificant, the negligible, the incidental. They concentrate upon the relevant, the significant, the important, the essential.

But what is the relevent, the significant, the important, the essential? The answer depends immediately

upon the insight, or set of insights, grounding the supposing, considering, thinking, defining, formulating. Ultimately, one will say that the answer depends on which insight, or set of insights, is right. But we are not yet ready to tackle ultimate questions. Accordingly, we have to acknowledge, for the present, that the relevant and the irrelevant, the significant and the insignificant, the important and the negligible, the essential and incidental, vary with one's insight. What at one time, one thinks important, later, in the light of fuller insight, one will think unimportant. Inversely, what one used to think insignificant, now one may think significant; and what makes the difference is the advent of further insight.

Still, even for the present, this relative pronouncement is not the whole story. For if we restrict ourselves to the insights possible in mathematics, physics, chemistry, biology, sensitive psychology, and such sciences, then there are elements or components in sensible data and in images that always are regarded as irrelevant, insignificant, negligible, incidental. Such elements or components may be named the empirical residue. They are given as a matter of fact. But they are always disregarded when one concentrates on whatever one happens to think essential.

On four aspects of this empirical residue, something must now be said. They are 1) the individual, 2) the continuum, 3) place and time, and 4) the actual frequency of events.

Individuality

5.1 1 Individuality pertains to the empirical residue. For whenever we understand anything, we would understand an exactly similar instance in exactly the same fashion. A different understanding would presuppose a difference in the data. It would presuppose the possibility of saying that the previous understanding would do, were it not for this aspect of the object. But, ex hypothesi, there is no aspect in which the second object differs from the first, and so there is no possibility of a different understanding. One may learn something new when one turns to the second object; but one automatically learns it about the first object as well.

Thus, a first motorcar off the assembly line may be understood in terms of certain principles of construction and of operation. A second motorcar, similar in all respects, cannot but be understood in exactly the same fashion.

Nor is the issue changed essentially when one understands instances that are unique. In this case, there is no possibility of apprehending a second object and understanding it in the same manner. But there is the possibility of apprehending the same object a second time; the data in the second apprehension will be similar to those of the first; because the data are similar, the understanding has to be the same. The fact that the similar data are of the same object does not alter the underlying principle that our knowledge is so constituted that similar data have to result in similar insights with the consequence that, what is grasped by insight, is independent of the individuality of

the data.

Thus, if the development of all life on this planet were comprehended in a single evolution, there would be no remainder of life on the planet to be understood in either the same or a different fashion. The object would be unique and unparalleled in our experience. None the less, the understanding would consist in grasping principles and laws in the combinations suitable for mastering the enormous ranges of data, while knowledge of the unique instance would consist in observing the data to be understood.

Again, what is grasped by insight, may be named an idea or form emergent in sensible presentations or imaginative representations. But it is one thing to say that grasp of such an idea or form is knowledge of individuality, and quite another to say that within our experience there is found only one instance in which the idea or form can be grasped. If grasp of the idea or form were knowledge of individuality, then the individual would be known by understanding and it would not pertain to the empirical residue. But the mere fact that in some cases there is but a single, observable instance, in which the idea or form can be grasped, provides no evidence for the intrinsic intelligibility of individuality.

In brief, nothing is explained by saying that it is this instance. Inversely, in so far as we grasp explanations, we know not instances but what may or may not be found in individual instances.

Continuity

5.2 / The continuum pertains to the empirical residue. Let us begin with a definition. A variable, x, will be said to be continuous in the range, $a < x < b$, if the values of x in every part of the range form non-countable infinite sets. Next, a function, f(x), will be said to be continuous in a range if 1) x is continuous in the range and 2) for every distinct value of x there is a corresponding ~~distinct~~ value of the function. Finally, continuous functions possess a number of distinctive properties: hence, through the verification of these distinctive properties, it may be possible to verify the existence of continuous functions and so conclude to the existence of continua.

That departs from ordinary mathematical usage to meet our present purpose.

Now a continuum, in this defined and verifiable sense (which does not suppose a non-countable infinite set of observations) includes what cannot be counted because it cannot be ordered or systematized. By this inclusion of the non-systematic, a continuum clearly pertains to the empirical residue.

Place and Time

5.3 / Place and time pertain to the empirical residue. Space is a continuum of individual positions. Time is a continuum of individual instants. No position is any other. No instant is any other. And of both there are non-countable infinite sets. But the individual and the continuum both pertain to the empirical residue. So also, then, must place and time in their basic aspects.

Hence, when different experimenters, performing the same experiment at different places or times, obtain different results, then no one dreams of explaining the

difference in the results by the differences in the place or by the difference in the time. The appeal always is, not to the place, but to something in the place, and not to the time, but to something at the time.

Indeed, if place or time made any difference, then each place and each time would have its own physics, chemistry, and biology. For if place were relevant, the laws in one place could not be the laws in another. If time were relevant, the laws at one time could not be the laws at another. Further, since places and times are non-countable sets, there would be non-countable sets of different physics, different chemistries, different biologies. Finally, none of the elements of these sets could be ascertained. For one cannot set up a whole physics, or a whole chemistry, or a whole biology, with the observations or experiments made at a point-instant.

However, it is only in their basic aspects that place and time pertain to the empirical residue. A place can be of singular importance, provided that importance rests not on a mere "there" but on a "something there". Such is the importance of the place occupied by a central mass in a gravitational field. Similarly, a time can be of singular importance, provided its importance rests not on a mere "then" but on what happened then. Such is the importance of the initial moment in certain theories of the expanding universe.

Actual Frequency

5.4 Actual frequency pertains to the empirical

residue.

The probability of tossing "heads" is 1/2. But in any series of actual tosses, one does not obtain a regular alternation of "heads" and "tails". Between the probability and the actual frequency, there is a divergence. Moreover, this divergence is random. It cannot be reduced to any law or mitigated by any reasonable expectation. It is non-systematic. It is to be known in each case only by actual observation. It too pertains to an empirical residue.

5.5 The Significance of the Empirical Residue.

Let us now recall our initial restriction. The empirical residue was defined as always irrelevant from the viewpoint of insights in mathematics and the natural sciences. Why was this restriction imposed? Quite clearly, because in such a science as the theory of knowledge the notion of the empirical residue attains a systematic significance. For in a study of knowledge one attends systematically, not only to what is concentrated upon in abstraction, but also to what is regularly abstracted from. Theory of knowledge is a higher level science that takes as its materials the whole of the knowledge in other sciences.

Indeed, the theoretical account of the empirical residue is of considerable significance.

It is because insight abstracts from the individual that science is of the universal. It is because science is of the universal, that the observation of similarities is of such great heuristic importance.

It is because insight abstracts from the continuum

by proceeding to the limit that the infinitesimal calculus is such a unique and powerful instrument in the construction of theories.

It is because insight abstracts from place and time that principles and laws are independent of place and time and that the expression of principles and laws is invariant with respect to transformations of certain groups of coordinate systems.

It is because insight abstracts from the random divergence of the actual frequency that probability theory has its place among the instruments of scientific knowledge

Generally, corresponding to each aspect of the empirical residue, there will be a remarkably powerful technique of intelligence in mastering the multiplicity of sensible data. Unfortunately, the discovery of the techniques has to be prior to the determination of the complementary aspect of the empirical residue. For while all aspects of the empirical residue are given on the level of observation, still one can grasp them as pertaining to the empirical residue only by understanding the corresponding techniques.

of pure gold without melting it down? What accounts for a wheel being round? What is arithmetic and how does one go on to algebra? In each case, there is an appropriate image or set of images that, under the stress of inquiry, results in an insight that expresses itself in some formulation called the answer.

Now attention has to be directed to a quite different case. There is the question. There is the answer. But the answer consists in showing the question to be misconceived, and it is grounded in an insight that grasps why the question, as conceived, cannot be answered.

4.1 Surds. How big is the square root of two? Clearly, it is greater than one, for the square of one is one; and it is less than two, for the square of two is four. It would seem, then, that it is some improper fraction lying between one and two.

Now an improper fraction is the quotient of some positive integer divided by some other, smaller positive integer. Moreover, it is always possible to reduce such a fraction to its lowest terms by removing all common factors. Let us suppose then, that:

$$\sqrt{2} = M/N$$

where M and N are positive integers with no common factors. Multiplying across by N and squaring, one obtains:-

$$2N^2 = M^2$$

It follows that M must be an even number and so twice, say, P.

Substituting and dividing by two, one obtains:

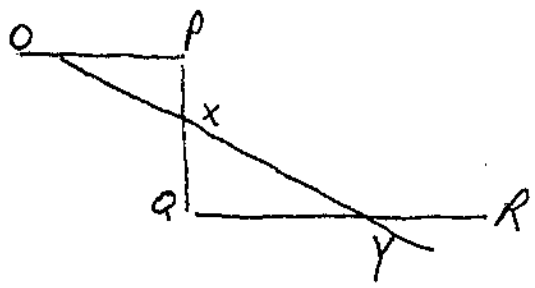
$$N^2 = 2P^2$$

so that N also must be an even number, which contradicts the assumption that all common factors were eliminated. It follows that there is no "rational" fraction, M/N, that is equal to the square root of two. Moreover, since any recurring decimal can be reduced to such a fraction, there is no recurring decimal equal to the square root of two. However, one can apply to 2 the ordinary method for taking the square root, and so it remains that the square root of two will be an infinite, non-recurring decimal. Finally, the foregoing argument can be generalized and applied to any surd. Thus, if

$$3N^2 = M^2$$

then 3 must be a factor of M, so that M can be replaced by 3P, whence, it will follow that 3 must be a factor of N.

4.2 Non-Countable Infinity. Again, to raise another, similar question. How many points are there in a straight line one inch long? Clearly, the number must be very large, for a point is position without magnitude. But, at least, one would be inclined to say that there are twice as many points in a straight line two inches long as in a straight line one inch long. Still, that would be erroneous, as appears from the following construction. Let the straight line, PQ, be perpendicular to the straight lines, OP and QR. Let QR be twice as long as PQ and let OXY be a straight line cutting PQ in X and QR in Y.



Then, from the construction, it is clear that for every point, Y, in QR, there is a corresponding, distinct point, X, in PQ. Indeed, this remains true if one produces QR to infinity in the direction of R. No matter where Y is taken on QR produced, there is always a corresponding and distinct point X in PQ. Hence, there are as many points in an inch of straight line as there are in two inches, or in a foot, or in a mile, or in as many light-years as you please.

However, we have not met the question. We have said there are <u>as many as</u>. . . . . We have not said <u>how many</u>. Accordingly, let us distinguish between the counted, the countable, and the non-countable. A set is counted when one says it contains N members, where N is some positive integer. A set is countable when it can be arranged in some determinate order that contains all its members once each and only once; for then there can be established a one-to-one correspondence between the members of the set and the positive integers. Finally, a set is non-countable when it is not possible to establish a one-to-one correspondence between its members and the positive integers.

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upon the insight, or set of insights, grounding the supposing, considering, thinking, defining, formulating. Ultimately, one will say that the answer depends on which insight, or set of insights, is right. But we are not yet ready to tackle ultimate questions. Accordingly, we have to acknowledge, for the present, that the relevant and the irrelevant, the significant and the insignificant, the important and the negligible, the essential and incidental, vary with one's insights. What at one time, one thinks important, later, in the light of fuller insight, one will think unimportant. Inversely, what one used to think insignificant, now one may think significant; and what makes the difference is the advent of further insight.

Still, even for the present, this relative pronouncement is not the whole story. For if we restrict ourselves to the insights possible in mathematics, physics, chemistry, biology, sensitive psychology, and such sciences, then there are elements or components in sensible data and in images that always are regarded as irrelevant, insignificant, negligible, incidental. Such elements or components may be named the empirical residue. They are given as a matter of fact. But they are always disregarded when one concentrates on whatever one happens to think essential.

On four aspects of this empirical residue, something must now be said. They are 1) the individual, 2) the continuum, 3) place and time, and 4) the actual frequency of events.

5.1 Individuality Individuality pertains to the empirical residue. For whenever we understand anything, we would understand an exactly similer instance in exactly the same fashion. A different understanding would presuppose a difference in the data. It would presuppose the possibility of saying that the previous understanding would do, were it not for this aspect of the object. But, ex hypothesi, there is no aspect in which the second object differs from the first, and so there is no possibility of a different understanding. One may learn something new when one turns to the second object; but one automatically learns it about the first object as well.

Thus, a first motor car off the assembly line may be understood in terms of certain principles of construction and of operation. A second motor car, similar in all respects, cannot but be understood in exactly the same fashion.

Nor is the issue changed essentially when one understands instances that are unique. In this case, there is no possibility of apprehending a second object and understanding it in the same manner. But there is the possibility of apprehending the same object a second time; the data in the second apprehension will be similar to those of the first; because the data are similar, the understanding has to be the same. The fact that the similar data are of the same object does not alter the underlying principle that our knowledge is so constituted that similar data have to result in similar insights with the consequence that, what is grasped by insight, is independent of the individuality of

the data.

Thus, if the development of all life on this planet were comprehended in a single evolution, there would be no remainder of life on the planet to be understood in either the same or a different fashion. The object would be unique and unparalleled in our experience. None the less, the understanding would consist in grasping principles and laws in the combinations suitable for mastering the enormous ranges of data, while knowledge of the unique instance would consist in observing the data to be understood.

Again, what is grasped by insight, may be named an idea or form emergent in sensible presentations or imaginative representations. But it is one thing to say that grasp of such an idea or form is knowledge of individuality, and quite another to say that within our experience there is found only one instance in which the idea or form can be grasped. If grasp of the idea or form were knowledge of individuality, then the individual would be known by understanding and it would not pertain to the empirical residue. But the mere fact that in some cases there is but a single, observable instance, in which the idea or form can be grasped, provides no evidence for the intrinsic intelligibility of individuality.

In brief, nothing is explained by saying that it is this instance. Inversely, in so far as we grasp explanations, we know not instances but what may or may not be found in individual instances.

5.2 Continuity The continuum pertains to the empirical residue. Let us begin with a definition that departs from ordinary mathematical usage to meet our present purpose. A variable, $\underline{x}$ will be said to be continuous in the range, $\underline{a} < \underline{x} < \underline{b}$, if the values of $\underline{x}$ in every part of the range form non-countable infinite sets. Next, a function, f(x), will be said to be continuous in a range if 1) $\underline{x}$ is continuous in the range and 2) for every distinct value of $\underline{x}$ there is a corresponding value of the function. Finally, continuous functions possess a number of distinctive properties; hence, through the verification of these distinctive properties, it may be possible to verify the existence of continuous functions and so conclude to the existence of continua.

Now a continuum, in this defined and verifiable sense (which does not suppose a non-countable infinite set of observations) includes what cannot be counted because it cannot be ordered or systematized. By this inclusion of the non-systematic, a continuum clearly pertains to the empirical residue.

5.3 Place and Time Place and time pertain to the empirical residue. Space is a continuum of individual positions. Time is a continuum of individual instants. No position is any other. No instant is any other. And of both there are non-countable infinite sets. But the individual and the continuum both pertain to the empirical residue. So also, then, must place and time in their basic aspects.

Hence, when different experimenters, performing the same experiment at different places or times, obtain different results, then no one dreams of explaining the

difference in the results by the differences in the place or by the difference in the time. The appeal always is, not to the place, but to something in the place, and not to the time, but to something at the time.

Indeed, if place or time made any difference, then each place and each time would have its own physics, chemistry, and biology. For if place were relevant, the laws in one place could not be the laws in another. If time ~~was~~ were relevant, the laws at one time could not be the laws at another. Further, since places and times are non-countable sets, there would be non-countable sets of different physics, different chemistries, different biologies. Finally, none of the elements of these sets could be ascertained. For one cannot set up a whole physics, or a whole chemistry, or a whole biology, with the observations or experiments made at a point-instant.

However, it is only in their basic aspects that place and time pertain to the empirical residue. A place can be of singular importance, provided that importance rests not on a mere "there" but on a "something there". Such is the importance of the place occupied by a central mass in a gravitational field. Similarly, a time can be of singular importance, provided its importance rests not on a mere "then" but on what happened then. Such is the importance of the initial moment in certain theories of the expanding universe.

5.4 Actual Frequency Actual frequency pertains to the empirical

residue.

The probability of tossing "heads" is 1/2. But in any series of actual tosses, one does not obtain a regular alternation of "heads" and "tails". Between the probability and the actual frequency, there is a divergence. Moreover, this divergence is random. It cannot be reduced to any law or mitigated by any reasonable expectation. It is non-systematic. It is to be known in each case only by actual observation. It too pertains to an empirical residue.

5.5 The Significance of the Empirical Residue

Let us now recall an initial restriction. The empirical residue was defined as always irrelevant from the viewpoint of insights in mathematics and natural sciences. Why was this restriction imposed? Quite clearly, because in such a science as the theory of knowledge the notion of the empirical residue attains a systematic significance. For in a study of knowledge one attends systematically, not only to what is concentrated upon in abstraction, but also to what is regularly abstracted from. Theory of knowledge is a higher level science that takes as its materials the whole of the knowledge in other sciences.

Indeed, the theoretical account of the empirical residue is of considerable significance.

It is because insight abstracts from the individual that science is of the universal. It is because science is of the universal, that the observation of similarities is of such great heuristic importance.

It is because insight abstracts from the continuum

by proceeding to the limit that the infinitesimal calculus is such a unique and powerful instrument in the construction of theories.

It is because insight abstracts from place and time that principles and laws are independent of place and time and that the expression of principles and laws is invariant with respect to transformations of certain groups of coordinate systems.

It is because insight abstracts from the random divergence of the actual frequency that probability theory has its place among the instruments of scientific knowledge.

Generally, corresponding to each aspect of the empirical residue, there will be a remarkably powerful technique of intelligence in mastering the multiplicity of sensible data. Unfortunately, the discovery of the techniques has to be prior to the determination of the complementary aspect of the empirical residue. For while all aspects of the empirical residue are given on the level of observation, still one can grasp them as pertaining to the empirical residue only by understanding the corresponding techniques.

INSIGHT

CHAPTER II

HEURISTIC STRUCTURES

1. Mathematical and Scientific Insights.

So far our illustrations of insight have been drawn from the field of mathematics. We have examined the definition of the circle, the transition from arithmetic to algebra, the distinction between different kinds of infinite sets. It is true that we began from the story of Archimedes' discovery of principles of displacement and specific gravity. But then we were content merely to indicate the more obvious features of insight and made no attempt to analyze the precise nature of the origin and development of scientific knowledge. Such an analysis must now be tackled.

1.1 Similarities. Galileo's determination of the law of falling bodies not only is a model of scientific procedure but also offers the attraction of possessing many notable similarities to the already examined process from the image of the cart-wheel to the definition of the circle.

In the first place, the inquiry was restricted to the immanent intelligibility of a free fall. Just as we ruled out of consideration the purpose of cart-wheels, the materials from which they are made, the wheelwrights that make them, and the tools that wheelwrights use, so also

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What is happening? Consider the algebraic procedure that we are generalizing and observe the isomorphism. Where before we said, Let x be the required number, now we say, Let $f(x, y, z, t) = 0$ be the required function. Where before we noted that, while the minute hand moves over x minutes, the hour hand moves over x/12 minutes, now we work out a differential equation that expresses mathematically certain very general features of the data, such as continuity, indestructibility, incompressibility, homogeneity, and so forth. Where before we appealed to the fact that at three o'clock the hour hand had a fifteen minute start on the minute hand, now we turn our attention to boundary conditions that restrict the range of functions satisfying the differential equation.

Restricted Invariance

2.5 1 Place and time, no less than individuality and continuity, pertain to the empirical residue. It follows that the function to be determined will hold independently of particular places and times for, as has been seen, particular places and times are, in their basic aspect, continua of individual differences.

Thus, Newton's first law of motion is to the effect that a body continues in its state of rest or of uniform motion as long as no external force intervenes. This law might be regarded as a positive correlation between zero acceleration and zero force. But directly it regards constant velocities and its contention is that such velocities pertain to the empirical residue, If there is an acceleration, mechanical analysis has to assign a corres-

ponding force. If there is no acceleration, then mechanical analysis does not have to bother about assigning any force. Like rest, constant velocity lies outside the range of problems envisaged by Newtonian mechanics. It is a residual feature that needs no positive explanation.

Indeed, there could be no positive explanation of a constant velocity. For it is mere change of place and mere change of time. One can account for change in velocity, and one does so by the law of force. One might account for the conservation of acquired velocity, but that would be, perhaps, a philosophic question rather than a mechanical one. But one cannot assign a positive explanation for every element in change of place for, since places are continuous, since a continuum is a non-countable infinite set of differences, there would be needed a non-countable infinite set of positive explanations for every instance of constant velocity. But a non-countable infinite set of positive explanations is impossible. Therefore, a single explanation has to serve for the whole duration of a constant velocity, and that is provided when one explains the acceleration that terminates in the constant velocity.

However, as is clear from its premise, the point we are making is more general than Newton's first law of motion. The argument rests on the impossibility of a non-countable infinite set of positive explanations. If it may underpin Newtonian mechanics, it may also underpin Maxwell's theory of the electro-magnetic field. Hence, if we may use the technical formulation of the postulate

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of the Special Theory of Relativity, we may conclude that the mathematical expression of the principles and laws of physics is invariant in form under transformations from one set of coordinate axes to another set moving with a relative constant velocity. (See Lindsay and Margenau, 151 f, 326 ff.).

2.6 1 Equivalence An even more general heuristic anticipation can be set forth.

The empirical inquirer measures and correlates the results of measurements to reach the functions that relate things directly to one another. There follows a principle of equivalence for all observers.

For, since the function sought relates things directly to one another, the relations of things to observers are omitted. Because the relations of things to observers are omitted, the functions cannot be modified by variations in the relations between the observers and the things. Because there cannot be any such modification, the functions must be the same for all observers.

It is to be noted that the principle of equivalence goes far beyond mere independence of particular places and particular times. Colors as observed vary with the position, velocity, acceleration, of the observer; they vary with the intensity of the light by which he views them; they vary with the condition of his eyes, such as his need of spectacles and his possible color-blindness. But colors as explained by a series of wave-lengths of radiation are necessarily the same for all observers; all conceive them in the same fashion; no one is handicapped by color-blindness.

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Now this principle of equivalence represents a property of the direct relations of things to one another. Such a property can be employed as a premise to determine what the relations are. How can such a premise be formulated? A partial formulation is to take the origin and orientation of coordinate axes as representing the observer, and to say that functions, representing principles ~~are~~ laws, satisfy the principle of equivalence if they remain invariant in form under the group of continuous transformations. For if the observer moves about, he does so in some continuous fashion. But the functions representing laws are independent of any such motion of the observer. And this independence is guaranteed to them by their invariance under continuous transformations.

Such is the postulate of the General Theory of Relativity, which has had some confirmation, and of the Generalized Theory of Gravitation, which as yet has not been put in a form that admits an empirical test.

Certain observations are in order.

First, scalars, vectors, and generally tensors are quantities that may be defined by their transformation properties. Thus, a set of n quantities forms a contravariant vector if they transform according to the same rule as the differentials of the coordinates. A set of n quantities forms a covariant vector if they transform in an opposite manner to the differentials of the coordinates. Contravariant and covariant tensors are sets of n^2 and higher orders of quantities that transform in a more complicated but analogous fashion. Hence, by expressing

physical principles and laws in covariant form, automatically there is attained invariance under the group of continuous transformations. [On the tensor calculus, the reader may consult for a brief outline the second chapter of E.C. McVittie's Cosmological Theory, London 1937, Methuen's Monographs on Physical Subjects.]

[ ] Footnote

Secondly, invariance will be obtained only in so far as there are expressed the relations of things to one another. As soon as equations are made more specific by appealing to observational data of any kind, there is introduced a determination from relations to observers; and then invariance is no longer to be expected. Perhaps this accounts for the fact that in the General Theory of Relativity, the equations remain invariant only as long as the coefficients, g_{ij}, remain in place. See Lindsay and Margenau, p. 368.

g_{ij} ~~$g_{\mu\nu}$~~

Thirdly, the same consideration seems relevant when one attempts to understand the apparent incompatibility of General Relativity and Quantum Mechanics. As will appear presently, Quantum Mechanics is concerned with observables. It seeks formulations of things in their relations to us while General Relativity rests on the relations of things to one another, and only in its applications turns to relations to us.

Fourthly, the heuristic significance of the principle of equivalence, interpreted as a principle of covariance, is not that it restricts the field of possible laws but rather that it gives a determinate meaning to the empirical investigator's preference for the simplest laws. As A. Einstein has advanced in his autobiography [(Albert

[ ] Footnote

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Schilpp

Einstein, Philosopher-Scientist, ed. P.A. Schilpp, Library of Living Philosophers, 1949 and 1951, New York, Tudor Publishing Company, p. 69.),] any law could, perhaps be expressed in covariant form but within the restriction of such a form one can begin by working out the simplest laws and, if they fail, advance to the more complex.

Fifthly, of interest in this connection is Einstein's conviction that data alone are insufficient to guide the constructive efforts of intelligence. There also is needed a formal principle that functions as does the negation of the possibility of a perpetuum mobile in thermodynamics. Such a formal principle Einstein believed he had found in his postulate of invariance, first in Special Relativity, and then in General Relativity. (See ibid., pp.53, 57, 69.)

Summary

2.7 1 Before we turn to the consideration of statistical laws, a summary would seem to be in order.

After noting the similarities between mathematics and empirical insights (1.1) and the differences between them (1.2), we raised the question of the origin and nature of the clues, hints, suggestions that lead up to insight.

As a clue for insight into clues we took the solution of a simple algebraic problem (2.1) and proceeded to generalize.

What is to be known, when the insight occurs, is anticipated by the mere fact of inquiry and is named the "nature of ....", the "such as to...", the "sort of thing

that....".

But similars are similarly understood. Hence, the "nature of ...", may be specified by means of a classification based on sensible similarity; and when insight occurs, this preliminary classification will yield place to a systematic account that speaks of things, not in terms of their relations to our senses, but in terms of their relations to one another. Thus, the "nature of..." is replaced by the more precise anticipation of an unspecified correlation to be specified, of an indeterminate function to be determined §(2.3).

Now functions can be determined, not only by the empirical process of reaching formulae that all known measurements satisfy, but also by appealing to quite general considerations and arguing from them to differential equations which restrict the group of possibly relevant functions. Quite obviously, both procedures can be combined and commonly are combined to obtain a scissors-like action that approaches a solution both from above and below (2.4).

Further, when differences form a non-countable infinite set, as is the case with place and time, there cannot be a distinct explanation for each element of difference. Hence constant velocity has to be regarded as residual and, in fact, it is so regarded in Newton's first law of motion. More generally, the mathematical expression of principles and laws has to be invariant under transformations between inertial systems in accordance with the postulate of Special Relativity (2.5).

Indeed, inasmuch as principles and laws express

the relations of things to one another and omit all reference to the relations of things to observers, it follows that the mathematical expression of principles and laws must be invariant in symbolic form under continuous transformations (2.6).

Finally, one may add that these considerations supply only an abstract scheme. In concrete inquiry they are employed not singly but together. As a science develops, all that already is known serves to render more determinate and precise the general heuristic anticipations that have been outlined.

Statistical Heuristic Structures

3.0 1 The fact of inquiry is an anticipation of something to be known by understanding. Hitherto, only one type of such anticipation has been considered, namely, the anticipation of a correlation, a function, a law, a system. The investigator measures, plots his results upon a graph, and expects to find a smooth curve or formula that will be satisfied, not only by the measurements he has made, but also by all the relevant measurements that he or anyone else ever will make.

Now it is well to encourage investigators in that expectation, to tell them that, if they do not discover any laws, then perhaps, they are measuring the wrong things, that they are not excluding some extraneous influence, that if only they are dogged enough, some day someone will discover the relevant correlation, function, law.

Still, encouragement must not be carried to the point of deception. As we have seen, there is an empirical

residue, and the insight relevant to it consists in grasping, not the system to which it conforms, but its ultimately non-systematic character. Hence, with respect to an aggregate of data or measurements, the anticipation implicit in the fact of inquiry is not a single assertion but rather a disjunction. The anticipation is, not that there must be some correlation to be grasped, but that either there is such a correlation or else there is not. The positive member of the disjunction has been considered in the foregoing account of anticipations of the systematic, and now we must endeavor to clarify the meaning of anticipations of the non-systematic.

3.1 The Non-Systematic.

To reach a classical correlation, function, rule, law, theory, system, there is needed an initial insight into some particular case. By that insight one may master an indefinite multitude of exactly similar cases. Still such universality is not enough. The significance of the initial insight is that it can lead to further insights that master ever more dissimilar particular cases until eventually one reaches a general case and brings under one's control a definable range of particular cases. So Galileo's understanding of the free fall regarded, not bodies of some determinate size, shape, and weight falling at some fixed inclination from the vertical, but bodies of any size, any shape, any weight, falling at any inclination from the vertical.

Now a heuristic anticipation of the non-systematic implies, not a denial of the possibility of concrete insight into particular cases, but a denial of the possibility of the abstract generalization that subsumes a range of particular cases under a general case. In other words, the non-systematic is not to be

identified with the non-intelligible. While the non-systematic excludes the generality of classical correlations, functions, rules, laws, theories, systems, it need not exclude the intelligibility to be reached by inspection and insight into particular cases.

For example, in a particular case, dice may be cast from a determinate receptacle in a determinate manner upon a determinate surface; sufficient information on the case could be attained with the help of a slow-motion film; insight could analyze the total movement into a sequence of mechanically homogeneous stages; each stage could be subsumed separately under known laws of motion, gravity, air resistance, impact, friction, and elasticity; andt the total movement would be no more than the sequence of the stages. Still, dice can be cast from any sort of receptacle, in any manner whatever, upon any type of regular or irregular, fixed or moving surface. There would be no point in attempting to repeat the above laborious procedure for the infinity of particular cases; and if casting dice is a non-systematic, there exists no general case of the classical type to provide an alternative to a pointless repetition of merely particular investigations.

3.2 Actual Frequency.

Where classical generality fails, statistical generality may be sought.

Let us say, then, that there an exists an ~~action~~ actual frequency if, from some determinate antecedent, O, there always follows one and only one of the alternatives, P, Q, R,.... For in any n occurrences of the antecedent, O, the alternative, P, will occur on a determinable p occasions, Q on q occasions, R on r occasions, etc. Accordingly, the actual frequency of P in

~~thus, say r, etc. Hence, the actual frequency of P in~~ a given n occurrences of O will be p/n, the actual frequency of Q will be q/n, the actual frequency of R will be r/n, etc., so that necessarily

$$n = p + q + r + \ldots\ldots$$

Finally, these actual frequencies will be non-systematic if it is not possible to define an Op, Oq, Or, P', Q', R', such that P' always follows Op, Q' always follows Oq, R' always follows Or, etc., so that the indeterminateness of the alternatives is eliminated.

It is to be noted that when a set of alternative consequents has been defined, then it is possible by combinations to construct further sets of alternatives. Thus, one can consider the actual frequency of the combination "either P or Q", or of the combination "P on a first occasion and Q on the second occasion", etc., etc.

One may add at once that the actual frequency of a number of alternatives taken together is the sum of their actual frequencies taken separately. Thus, the actual frequency of "either P or Q" will necessarily be (p + q)/n. Similarly, the actual frequency of the total set of alternatives will necessarily be n/n or unity.

A Generic Notion of Probability

3.3 Let us now generically define a probability as the proper fraction from which actual frequency does not diverge systematically.

The definition posits an ideal proper fraction, which it names a probability. It admits that this ideal proper fraction will not be coincident with actual frequen-

cies. It denies that the divergence between the ideal and the actual will be systematic.

Suppose, for instance, that the probability of casting a "six" with a single die is 1/6. Then, on the first six throws, a "six" may occur twice, on a second once, on a third, not at all, etc... The actual frequency hops about in random fashion while the probability always remains the same 1/6. There is then a divergence between the actual and the ideal. But this divergence is non-systematic so that the difference between the actual and the ideal cannot be reduced to any rule or law.

Certain clarifications are in order.

First, the reason for the definition is, perhaps, obvious enough. Actual frequencies are non-systematic; they vary from case to case; and their variation is not subject to any rule or law. But a probability is an ideal fraction; it is the same for every case of a given kind; it is the representative of the universal, abstract, necessitating, systematizing tendencies of understanding. Hence, if probability and actual frequency coincided, then either both would be systematic or both would be non-systematic. If they diverged and the divergence were systematic, then the actual frequency would have to be the systematic resultant of the systematic probability and the systematic divergence from probability. One meets the requirements of the problem only if 1) the actual frequency is non-systematic, 2) the probability is somehow systematic, 3) the actual frequency may diverge non-systematically from the probability, and 4) the actual frequency cannot diverge

systematically from the probability.

Secondly, it follows that the probability of a set of alternatives is the sum of the probabilities of the alternatives taken singly. For, as we have seen, the actual frequency of such a set is the sum of the actual frequencies of the members of the set (3.2) and, moreover, there cannot be a systematic divergence between actual frequency and probability. But there would be such a systematic divergence if the probability of the set were not the sum of the probabilities of the members of the set. Accordingly, one must deny the consequent and its antecedent to affirm that the probability of a set of alternatives is the sum of the probabilities of the alternatives taken singly.

Thirdly, a probability is not the mathematical limit of a series of actual frequencies. For a series of terms tends to a mathematical limit inasmuch as divergence from that limit can be made as small as one pleases. But actual frequencies do not converge upon probability. They hop about at random. They approach the probability only to recede. Instead of converging, they diverge. But they cannot make their divergence effective, for they cannot get any system into it.

Fourthly, though a probability is not a mathematical limit, there are unobjectionable assumptions that may be introduced so that the non-systematic divergence of probability becomes virtually equivalent to the convergence characteristic of the mathematical limit. (See Lindsay and Margenau, pp. 165 ff.)

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Fifthly, our procedure will be to distinguish two radically different meanings of the term, probability. As defined, probability is an ideal proper fraction from which actual frequencies can diverge but not systematically. However, one also speaks of the probability of opinions and then one does not mean that there is some fraction relevant to the opinion. What is probability in this second sense and what is its relation to probability in the first sense, are questions that must for the moment be postponed.

3.4, Specific Differences It is one thing to calculate the probability of throwing a "four" with a single, unbiassed die, another to make the same calculation when a pair of dice are used, and a third, to do so when the dice are "loaded". In all three cases there is the same generic element: actual frequency diverges non-systematically from the proper fraction named probability. But this genus divides into three distinct species, and the basis of the division resides in the manner in which probability is determined.

The first species is equiprobability. Its conditions are that 1) when an antecedent,), occurs, then there occurs one and only one of a set of n alternatives, and 2) there is no systematic favoring of any of the n alternatives. From the conditions it follows that the probability of the occurrence of any given alternative will be 1/n. For were the probability some other fraction, say, a/n, where a is less or greater than unity, then that alternative could not diverge systematically from a/n and so must suffer systematic discrimination, if a is less than

unity, or receive systematic favoring, if a is greater than unity.

The second species is a derivative of the first. Its conditions are that 1) when an antecedent, O, occurs, then there occurs one and only one of a set of n alternatives, 2) there is a systematic favoring of some alternatives, but 3) this systematic favoring can be reduced to a case in which there is no systematic favoring.

Thus, when a pair of dice are cast, there are eleven possible results, of which some regularly occur more frequently than others. However, this favoring can be eliminated by considering the thirty-six alternatives constituted by combining each of the six faces of one die with each of the six of the other. No one of thirty-six alternatives is favored in any systematic manner, and so the second species is reduced to the first.

The second species of probability is investigated at length by applying the mathematical theory of combinations. The basic formula assigns the probability, P, of r successes in n tries, when p is the probability of one success in one try. This formula is worked out in any suitable case and along with it the reader will find the approximations developed by Laplace, Poisson, and Gauss.

The third species does not admit reduction to the first or to the second. There is an antecedent followed by one and only one of a non-systematic set of alternatives. But one cannot settle by inspection what the alternatives are; and their respective probabilities neither are equal nor are reducible to the case of equiprobability.

Thus, when dice are loaded, some combinations might never occur; moreover, the occurrence of any given face of a loaded die is not equal in probability with the occurrence of any other face, for there is some systematic favoring.

The third species may be described as involving a systematic element which, however, does not succeed in completely dominating the results. There is a systematic element, otherwise the alternatives would be equiprobable. But the systematic element does not succeed in dominating the results, for they are found to be non-systematic.

To meet the problem set by the third species, the relevent technique would seem to be 1) to loosen the heuristic anticipations for dealing with data that can be reduced to system and 2) to compensate for this loosening by introducing probabilities in place or precise predictions.

What would such loosening be? First, anticipations of the systematic are 1) that the data will satisfy some one law or function, 2) that this function will be a solution of the differential equations that represent general features of the problem. Secondly, these anticipations can be loosened. Instead of expecting one function to cover all the data, one may expect a series of eigen functions, say ψ_λ, and a corresponding series of eigenvalues, say p_λ. Again, instead of expecting the single function to be a solution of a differential equation, one may expect the eigenfunctions and eigenvalues to be the solutions of an operator equation, say,

$$P\psi_\lambda = p_\lambda \psi_\lambda$$

where P is the operator, that is, a mathematical entity

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that changes one function into another.

What is the compensating? The foregoing yields a set of observables, the eigenvalues, p_λ . Those that occur will possess some probability, else they would not occur; and they will not possess more than probability, else a systematic solution would work. There exists then some state function from which the probabilities can be calculated; and one may expect the eigenfunctions to lead to the determination of the state function, for if they succeed in selecting the observables with some probability, they should be able to contribute to the determination of the respective probabilities.

Is this guess-work? Certainly, it is not a rigid deduction. On the other hand, it is not purely arbitrary. It is the fruit of an insight based upon clues where, as is always the case, the insight takes one beyond the clues. There must be some loosening of systematic anticipations, for the data dealt with are only partially under the influence of what one might name a systematic component. There must be some compensation for this loosening, else there would be no conclusions at all. But the exact course of the loosening and the compensating is guided by insights into mathematical possibilities and, however strangely, the resulting postulates of Quantum Mechanics have proved highly successful.

Foot note. For an insight into the nature of the operators, P, see G. Temple, The General Principles of Quantum Theory, Methuen Monographs on Physical Subjects, London, 1951.

3.5 Summary.

Classical method is not content with mastery of particular cases but goes beyond them to the abstract generality expressed in correlations, functions, laws, theories, systems. However, there is an empirical residue; particular cases can consist in coincidental manifolds of distinct instances of general cases; and corresponding to such coincidental manifolds there is no general case of the classical type. Still this negation of systematic generality is not the negation of all generality. For if one supposes data to be involved in the non-systematic, one cannot suppose that they diverge systematically from ideal norms.

Among such ideal norms the most familiar is the probability of the occurrence of one of alternative possibilities; and the mode of its determination also supplies its subdivision. If there is no systematic favoring of any of the alternatives, there is equiprobability. If there is systematic favoring that can be reduced to equiprobability, Bayes's formula becomes the relevant anticipation. Finally, when there is systematic favoring that cannot be reduced to equiprobability, then some special axiomatic structure has to be invoked.

There is, then, a statistical heuristic structure and it complements classical structure. In any selected field of inquiry, experiments are performed, measurements are made, and the results are tabulated. In so far as the general intelligibility of the measurements is systematic, classical procedure is relevant. In so far as the general intelligibility of the measurements is not systematic, a probability function is to be sought. Finally, since antecedently the general intelligibility of measurements may be either systematic or non-systematic, a general theory of measurements

must envisage both alternatives. May I ask whether this requirement, rather than particular hypotheses on the accuracy or the distorting effect of measuring, can be regarded as the ultimate basis of the insight into operators that is offered by G. Temple in The General Principles of Quantum Theory [Methuen's Monographs on Physical Subjects, London 1951]?

May further suggestions be made? As long as physicists were engaged in introducing ever more complex modifications of Bohr's image of the atom, they were endeavoring to mount through particular cases to the general case. When they decided to limit their operations to observables (i.e., variables admitting experimental control), they surrendered not generality but systematic generality. Again, in so far as Quantum Theory may be said not to offer insight into particular cases, it suffers on that lower level a perhaps irremediable incompleteness; on the other hand, interpreted as a statistical theory, it possesses fully the completeness of the non-systematic general case.

If such suggestions are to be tried out, it is not to be forgotten that our account of probability supposes an explicit advertence to insight, that other accounts do not, and that the other accounts not only possess the field but also penetrate the interpretation of scientific results. Only a critical and creative effort, meticulously separating methodological assumptions from scientific hypotheses, can determine adequately the relevance of the present analysis to the problems in which scientists are involved; or in the simpler words of Einstein's rather celebrated remark, the cognitional theorist has to attend, not to what scientists say, but to what they do.

Appendix to Chapter 11.

ON THE USE OF THE TERMS "CLASSICAL" AND "STATISTICAL"

In ordinary usage, "classical" and "statistical" are not opposed. The opposite to "classical" is "quantum", and the opposite to "statistical" is "mechanical". This usage may be illustrated by the fourfold classification of 1) classical mechanics (Newton), 2) classical statistics (Boltzmann), 3) quantum mechanics (Schrödinger, Heisenberg), and 4) quantum statistics (Bose-Einstein, Fermi-Dirac).

The trouble is that this fourfold classification seems incomplete. For relativity mechanics is opposed to classical mechanics and, while special relativity enters into combination with quantum mechanics (Dirac), general relativity seems as opposed to it as Einstein himself. Further, if these complications are not to be neglected, it is necessary to go behind the terminology to a systematic conception of the conceptions entertained by interpreters of physical theory. As is obvious, however, the purpose of this appendix is not to expound and to justify a systematic view but simply to clarify the linguistic usage that we have found convenient by contrasting its assumptions with the assumptions that seem to underlie more common modes of speech.

From our viewpoint, then, the fundamental disjunction regards the interpretation of laws of the Newtonian and Einsteinian type. Such laws will be said to be interpreted concretely if they are taken to relate imaginable

terms. The same laws will be said to be interpreted abstractly if they are taken to relate terms that are defined implicitly by the laws themselves.

On the first alternative of concrete interpretation, the law is completely determinate in principle. It is true enough that the law is expressed by a mathematical formula of wide generality and that further determinations will have to be added before any application to concrete instances can occur. It also is true that the further determinations cannot be deduced from the law as a mathematical or as a physical formula. But on concrete interpretation the law is not simply a physical formula; it relates imaginable terms: and because terms are imaginable inasmuch as their various dimensions are assignable, it follows that for concrete interpretation the law is fully determinate in principle.

However, those that accept the first alternative split into two groups. The first group not only affirms concrete interpretation but also affirms that concretely interpreted laws of the Newtonian type exist. The second group agrees with the first in admitting concrete interpretation but differs from it by affirming that, if any such laws seem to be verified, the verification is mere macroscopic appearance. The agreement and difference of this first and this second group seem to me to correspond to the agreement that unites and the difference that separates ordinary conceptions of classical statistics and quantum mechanics.

On the second alternative of abstract interpretation, the foregoing debate is replaced by a distinction. Concretely interpreted laws of the Newtonian and Einsteinian type are resolved into abstract and concrete components. The abstract component is the verified correlation of implicitly defined correlatives. The concrete component is the schematic or non-schematic situation.

The abstract component is determinate but not fully determinate. It is determinate in its own abstract order as an element in abstract system. But it becomes fully determinate only when it is applied successfully to concrete situations. Such application calls for two further types of information: first, one must know which laws in what combination are relevant to the given situation; secondly, one must know what numerical values are to be substituted for the variables and general constants of the abstract laws.

Now while there are well-known difficulties in obtaining accurate numerical values by measurement, a far more radical difficulty arises when one does not know exactly which combination of laws is relevant to a given situation, for then one is unable to go about the task of measuring in any orderly and economical manner. Fortunately, however, there do exist schematic situations in which a happy constellation of circumstances and an appropriate combination of laws have the encouraging implication that the same laws will be applied over and over again in an indefinite sequence. Such, for example, is our planetary system, which has provided the most striking instances of accurate deduction and long-term prediction.

Unfortunately, there also are non-schematic situations. Then the task of applying abstract laws to concrete

situations is at the mercy of circumstance, and the relevant circumstances form a diverging and scattering series of ever more numerous and more remote conditions. For example, a planetary system has a beginning and may come to an end; either event can occur only once for any given system; and then it can occur in any of a notable range of different manners.

Still the existence of non-schematic situations, so far from blocking human intelligence, gives it a new impetus. Statistical investigation becomes the key to an account of the emergence and survival, the numbers and distribution, the differentiation and development of schematic situations. Classical anticipation of the systematic and statistical anticipation of the non-systematic cease to be disparate alternatives. They become complementary techniques in gaining insight into a universe in which the thrust of probability generates from the non-schematic ever more numerous and more developed instances of the schematic.

Accordingly, our contrast between classical and statistical rests not on current issues but on their transposition. On the basis of cognitional analysis the opposition between determinism and indeterminism is sublated in favor of a more comprehensive structure. Classical laws are reinterpreted so that Einstein's differential equations are regarded, not as statements about events at point-instants, but as mathematical expressions of the abstractness of classical laws. Statistical laws are reinterpreted so that indeterminacy has its root in the abstractness of classical laws, its factual ground in the existence of non-schematic situations, and its significance in the type of explanation associated not with the name of Laplace but with the name of Darwin.

observations to assemble into the conditions at some nth remove for some specific event.

The non-Systematic Aggregate of Diverging Series

6.53$_1$ Unfortunately, there is no system to the aggregate of concrete patterns of diverging series of conditions for all kinds of events. Full and exact knowledge of all classical laws assures only a systematic unification of the laws. Such a systematic unification is not an imaginative synthesis. On the other hand, each of the concrete patterns of diverging series is an imaginative synthesis. It follows that singly and together these concrete patterns are non-systematic, for the totality of systematic relations is included in the totality of abstract laws.

Now this general argument can be set forth in more concrete fashion inasmuch as the reader can be offered the materials for two insights. The first insight will be a grasp of the non-systematic in a familiar case. The second will be a grasp of the same lack of system in the aggregate of concrete patterns of diverging series of conditions.

The familiar case may be defined by the question, How many ways are there to cast a "five" with a single die? One might attempt to answer this question empirically. One would get a high-speed camera, suitable lighting, a transparent box, and proceed to take pictures. Next, one would study the pictures of all cases in which a "five" was thrown and calculate the linear and angular

momenta in each movement of the die. The more diligent one was, the greater would be the number of known distinct manners in which a "five" can be thrown. But no matter how great one's industry, one could hardly arrive at the point where one could say one knew all of the ways in which a "five" could be thrown with this die from this box on this surface. Accordingly, one would shift to an a priori method. One would work out a formula that gave the maximum and minimum initial momenta for the last stage of a throw, and the formula would contain constants that received different numerical values for different surfaces and different dice. From the formula one could list all the possible combinations of specifications for the last stage of throwing a "five". By introducing a convenient supposition to prevent the list from containing a non-countable infinite multitude of cases, one could proceed to the second last stage of the process; it would end in any of the manners in which the last could begin; and a further formula would enable one to assign a multitude of ways in which the second last could begin for each way in which the last could begin. With this multitude of multitudes on one's hands, one could turn to the third last stage, and so forth.

Now we happen to know that throwing a "five" is a non-systematic process. While each movement in the process is determinate, while the relations between successive movements are determinate, still these relations depend upon concrete circumstances no less than abstract law. ~~cannot be subsumed under any rule or law~~. The purpose of the preceding paragraph was, not to show that throwing a

"five" is non-systematic, but to grasp in that instance of the non-systematic some of its distinctive characters or symptoms. Our first discovery, then, was that an empirical method of observation and analysis could reveal a great number of ways in which the result might occur, but it offered no promise of providing a complete list of all the ways. Our second discovery was that an a priori method yielded an unmanageable variety of different combinations of distinct alternatives. Even though distinct stages of the process were summed up in formulae, still every possible combination of numerical values satisfying the formulae offered a different alternative, and combinations of these alternatives defined the different ways.

Let us now turn to the second insight. Consider any event, X, and let it be defined as a determinate numerical value of some variable in some classical law.

Next, consider all the laws in which this variable occurs, and list all the alternative combinations of numerical values for the other variables in these laws when the event, X, is occurring.

Thirdly, consider the different manners in which each of the alternative combinations may be approached. Thus, if there are n variables involved and they may have the numerical values, a, b, c,.... when the event, X, is occurring, then the a, b, c, ... specify one of the alternative combinations. Now there are different combinations of rates of change in these variables, such that the rates of change are compatible and, as well, they bring the

variables to the values, a, b, c, ... A complete list of such combinations of rates of change, first, when the rates are regular, and secondly when they are not, would serve to define the different approaches to one of the alternative combinations.

Fourthly, repeat the foregoing performance for all kinds of events. Then, one will have worked out all the manners in which one may approach at all possible combinations of rates of change all the alternative combinations of numerical values for the other relevant variables when each variable in each law assumes every possible numerical value.

Fifthly, by comparing different processes, one can draw up a list of incompatible events.

Sixthly, by combining compatible processes in all possible manners, one can list ~~abstract~~ diverging series of positive conditions for all kinds of events to as many removes as one pleases.

Perhaps this is enough. One is working out a plan of setting up an unmanageable variety of different combinations of distinct alternatives. The intelligent procedure in dealing with such combinations of alternatives is to acknowledge their non-systematic character and turn to the calculation of probabilities. For an a priori method of working out diverging series of conditions yields the concrete patterns that occur, not only in this visible universe, but also in every possible universe subject to the same laws. On the other hand, an a posteriori method would be both impracticable and inconclusive.

Summary

6.54 1 Such is the argument in the general case.

Classical laws hold in the concrete only if conditions are fulfilled. To invoke the same or different laws to show that conditions will be fulfilled, merely sets up a diverging series of conditions. The further one goes back along the series, the more numerous become the conditions and the more they are dispersed not only in space but also in time. Even if one knew the patterns of the diverging series, and the fulfilment of all conditions at some nth remove, the only possible deduction would be in virtue of the inverse, converging series. Finally, such patterns form a non-systematic aggregate; they are an enormous series of different combinations of distinct alternatives; their intelligibility is reached, not by working them out in detail, but by acknowledging their non-systematic character and turning to probabilities.

The Possibility of Accurate Prediction

6.55 1 However, besides the foregoing general case, there is also a range of particular cases. In the last analysis, they reduce to the general case. But the last analysis is not reached at once and, in the meantime, there is the possibility of the accurate deduction and prediction of fully determinate events. Accordingly, we have to define the particular case, show how it escapes the logic of the diverging series of conditions, and finally argue that this escape is never complete.

The particular case will be named a scheme. Its abstract or theoretical component is some classical law or combination of laws, such that there arises a

mutual fulfilment of conditions. Its concrete or factual component is such a conjunction of things or events that, in virtue of the law or laws, the conjunction leads to another, the other leads to a third, the third leads to a fourth, until eventually the initial conjunction recurs. Such schemes may be extremely simple or extremely complex. They may involve any number of intermediaries or, in the case of the straight-forward continuity, none at all. Moreover, schemes may be combined, so that all will function if any one or two or n function. Finally, schemes may emerge in a conditioned series, such that the later become possible when the earlier are functioning.

The conspicuous example of ~~this~~ schemes of recurrence is of course the planetary system. But the whole of nature seems full of oscillations, rhythms, alternations, recurrences, from the elementary processes of physics to the technological, economic, and political inventions and routines of men. Finally, when such patterns of recurrent activity are submitted to analysis, they are found to involve the two elements of a scheme, the theoretical component of interrelated laws and the factual component of a conjunction that through the laws brings forth its own recurrence.

Clearly, such schemes do not suppress the principle that no event is unconditioned. Nor do they prevent each event from having many conditions. None the less, though the diverging series of conditions remains, it has been brought to heel. For the scheme itself takes care of its positive conditions, all of which are included

within classes of events, and every event within the classes keeps recurring because the others do in a perpetual vicious circle.

There is, then, an escape from the unpredictability implicit in the diverging series of conditions. Were astronomers merely in possession of full and exact knowledge of all natural laws, they still would be stuck with their 3-body problem, that is, with the task of finding a general solution to the problem of determining the trajectories of three bodies when their initial positions and momenta were given. In fact, astronomers operate in the light of an imaginative synthesis; Ptolemy's mistaken imaginative synthesis yielded fair results; Copernicus' simpler imaginative synthesis combined with a more accurate knowledge of laws enables men to predict with remarkable accuracy the movements, not merely of three bodies, but of the sun, the planets, their satellites, the comets, and even asteroids.

Still, this escape is not complete. The periodicity of our planetary system offers no guarantee against internal disruption of its members or against the intrusion of some external body like a bull into our china shop. The planetary system secures its own perpetuity only if certain negative conditions are fulfilled, and over those negative conditions it exercises no control. Moreover, just as the planetary system is not a proof of its own survival, so it is not the ground of its own emergence. A scheme is a matter, not merely of a combination of laws, but also of a happy conjunction of things or events. That

conjunction has to take place before the scheme can begin to function, and so the scheme has its origin in a combination which it did not generate.

Now one might like to suppose that, just as there are schemes, so too there is an over-all scheme, an ultimate imaginative synthesis, on which there could be based accurate predictions of the emergence and survival of lesser schemes. Such would be the affirmation of mechanist determinism. But, as we have seen, complete and exact knowledge of all laws would include a systematic unification of laws without involving an imaginative synthesis either of the concrete unfolding of this universe or of any other subject to the same laws. Moreover, an over-all scheme would have not only a theoretical component, constituted by laws in combination, but also a factual component, constituted by an initial conjunction that the over-all scheme itself could not bring about. Finally, the issue before us is to be settled, not by what one might like to think, but by the evidence; and the evidence is that the concrete, historical unfolding of this world process involves a conspicuous use of the statistical techniques of large numbers and long intervals of time. It seems to follow that the over-all intelligibility of our world process is, not in accord with the assumptions of mechanist determinism, but, in accord with some different view that assigns a due place to statistical laws. After all, machines are constructed and function within political, economic and technological schemes,

and ~~such~~ SUCH schemes emerge, survive, and are superseded without systematic divergence from the probabilities.

The Indeterminacy of the Abstract

6.56 We have been endeavoring to indicate in precise terms both the indeterminacy of the abstract and the consequent statistical residues.

In brief, the indeterminacy of the abstract is the indeterminacy of the blanket proviso, "other things being equal". Classical laws are said to hold in the concrete, provided other things are equal, but no one specifies what the other things are or in what their equality consists.

There is good reason for this omission. For a fully determinate event in the general case depends upon the fulfilment of a diverging series of positive and negative conditions. The conditions at each remove in the series not only become more numerous but also scatter in space and time. Finally, the patterns of such diverging series form an enormous, non-systematic aggregate.

It is true that there are schemes of recurrence. Granted any of a long series of suitable initial conjunctions, the operation of classical laws will tend to repeat the initial conjunction indefinitely. Still, there is only a tendency and not an absolute necessity, for here too there rules the blanket proviso, other things being equal, or unequal but negligibly so. Nor is there any evidence to support the affirmation of some over-all scheme to regularize the emergence and the survival of lesser schemes.

The general case, then, is the universal

case. In the last analysis, events depend upon a non-systematic aggregate of patterns of diverging series of conditions. Because that aggregate is non-systematic, it is a residue abstracted from by the totality of classical laws. Because the non-systematic is the premise of statistical inquiry, this residue may be named statistical. Hence, the canon of statistical residues may be said to affirm the non-systematic character of the aggregate of patterns of diverging series of conditions that govern concrete events.

A Mathematical Analogy

6.57 A mathematical analogy may exist. For combinations of differential equations are likely to be soluble only through the introduction of special suppositions and, even then, only by a method of approximations. Hence, if one said that classical laws corresponded to differential equations, that concrete problems demanded combinations of such equations, and that the totality of sets of special suppositions and approximate solutions was non-systematic, one would have in the field of mathematics an analogy to the canon of statistical residues.

The General Character of Statistical Theories.

6.6 Finally, the canon of statistical residues, in conjunction with the other canons of empirical method, makes it possible to complement our account of the notion of probability (Chapter 11, 4) with a derivation of the general characteristics of statistical theories.

Events.

6.61 First, statistical theories will deal

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4.1 The Elementary Paradox

Let $(\underline{x}_1, \underline{t}_1)$ and $(\underline{x}_2, \underline{t}_2)$ be the coordinates of a pair of point-instants, P and Q, in a reference frame, K.

Let $(\underline{x'}_1, \underline{t'}_1)$ and $(\underline{x'}_2, \underline{t'}_2)$ be the coordinates of the same of point-instants in a relatively moving frame, K', and let them from this view-point be named, P' and Q'.

On the Lorentz-Einstein transformation, writing

$$\underline{H} = 1/(1 - \underline{u}^2/\underline{c}^2)^{\frac{1}{2}}$$

one relates the coordinates by the equations

$$\underline{x'}_1 = \underline{H}(\underline{x}_1 - \underline{u}\underline{t}_1) \qquad (1)$$

$$\underline{x'}_2 = \underline{H}(\underline{x}_2 - \underline{u}\underline{t}_2) \qquad (2)$$

$$\underline{t'}_1 = \underline{H}(\underline{t}_1 - \underline{u}\underline{x}_1/\underline{c}^2) \qquad (3)$$

$$\underline{t'}_2 = \underline{H}(\underline{t}_2 - \underline{u}\underline{x}_2/\underline{c}^2) \qquad (4)$$

Now consider two particular cases. So far, P and Q are any point-instants whatever: but in our first particular case we suppose that P and Q are the simultaneous positions of the ends of a standard measuring rod in the frame, K. Since the length of the rod is unity, and since the positions are simultaneous, we have

$$\underline{x}_1 - \underline{x}_2 = 1 \qquad (5)$$

$$\underline{t}_1 - \underline{t}_2 = 0 \qquad (6)$$

By subtracting equation (2) from (1) and equation (4) from (3) and substituting the values from equations (5) and (6), we have

$$\underline{x'}_1 - \underline{x'}_2 = \underline{H} \qquad (7)$$

$$\underline{t'}_1 - \underline{t'}_2 = -\underline{H}\underline{u}/\underline{c}^2 \qquad (8)$$

so that, clearly, a unit length between simultaneous positions becomes on transformation a length that is not unity between positions that are not simultaneous.

In our second particular case, we suppose that P and Q are the point-instants of successive seconds in a standard clock stationary relative to the frame K. Clearly,

$$\underline{x}_1 - \underline{x}_2 = 0 \qquad (9)$$

$$t_1 - t_2 = 1 \qquad (10)$$

whence, as before, by appealing to equations (1) to (4) and by substituting from (9) and (10), one obtains,

$$\underline{x'}_1 - \underline{x'}_2 = -\underline{v} \qquad (11)$$

$$t'_1 - \underline{t'}_2 = \underline{\gamma} \qquad (12)$$

so that a distance that is zero has been transformed into a distance that is not zero, and a time that is unity has been transformed into a time that is not unity.

Still, though distances and times are relative to reference frames, the four-dimensional interval is invariant. Let us name the interval, $\underline{s}$, where

$$\underline{ds}^2 = \underline{dx}^2 - \underline{c}^2\underline{dt}^2 \qquad (13)$$

and in the present cases

$$\underline{s}^2 = (\underline{x}_1 - \underline{x}_2)^2 - \underline{c}^2(\underline{t}_1 - \underline{t}_2)^2 \qquad (14)$$

On substituting from equations (5) and (6), one finds that the interval of the rod in K according to the account in K is unity. Likewise, on substituting from equations (7) and (8), one finds that the interval of the rod in K according to the account in K' is unity. Again, on substituting from equations (9) and (10), one finds that the interval of the clock in K

according to the account in K is ic [$1 = \sqrt{-1}$].

Likewise on substituting from equations (11) and (12), one finds that the interval of the clock in K according to the account in K' is also ic.

Thus we have arrived both at the elementary paradox and at its solution. The elementary paradox arises from the contrast of equations (5) and (7) and again from the contrast of equations (10) and (12). The first contrast shows that the length of a rod in K is unity on the account in K but on the account in K' is greater than unity; and if K' finds a unit rod greater than unity, it seems to follow that his own rod is shorter. The second contrast shows that the length of a standard duration in K is unity in the account in K but is greater than unity in the account in K'; and if a unit of duration in K is found to be greater than unity in K', it seems to follow that the unit in K' must be shorter.

However, if we began from rods and clocks in the system, K', we could establish the opposite conclusions with equal validity; for then it would seem to follow that the shorter units were in the system, K. Such is the elementary paradox.

What the paradox overlooks is the fact that, in the context of Special Relativity, one is not dealing with rods that are merely spatial or with clocks that are merely temporal. For, as has been seen, a standard rod determines an invariant four-dimensional interval of magnitude, unity; and a standard clock determines an invariant four-dimensional interval of magnitude, ic. Rods that determine an invariant four-

dimensional interval must have a temporal component, and clocks that determine an invariant four-dimensional interval must have a spatial component.

Indeed, as appears from equations (5) and (6), in the reference frame, in which a rod lies between simultaneous point-instants, the invariant interval has a spatial component of magnitude, unity, and a temporal component of magnitude, zero. As appears from equations (7) and (8), in other relatively moving reference frames, the same rod determines the same four-dimensional interval, which, however, now has a spatial component of magnitude, $\underline{H}$, and a temporal component of magnitude, $-\underline{H}u/\underline{c}^2$. Concomitant with the variation of the spatial components, there is a variation of the temporal components. The rod in K by the account in K lies between simultaneous point-instants. The same rod in K by the account in K' lies between non-simultaneous point-instants. The spatial and temporal components, say $\left[1, 0\right]$, transform to spatial and temporal components, $\left[\underline{H}, -\underline{H}u/\underline{c}^2\right]$. Inversely, the rod in K' by the account in K' will lie between simultaneous point-instants. But the same rod in K' by the account in K will lie between non-simultaneous point-instants. In this case, spatial and temporal components, $\left[1, 0\right]$, transform to spatial and temporal components, $\left[\underline{H}, \underline{H}u/\underline{c}^2\right]$, for the sign of the relative velocity, $\underline{u}$, changes.

Again, as appears from equations (9) and (10), in the reference frame, in which the beginning and the end of a standard duration occur in relatively the same position, the invariant interval of magnitude, $\underline{ic}$, has a spatial com-

ponent of magnitude, zero, and a temporal component of magnitude, unity. As appears from equations (11) and (12), in other relatively moving frames of reference, the same duration determines the same invariant interval, which, however, now has a spatial component of magnitude, -Hu, and a temporal component of magnitude, H. Again, there is concomitant variation of spatial and temporal components. A standard duration in K by the account in K has components [0, 1]; the same duration in K by the account in K' has components [-Hu, H] . Inversely, a standard duration in K' by the account in K' will have components [0, 1]; but this duration in K' by the account in K will have components [Hu, H].

The elementary paradox results from a cumulation of oversights. It disregards the invariant interval fixed by any rod for all reference frames and the invariant interval fixed by any clock for all reference frames. It disregards four accounts of two rods to consider only two rods, and it disregards four accounts of two clocks to consider only two clocks. Finally, it disregards the temporal component that pertains to rods and the spatial component that pertains to clocks.

Still, if the elementary paradox is to be set aside as a gross over-simplification, there remains in its entirety the problem of working out a coherent account of the notion of measurement compatible with the complexity of Special Relativity. To this task we must now address our attention.

are invariant under permissible transformations, and so measurements valid in one reference frame are valid in all permissible frames.

On the suppositions of the Special Theory of Relativity, some revision is necessary. We shall consider how it affects 1) lengths of standard units, 2) lengths of measurable objects, 3) measurements, and 4) sizes.

First, a length results from fitting a size into a geometrical construction. On the Special Theory of Relativity, the relevant geometry is that of Minkowski space. The following characteristics of the lengths of standard units follow from the properties of this space or, what comes to the same thing, from the Lorentz-Einstein transformation.

1. In all inertial frames of reference a standard rod determines a four-dimensional interval of magnitude, unity. Similarly, in all inertial frames of reference a standard clock determines a four-dimensional interval of magnitude, ic, where i is the square root of minus one, and c is the velocity of light in *vacuo*.

2. A reference frame will be said to be normal to a standard rod, when the rod in the frame determines an interval with spatial component of magnitude, unity, and with temporal component of magnitude, zero.

Similarly, a reference frame will be said to be normal to a standard clock, when the clock in the frame determines an interval with a spatial component of magnitude, zero, and a temporal component of magnitude, unity.

3. Reference frames that are not normal to standard rods or standard clocks are in relative motion to

normal reference frames.

Inversely, in reference frames in relative motion to normal frames, standard rods determine the same invariant interval but now possess spatial components, H, and temporal components, $-Hu/c^2$ or Hu/c^2, according to the direction of the relative motion.

Similarly, in reference frames in relative motion to normal frames, standard clocks determine the same invariant interval, which, however, now possesses a spatial component, -Hu or Hu, and a temporal component, H.

Secondly, there are to be determined the characteristics of the lengths of other measurable objects. Clearly, these lengths will have the same properties as the lengths of standard units. For both sets of lengths are subject to the same transformation equations.

Accordingly, for every measurable spatial object there is a group of normal reference frames, relatively at rest, and in them the object determines an interval with spatial component, A, and with temporal component, zero. In other reference frames in relative motion, the same object will determine an interval of the same magnitude but with spatial component, AH, and with temporal component, $-AHu/c^2$ or AHu/c^2, according to the direction of the relative motion.

Similarly, for every measurable temporal object, there is a group of normal reference frames, relatively at rest, and in them the object determines an interval with spatial component, zero, and with temporal component, B. In other inertial frames in relative motion, the same object

will determine the same invariant interval, namely icB, but with a spatial component, -BHu or BHu, and with a temporal component, BH.

In the third place, measurements are to be considered, and they offer two distinct aspects.

For, in so far as measurements are numbers to be substituted into equations or to be derived by solving equations, they are identical with lengths. This follows from the nature of the coordinate system which, in the present case, deals only with measured lengths. Accordingly, all that has been said about lengths may now be repeated about measurements. A spatial magnitude will determine an invariant interval, A, with components, $[\, AH,\ -AHu/c^2 \,]$, and a temporal magnitude will determine an invariant interval, icB, with components, $[\, BH,\ -BHu \,]$. In normal reference frames, H becomes unity, and u becomes zero, so that the components are $[\, A,\ 0 \,]$ and $[\, 0,\ B \,]$ respectively. Finally, in transformations to the left, the sign of u changes.

However, there is a further aspect to measurements. The numbers substituted into equations have to be derived from data, and the numbers derived from equations have to be verified in data. Thus, there arises the question whether Special Relativity modifies the concrete operation of measuring.

The general answer would seem to be that it does not. A measurement remains the number that stands to unity as the measurable object stands to a standard unit. However, within the frame-work of that general answer it will be well to advert to particular cases.

Ordinarily, simultaneity is determined in the same manner in selecting the point-instants at the ends of the standard unit and in selecting those at the ends of the measurable object. It will follow that spatial measurements ordinarily occur with the standard unit and the measurable object in the same reference frame and, since A: 1 : : AH : H, the result of measuring will be the number, A.

Still, this is not inevitable. Further, it may be fairly common to use a clock, stationary in a reference frame, to time a process that begins at one place in the frame and ends at another. Hence, besides the measurements that result when the object and the standard are taken in the same frame, namely, A/1, AH/H, B/1, BH/H, there are the measurements that result when they are in different frames. If one of these frames is normal, the results will be AH/1, A/H, BH/1, B/H; if neither frame is normal, one must distinguish two values of H, say H' and H", so that the results may be AH'/H", AH"/H', BH'/H", BH"/H'. In other words, the actual process of measuring can involve the same ambiguities as are contained in the elementary paradox and, indeed, even more elaborate ambiguities.

Accordingly, we are brought to the conclusion that, while Special Relativity demands an operation of measuring that fundamentally is similar to measuring under Newtonian assumptions, still it adds new rules that either eliminate or correct some results which, on Newtonian assumptions, would be valid.

Chapter V: Space and Time. §4. Rods and Clocks.

Foot-note to p. ~~259~~. 277

The questions should be clarified. "Size" has been defined as an experiential conjugate that varies both from inner change in the object and from change of position of the observer. In the text I do not mean to deny perspectival variation of size. Similarly, I do not mean either to affirm or to deny what I regard as meaningless, namely, that there is or is not an inner change of the object as referred to some absolute space. The question is whether an acceptance of special relativity logically entails any change in rods or clocks, and my answer is that no such change can be deduced. "Lengths" vary because reference frames vary; and reference frames vary because modes of determining simultaneity vary.

In the fourth place, there are the sizes of spatial and of temporal magnitudes. Do rods contract or expand? Do clocks run slow or fast?[1] Our answer will be negative, and our reasons run as follows:

[1] Footnote on previous sheet.

First, it is difficult to suppose that rods and clocks should undergo such variation without a proportionate variation occurring in the objects that they measure; and if the proportionate variation occurs, then no explanation is provided for the relativity of lengths to reference frames.

Secondly, even if rods and clocks varied while other sizes do not vary, the required explanation would not be forthcoming. For rods and clocks and other sizes determine intervals that are invariant for all inertial reference frames. Moreover, these intervals exhibit temporal components for rods and other spatial magnitudes; and they exhibit spatial components for clocks and other temporal magnitudes. How does a contracting rod generate a temporal component? How does a decelerated clock generate a spatial component?

Thirdly, the evidence for contracting rods and decelerated clocks lies in the elementary paradox. Now we have no doubt that, on the suppositions of Special Relativity, it would be possible to reach such measurements as A/B, AB/1, B/H, BH/1, which are the lengthened and shortened rods and the faster and slower clocks. But the obvious explanation lies, not in any variation of the sizes of rods or clocks, but in the relativity of lengths and in the use of a standard unit in one reference frame to measure an object in another, significantly different, frame.

Fourthly, there is no aspect of the Special Theory of Relativity that is not accounted for by distinguishing between size and length, where length is constructed in accord with the geometry of Minkowski space. There follow immediately both the invariant intervals and the relativity of spatial and temporal components to reference frames. Moreover, this construction of length presupposes, not a variation in size, but a relativity of simultaneity. It was from a relative solution to the problem of synchronization that Special Relativity was evolved: and whenever such a solution is adopted, Special Relativity will follow even though no variation in size is admitted.

This point is worth illustrating. Suppose two planes flying in the same direction with the same constant velocity, so that the distance between them is constant. Let that distance be regarded as the standard unit, and suppose two observers, K and K', that determine simultaneity differently. Now consider the instant at which the first plane is at a point, P. Let us say that for K the second plane at the same instant is at some point, R. Then for K', since he determines simultaneity differently, the second plane must be at some nearer or further point, S, at the instant when the first plane is at P. Accordingly, though there is only one size, though this size is constant, though both observers agree that there is only one size and that it is constant, none the less, in virtue of different determinations of simultaneity, there are two lengths, PR and PS, and they are unequal with an inequality in some proportion to the relative velocities of the

planes and the divergence between the two determinations of simultaneity.

While this illustration is, I believe, to the point, still it is only an illustration. One cannot take a relativity of simultaneity as postulate and from it deduce the Special Theory of Relativity. On the contrary, a relativity of simultaneity merely sets a problem: confronted with that problem, one adverts to the invariance of principles and laws: and it is by postulating the invariance of principles and laws under inertial transformations that one reaches the basic premise from which Special Relativity follows.

4.4 Summary

The aim has been to work out a general theory of measurement and thereby clarify the notions of measurable object, standard unit, measuring, and measurement peculiar to Special Relativity.

Measurement was seen to be the technique by which the scientist moves from the description of things as related to our senses to the explanation of things as related to one another.

Standard units were conceived as measurable objects that intrinsically stand on the same footing as other measurable objects but conventionally are given a unique status to simplify and systematize the formulation of the relations of things to one another.

The definitions of measurable objects of various kinds, the standardization of their respective units, the rules of measuring, and the nature of measurement were seen to depend on abstract presumptions and laws and, therefore, to be subject to revision along with revisions of the presumptions and the laws.

This generic notion of measurement was then applied to measurements of spatial and temporal magnitudes.

A basic distinction was drawn between the experiential conjugate, size, and the pure conjugate, length. The former is correlative to our experience. The latter is implicit in a geometrical structure of definitions, postulates, and inferences.

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The transition from Newtonian to Einsteinian physics is a transition from length, as implicit in Euclidean geometry, to length, as implicit in Minkowski space. It drops invariant spatial and temporal lengths. It introduces invariant four-dimensional intervals with variable spatial and temporal components. While it grants no special significance to reference frames at rest, still it does imply a position of privilege for normal reference frames, in which spatial magnitudes have a zero temporal component and temporal magnitudes have a zero spatial component. Thus, an interval, $\underline{A}$, which is a real number, has the components $[\underline{AH}, -\underline{AHu}/c^2]$ which become $[\underline{A}, 0]$ in a normal reference frame; and an interval, $i\underline{B}$, which is an imaginary number, has the components $[-\underline{BHu}, \underline{BH}]$, which become $[0, \underline{B}]$ in a normal reference frame. It is to be noted that the distinction between the spatial and the temporal is as sharp as the distinction between real and imaginary numbers, that the lengths of standard units are but particular cases of the lengths of other measurable objects, that the transformation properties of unit and of other lengths are the same, that in a Minkowski manifold lengths are already measured so that measurements are coincident with lengths, that in the operation of measuring, there arise in Special Relativity ambiguities that do not exist and so do not have to be solved on Newtonian suppositions.

However, while Special Relativity involves a revision of the notions of lengths and of measurements and while it introduces a new caution in the operation of measuring,

it does not imply the expansion or contraction of rods or the acceleration or deceleration of clocks. In other words, the unit divisions of the axes in the coordinate systems are constituted, not by the size, but by the length of standard distances and standard durations. Such lengths are relative to reference frames, but this relativity of length arises, not from change of size, but from the interdependence of determinations of length and of similtaneity. That corresponds to change of size is, not a mere transformation of reference frames, but a variation in the intervals, A or icB. A variation in some of these intervals corresponds to a variation in some measurable objects; a proportionate variation in all of these intervals suggests that the standardization of units needs to be corrected and revised.

Might I suggest that, on this showing, there vanishes the arbitrary division of the world of physics into rods and clocks and, on the other hand, all other objects? Such arbitrariness is noted and regretted by Prof. Einstein in his Autobiography. (Albert Einstein, Philosopher-Scientist, ed. P.A.Schilpp, The Library of Living Philosophers, New York, 1949 and 1951, p. 59)

It would seem to vanish 1) inasmuch as physics is set the task of assigning invariantly expressed abstract relations to account not only for experienced colors and sounds but equally for experienced extensions and durations, 2) inasmuch as these relations are reached by formulating and verifying hypotheses, 3) inasmuch as notions of length and

measurement and the standardization of units form internal parts of the hypothesis to be verified, 4) inasmuch as the hypothesis assigns the same properties to lengths of standard units as to lengths of other measurable objects, and 5) inasmuch as frames of reference have their units constituted, not by the sizes of rods and clocks, but by their theoretically defined lengths.

Finally, it would seem that the foregoing account of rods and clocks in Special Relativity might easily be adapted to the requirements of General Relativity. In General Relativity there remains the invariant four-dimensional interval; there remain its spatial and its temporal components; there remains the covariance of these components in different reference frames. The basic differences are that the components now are curvilinear and that specifications of coordinates are not virtual measurements of distance or duration.

On the other hand, it is not to be claimed that our account of measuring is completely general. Rather that distinction seems to pertain to Quantum Theory viewed as a theory of measurements. For if it is true that all measuring is abstractive both in the sense that it replaces sets of data by series of approximate numbers and in the sense that it relates the numbers not to our senses but to one another, still the relations may be systematic or non-systematic; and non-systematic relations, no matter what their origin, can be manipulated theoretically only in a context that envisages statistical laws.

Chapter X

REFLECTIVE UNDERSTANDING

Like the acts of direct and introspective understanding, the act of reflective understanding is an insight. As they meet questions for intelligence, it meets questions for reflection. As they lead to definitions and formulations, it leads to judgments. As they grasp unity, or system, or ideal frequency, it grasps the sufficiency of the evidence for a prospective judgment.

When Archimedes shouted his Eureka, he was aware of a significant addition to his knowledge, but it is not likely that he would have been able to formulate explicitly just what a direct insight is. Similarly, we perform acts of reflective understanding, we know that we have grasped the sufficiency of the evidence for a judgment on which we have been deliberating, but without prolonged efforts at introspective analysis we could not say just what occurs in the reflective insight. What we know is that to pronounce judgment without that reflective grasp is merely to guess: again, what we know is that, once that grasp has occurred, then to refuse to judge is just silly.

Accordingly, the present section will be an effort to determine what precisely is meant by the sufficiency of the evidence for a prospective judgment. There is presupposed a question for reflection, "Is it so?". There follows a judgment, "It is so.". Between the two there is a marshalling and weighing of evidence. But what are the scales on which evidence is weighed? What does ~~it~~ evidence have to weigh, if one is to pronounce a "Yes" or a "No"?

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Unfortunately, the more complex judgments become, the more complex is the analysis of the grounding act of reflective understanding. The whole answer cannot be given at once and partial answers are incomplete. Hence, we shall begin from a very general statement and then illustrate its meaning from the form of deductive inference. Next, we shall turn to the concrete judgments of every day life, and consider in turn concrete judgments of fact, judgments on the correctness of insights into concrete situations, and finally the occurrence of analogies and generalizations. In the third place there will be considered the judgments of empirical science, the radical difference of such judgments from those of ordinary living, the nature of scientific generalization and verification, and what is meant by the probability of scientific opinions. Fourthly, analytic propositions and principles are distinguished and their criteria investigated. Fifthly, the nature of mathematical judgments is considered. Finally, we may add that philosophic judgments are not treated in this chapter, ~~section~~, for they can be examined satisfactorily only after further elements in the problem have been set forth.

1. THE GENERAL FORM OF REFLECTIVE INSIGHT

grasp/~
evidence/~

To grasp evidence as sufficient for a prospective judgment is to grasp the prospective judgment as virtually unconditioned.

Distinguish then, between the formally and the virtually unconditioned. The formally unconditioned has no conditions whatever. The virtually unconditioned has conditions indeed but they are fulfilled.

Accordingly, a virtually unconditioned involves three elements, namely. 1) a conditioned, 2) a link between the conditioned and its conditions, and 3) the fulfilment of the conditions. Hence, a prospective judgment will be virtually unconditioned if 1) it is the conditioned, 2) its conditions are known, and 3) the conditions are fulfilled. By the mere fact that a question for reflection has been put, the prospective judgment is a conditioned: it stands in need of evidence sufficient for reasonable pronouncement. The function of reflective understanding is to meet the question for reflection by transforming the prospective judgment from the status of a conditioned to the status of a virtually unconditioned; and reflective understanding effects this transformation by grasping the conditions of the conditioned and their fulfilment.

reflection/~

Such is the general scheme and we proceed to illustrate it from the form of deductive inference. Where A and B each stand for one or more propositions, the deductive form is:

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If A, then B.
But A.
Therefore B.

For instance:

If X is material and alive, X is mortal.
But men are material and alive.
Therefore, men are mortal.

Now the conclusion is a conditioned, for an argument is needed to support it. The major premise links this conditioned to its conditions, for it affirms, If A, then B. The minor premise presents the fulfilment of the conditions, for it affirms the antecedent, A. The function, then, of the form of deductive inference is to exhibit a conclusion as ↓ virtually unconditioned. Reflective insight grasps the pattern, and by rational compulsion there follows the judgment.

However, deductive inference cannot be the basic case of judgment, for it presupposes other judgments to be true. For that reason we have said that the form of deductive inference is merely a clear illustration of what is meant by grasping a prospective judgment as virtually unconditioned. Far more general than the form of deductive inference is the form of reflective insight itself. If there is to be a deduction, the link between the conditioned and its conditions must be a judgment, and the fulfilment of the conditions must be a further judgment. But judgments are the final products of cognitional process. Before the link between conditioned and conditions appears in the act of judgment, it existed in a more rudimentary state within cognitional process itself.

the

Before the fulfilment of conditions appears in another act of judgment, it too was present in a more rudimentary state within cognitional process. The remarkable fact about reflective insight is that it can make use of those more rudimentary elements in cognitional process to reach the virtually unconditioned. Let us now see how this is done in various cases.

2. CONCRETE JUDGMENTS OF FACT.

Suppose a man to return from work to his tidy home and to find the windows smashed, smoke in the air, and water on the floor. Suppose him to make the extremely restrained judgment of fact: Something happened. The question is, not whether he was right, but how he reached his affirmation.

The conditioned will be the judgment that something happened.

The fulfilling conditions will be two sets of data: the remembered data of his home as he left it in the morning; the present data of his home as he finds it in the evening. Observe that the fulfilling conditions are found on the level of presentations. They are not judgments, as is the minor premise of syllogism. They involve no questions for intelligence nor insights nor concepts. They lie simply on the level of past and present experience, of the occurrence of acts of seeing and smelling.

The link between the conditioned and the fulfilling conditions is a structure immanent and operative within cognitional process. It is not a judgment. It is not a formulated set of concepts, such as a definition. It is simply a way of doing things, a procedure within the cognitional field.

The general form of all such structures and procedures has already been outlined in terms of the three levels of presentations, intelligence, and reflection. Specializations of the general form may be exemplied by the classical and statistical phases of empirical method, by the notion of the thing, and by the differences between description and explanation. However, such accounts of the general form and its specializations pertain to introspective analysis. Prior to such an investigation and formulation, the structures and procedures exist and operate; nor, in general, do they operate any better because the analysis has been effected.

Now, in the particular instance under consideration, the weary worker not only experiences present data and recalls different data but by direct insights he refers both sets of data to the same set of things which he calls his home. The direct insight, however, fulfils a double function. Not merely are two fields of individual data referred to one identical set of things but a second level of cognitional process is added to a first. The two together contain a specific structure of that process, which we may name the notion of knowing change. Just as knowing a thing consists in grasping an intelligible unity-identity-whole in individual data, so knowing change consists in grasping the same identity or identities at different times in different individual data. If the same thing exhibits different individual data at different times, it has changed. If there occurs a change, something has happened. But these are statements. If they are affirmed, they are judgments. But prior to being either statements or judgments, they exist as

unanalysed structures or procedures immanent and operative within cognitional process. It is such a structure that links the conditioned with the fulfilling conditions in the concrete judgment of fact.

The three elements have been assembled. On the level of presentations there are two sets of data. On the level of intelligence there is an insight referring both sets to the same things. When both levels are taken together, there is involved the notion of knowing change. Reflective understanding grasps all three as a virtually unconditioned to ground the judgment, Something happened.

While our illustrative instance was as simple as it could be, still it provides the model for the analysis of more complex instances of the concrete judgment of fact. The fulfilling conditions may be any combination of data from the memories of a long life, and their acquisition may have involved exceptional powers of observation. The cognitional structure may suppose the cumulative development of understanding exemplified by the man of experience, the specialist, the expert. Both complex data and a complex structure may combine to yield a virtually unconditioned that introspective analysis could hardly hope to reproduce accurately and convincingly. But the general nature of the concrete judgment of fact would remain the same as in the simple case we considered.

However, the reader probably is asking how we know whether the insights that constitute the pivot of such structures are themselves correct. To this point we have now to turn.

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3. INSIGHTS INTO CONCRETE SITUATIONS

Direct and introspective insights arise in response to an inquiring attitude. There are data to be understood; inquiry seeks understanding; and the insight arises as the relevant understanding. But a mere bright idea is one thing, and a correct idea is another. How do we distinguish between the two?

The question is asked, not in its full generality but with respect to concrete situations that diverge from our expectations and by that divergence set us a problem. Thus, to retain our former illustration, the man on returning home might have said: There has been a fire. Since any fire ~~there might have been~~, was extinguished, that judgment would suppose an insight that put two and two together. Our question is on what grounds such an insight could be pronounced correct.

First, then, observe that insights not only arise in answer to questions but also are followed by further questions. Observe, moreover, that such further questions are of two kinds. They may stick to the initial issue, or they may go on to raise distinct issues. What started the fire? Where is my wife? Observe, thirdly, that the transition to distinct issues may result from very different reasons; it may be because different interests supervene to draw attention elsewhere: but it may also be because the initial issue is exhausted, because about it there are no further questions to be asked.

Let us now distinguish between vulnerable and invulnerable insights. Insights are vulnerable when there are further questions to be asked on the same issue. For the further questions lead to

further insights that certainly complement the initial insight, that to a greater or less extent modify its expression and implications, that perhaps lead to an entirely new slant on the issue. But when there are no further questions, the insight is invulnerable. For it is only through further questions that there arise the further insights that complement, modify, or revise the initial approach and explanation.

Now this reveals a law immanent and operative in cognitional process. Prior to our conceptual distinction between correct and mistaken insights, there is an operational distinction between invulnerable and vulnerable insights. When an insight meets the issue squarely, when it hits the bull's eye, when it settles the matter, there are no further questions to be asked and so there are no further insights to challenge the initial position. But when the issue is not met squarely, there are further questions that would reveal the unsatisfactoriness of the insight and would evoke the further insights that put a new light on the matter.

Such, then, is the basic element in our solution. The link between conditioned and its conditions is a law immanent and operative in cognitional process. The conditioned is the prospective judgment, This or that direct or introspective insight is correct. The immanent law of cognitional process may be formulated from our analysis. Such an insight is correct, if there are no further, pertinent questions.

At once it follows that the conditions for the prospective judgment are fulfilled when there are no further, pertinent questions.

Note that it is not enough to say that the conditions are fulfilled when no further questions occur to me. The mere absence of further questions in my mind can have other causes. My intellectual curiosity may be stifled by other interests. My eagerness to satisfy other drives may refuse the further questions a chance to emerge. To pass judgment in that case is to be rash, to leap before one looks.

As there is rash judgment, so also there is mere indecision. As the mere absence of further questions in my mind is not enough, so it is too much to demand that the very possibility of further questions has to be excluded. If, in fact, there are no further questions, then, in fact, the insight is invulnerable; if, in fact, the insight is invulnerable, then, in fact, the judgment approving it will be correct.

But how is one to strike this happy balance between rashness and indecision? How is one to know when it is reached? Were there some simple formula or recipe in answer to such questions, then men of good judgment could be produced at will and indefinitely. All we can attempt is an analysis of the main factors in the problem and an outline of the general nature of their solution.

In the first place, then, one has to give the further questions a chance to arise. The seed of intellectual curiosity has to grow into a rugged tree to hold its own against the desires and fears, conations and appetites, drives and interests, that inhabit the heart of man. Moreover, every insight has its retinue of presuppositions, implications, and applications. One has to take

the steps needed for that retinue to come to light. The presuppositions and implications of a given insight have to knit coherently with the presuppositions and implications of other insights. Its possibilities of concrete application have to enter into the field of operations and undergo the test of success or failure. I do not mean, of course, that concrete living is to pursue this logical and operational expansion in the explicit, deliberate, and elaborate manner of the scientific investigator. But I do mean that something equivalent is to be sought by intellectual alertness, by taking one's time, by talking things over, by putting viewpoints to the test of action.

In the second place, the prior issue is to be noted. Behind the theory of correct insights, there is a theory of correct problems. It was to dodge this prior issue that we supposed a concrete situation that diverges from our expectations and by that divergence defines a problem. In other words, there has been postulated an inquirer that understands the background of the situation and so knows what is to be expected; there also has been postulated a problem that exists, that is accurately defined by the divergence of the situation from correct expectations, that in turn provides a definition of the pertinence of any further questions.

Now this amounts to saying that good judgment about any insight has to rest on the previous acquisition of a large number of other, connected, and correct insights. But before attempting to break this vicious circle, let us assure ourselves of the fact of its existence. Children ask endless questions; we have no

doubt about their intellectual curiosity; but so far from crediting them with good judgment, we do not suppose them to reach the age of reason before their seventh year. Young men and women have the alertness of mind that justifies their crowding into schools and universities, but the law doubts the soundness of their judgment and regards them as minors, while Aristotle denied they had enough experience to study ethics with profit. Nor is there merely the initial difficulty of acquisition but, as well, there is the subsequent necessity of keeping in touch. The man that returns to a field of commerce ~~and~~ or industry, to a profession or a milieu, in which once he was completely at home, may try to carry on from where he left off. But unless he learns to be more wary from mistakes and minor ineptitudes, he is merely inviting blunders and disaster. Good judgment about concrete insights presupposes the prior acquisition of an organized set of complementary insights.

In the third place, then, there is the process of learning. It is the gradual acquisition and accumulation of insights bearing on a single domain. During that process one's own judgment is in abeyance. It is being developed and formed but it has not yet reached the maturity needed for its independent exercise. For the gradual acquisition and accumulation of insights are not merely a matter of advancing in direct or introspective understanding. At the same time, intellectual curiosity is asserting itself against other desires. At the same time, the logical retinues of presuppositions and implications of each insight are being expanded either to conflict and provoke further questions or else to mesh into coherence. At the same time, operational possibilities are

envisaged to be tested in thought experiments, to be contrasted with actual practice, to be executed in ventures that gradually increase in moment and scope to enlighten us by failures and to generate confidence through success.

So it is the process of learning that breaks the vicious circle. Judgment on the correctness of insights supposes the prior acquisition of a large number of correct insights. But the prior insights are not correct because we judge them to be correct. They occur within a self-correcting process in which the shortcomings of each insight provoke further questions to yield complementary insights. Moreover, this self-correcting process tends to a limit. We become familiar with concrete situations; we know what to expect; when the unexpected occurs, we can spot just what happened and why and what can be done to favor or to prevent such a recurrence; or, if the unexpected is quite novel, we know enough to recommence the process of learning and we can recognize when, once more, that self-correcting process reaches its limit in familiarity with the concrete situation and in easy mastery of it.

In the fourth place, rashness and indecision commonly have a basis in temperament. Apart from occasional outbursts, that we view as out of character, the rash man nearly always is quite sure and the indecisive man regularly is unable to make up his mind. In such cases it is not enough to point out that learning is a self-correcting process that tends to a limit or that, while the limit is not marked with a label, still its attainment is revealed by a habitual ability to know just what is up. For unless a special effort is made to cope with temperament itself, the rash man

continues to presume too quickly that he has nothing more to learn, and the indecisive man continues to suspect that deeper depths of shadowy possibilities threaten to invalidate what he knows quite well.

Finally, we note that we leave to another occasion a discussion of the philosophic solutions that no one ever can be certain. Our immediate purpose is to explain the facts. Human judgments and refusals to judge oscillate about a central mean. If the precise locus of that divide can hardly be defined, at least there are many points on which even the rash would not venture to pronounce and (to) many others on which even the indecisive would not doubt. What, then, is the general form of such certitude of ignorance and such certitude of knowledge?

Our answer is in terms of the virtually unconditioned. There occurs a reflective insight in which at once one grasps 1) a conditioned, the prospective judgment that a given direct or introspective insight is correct, 2) a link between the conditioned and its conditions, and this on introspective analysis proves to be that an insight is correct if it is invulnerable and it is invulnerable if there are no further, pertinent questions, and 3) the fulfilment of the conditions, namely, that the given insight does put an end to further, pertinent questioning and that this occurs in a mind that is alert, familiar with the concrete situation, and intellectually master of it.

4. CONCRETE ANALOGIES AND GENERALIZATIONS

Two brief corollaries have to be drawn.

An argument from analogy assumes that some concrete situation, A, is correctly understood. It argues that some other similar situation, B, is to be understood in the same fashion.

A generalization makes the same assumption to argue that any other similar situation, X, is to be understood in the same fashion.

In both cases what is at work is the law, immanent and operative in cognitional process, that similars are similarly understood. Unless there is a significant difference in the data, there cannot be a difference in understanding the data. This point has already been made in discussing the heuristic procedure of the classical phase of empirical method. Clearly enough, it holds not merely for regularities, rules, laws, correlations but also for ideal frequencies and for things. A second look does not necessarily mean one is looking at a second thing. A second actual frequency does not necessarily mean that one will establish a second ideal frequency. For there to be a second thing or a second ideal frequency an appropriate difference in the data has to be supposed.

In the simplest possible manner then, our analysis resolves the so-called problem of induction. It makes the transition from one particular case to another or from a particular case to the general case an almost automatic procedure of intelligence. We appeal to analogies and we generalize because we cannot help

understanding similars similarly. This solution, be it noted, squares with the broad fact that there is no problem of teaching men to generalize. There is a problem of teaching them to frame their generalizations accurately: indeed, the whole point of the analogy is that it absolves one from that conceptual task and the complexities it involves. There is, above all, a problem of preventing men from generalizing on insufficient grounds, and very easily such grounds are merely putative.

For if our view makes generalization an easy matter, it also clips the generalizer's wings. There must be a correct insight with respect to the basic situation. Before similars can be similarly understood, there is needed an act of understanding: and if that act is mistaken in the first instance, it will be equally mistaken in the second. But, as we have seen, to know one's insights are correct presupposes a process of learning and the attainment of familiarity and mastery. Further, the analogous or the general situation must be similar. If there is any significant dissimilarity, then further, pertinent questions arise to complement, to modify, perhaps to revise the basic insight. Finally, and this is the real catch, what differences are significant? My familiarity and mastery of the initial situation enables me to tell whether further questions there are pertinent. Another's familiarity and mastery of the analogous situation would enable him to tell whether further questions are pertinent in that situation. But unless the two situations are similar in all respects, my familiarity with one does not enable me to tell whether or not further questions arise when my insight is transferred to the other.

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To conclude, analogy and generalization are essentially valid procedures. But when their basis is an insight into a concrete situation, the conditions of their proper use can become so stringent as to render them almost useless. It is this fact that grounds the suspicion with which men greet arguments from analogy and generalizations. But, at the same time, there is a compensating factor that arises from human collaboration in the process of learning. To this we have now to turn our attention.

5. COMMON SENSE

Common sense is that vague name given to the unknown source of a large and floating population of elementary judgments which everyone makes, everyone relies on, and almost everyone regards as obvious and indisputable. Though some repetition will be involved, three points, I think, call for our attention: 1) the source of these judgments, 2) their proper object or field, and 3) their relation to empirical science.

5.1 The Source of Common-Sense Judgments.

The proximate ground and source of common-sense judgments lie in the procedures just described, of concrete judgments of fact, judgments on the correctness of insights into concrete situations, and concrete analogies and generalizations. The remote source is more complex. One has to envisage these procedures carried out, not by isolated individuals, but by members of families, of tribes, of nations, over the face of the earth for generation after generation. One has to take into account the diffusion of judgments by communication and their transmission by tradition. Finally,

one has to note that there results not merely an enlargement but also a unification and transformation of the self-correcting process of learning.

If I may repeat myself, besides the hard way of finding things out for oneself, there is the comparatively easy way of learning from others. Archimedes had to rack his brains to discover what every schoolboy can be taught. For teaching is a vast acceleration of the process of learning. It throws out the clues, the pointed hints, that lead to insights: it cajoles attention to remove the distracting images that obstruct them: it puts the further questions that reveal the need of further insights to complement and modify and transform the acquired store; it grasps the seriation of acts of understanding to begin from the simple and work towards the more complex. But what is done explicitly and deliberately by professional teachers, also is done implicitly and unconsciously by parents with their children and by equals among themselves. Talking is a basic human art; by it each reveals what he knows and provokes from others the further questions that direct his attention to what he had overlooked. More general and more impressive than talking is doing: deeds excite our admiration and stir us to emulation: we watch to see how things are done: we experiment to see if we can do them ourselves; we watch again to discover the oversights that led to our failures. Thus it is that what anyone discovers passes into the possession of many, to be checked against their experience and to be confronted with the test of their further questions. Thus too, it is that the discoveries of different individuals enter into single, cumulative series; that the later

presupposes and improves upon the earlier; and that the starting-point of each generation is where its predecessor left off.

The remote source then of common-sense judgments is a collaboration. The self-correcting process of learning goes on in the minds of individuals, but the individual minds are in communication. The results reached by one are checked by many, and new results are added to old to form a common fund from which each draws his variable share measured by his interests and his energy.

There is another side to the story. It is human to err, and common-sense judgments are very human. They rest upon the self-correcting process of learning as transformed by communication and collaboration. But men share not only in intellectual curiosity but also in more earthy passions and prejudices. The mixed character of human drives can generate a common deviation from the pure product of intelligence and even a common dishonesty in refusing to acknowledge the effective pertinence of further, pertinent questions. So it is that we find each tribe and nation, each group and class, prone to develop its own brand of common sense and to strengthen its convictions by pouring ridicule upon the common nonsense of others. From the contradictory varieties of common sense, men have appealed to the common consent of the human race. But one may well doubt that such a procedure goes quite to the root of the matter. If one must suspect the collaboration of groups and classes, of tribes and nations, it does not follow that one cannot suspect the collaboration of mankind. Error is not primarily a class product or a national product. It is human. The group or class, the tribe or nation, only gives a more

specific twist to the mixed motives of human effort. Undertake to select the judgments on which all men agree, and you have no guarantee either that when all men agree they will do so from the pure and detached motives of intelligence and reason or, indeed, that you yourself in your investigation and selection have operated exclusively from that unmixed drive. (1)

The collaboration, named common sense, not only offers enormous benefits and advantages but it also intertwines them with more than a danger (2) of deviation and aberration. Nor do we ourselves stand outside this collaboration as spectators. We were born into it. We had no choice but to become participants, to profit by its benefits, and to share in its errors. We have no choice about withdrawing from it, for the past development of one's own intellect can no more easily be blotted out than the past growth of one's body, and future development will have to take place under essentially the same conditions and limitations as that of the past. There is, then, a fundamental problem, and how it is to be met, we cannot discuss at once. Our immediate objective has to be confined to discerning the field or domain within which common sense might be expected to operate successfully. This brings us to our second topic.

5.2 The Object of Common-sense Judgments.

Already a distinction has been drawn between description and explanation. Description deals with things as related to us. Explanation deals with the same things as related among themselves. The two are not totally independent, for they deal with the same things and, as we have seen, description supplies, as it were, the tweezers by which we hold things while explanations are being discovered or verified, applied or revised. But despite their

intimate connection, it remains that description and explanation envisage things in fundamentally different manners. The relations of things among themselves are, in general, a different field from the relations of things to us. There is an apparent overlapping only when we consider the relations of men among themselves; and then the different procedures of description and explanation prevent the overlapping from being more than apparent, for description is in terms of the given while explanation is in terms of the ultimates reached by analysis.

Not only are description and explanation distinct, but there are two main varieties of description. There are the ordinary descriptions that can be cast in ordinary language. There are also scientific descriptions for which ordinary language quickly proves inadequate and so is forced to yield its place to a special, technical terminology. Nor is it difficult to discern behind these linguistic differences a more fundamental difference. Both ordinary and scientific description are concerned with things as related to us, but both are not concerned with the same relations to us. The scientist selects the relations of things to us that lead more directly to knowledge of the relations between things themselves. Ordinary description is free from this ulterior preoccupation. As it begins, so also it ends with human apprehensions and interests as its center.

There exists then a determinate field or domain of ordinary description. Its defining or formal viewpoint is the thing as related to us, as it enters into the concerns of man. Its object is what is to be known by concrete judgments of fact, by judgments on the correctness of insights into concrete situations, by concrete

analogies and generalizations, and by the collaboration of common sense. It is as much an object of knowledge as any other, for it is reached by beginning from the level of presentations, by advancing through inquiry, insights, and formulation, by culminating in the critical inquiry of reflective understanding, the grasp of the unconditioned, and the rationally compelled pronouncement of judgment. To anticipate a later vocabulary, the domain of ordinary description is a section of the universe of being, of what intelligently is grasped and reasonably is affirmed. How much of that section really is reached by ordinary description, is of course, a further question. At least, it is something to know the goal at which it aims, and that has been our restricted topic.

But before going on to our third topic, it may be well to preclude possible misconceptions. First, then, the human collaboration that results in a common sense involves belief. The analysis of belief cannot as yet be undertaken. But the type of belief that is essential in this collaboration resembles that of the pupil, who believes his teacher only that later he himself may understand and be able to judge for himself. It resembles that of the scientist, who does not insist on exploring for himself all the blind allies down which his predecessors wandered but is content to test their final results either directly, by repeating experiments, or, more commonly, by operating on the principle that, if those results were erroneous, the error would be revealed indirectly in the experiments he himself does perform. Hence it is that a man pronouncing a common-sense judgment is convinced that he is uttering,

not what someone else told him, but what he himself knows.

Secondly, the human collaboration that results in a common sense is under the dominance of practical considerations and pragmatic sanctions. The further questions that arise and are considered pertinent, do not come from any theoretical realm, and the tests that are employed move within the orbit of human success and failure. Still that dominance, so far from vitiating the results, is dictated by the object to be known, by the thing as it is related to us and as it enters into the concerns of men. It was a philosophic school that invented the notion that ideas are true because they happen to work. Despite its practicality, common sense is convinced that ideas work only if they are true. Nor is this surprising, for the practical further question is a further question that leads to the modification or revision of an insight; and the pragmatic criterion of success is the absence of the failure that would reveal the necessity of thinking things out afresh.

Thirdly, the human collaboration that results in a common sense is subject to the deviations and aberrations that have their root in the mixed motives of man. But it is only in so far as I myself share in those mixed motives that my understanding and my judgment will suffer the same bias and fall in line with the same deviations and aberrations. As long as I share in them, my efforts at correction and selection will be just as suspect as the judgments I wish to eliminate. It is only when I go to the root of the matter and become efficaciously critical of myself that I can begin to become a reliable judge; and then that becoming will consist in the self-correcting process of learning ~~them~~ which has

already been described.

5.3 Common-sense Judgment and Empirical Science.

Our third main topic was the relation of common sense to science, and our fundamental assertion is that the two regard distinct and separate fields. Common sense is concerned with things as related to us. Science is concerned with things as related among themselves. In principle, they cannot conflict, for if they speak about the same things, they do so from radically different viewpoints.

When I say that in principle they cannot conflict, I mean of course, that in fact they can and do. To eliminate actual conflict, it is necessary to grasp the principle and to apply it accurately.

The basic difficulty has been to grasp the principle. The scientists of the Renaissance were quite aware that there was some difference in principle, but they expressed it by a distinction between primary and secondary qualities. Science is concerned with things and their primary qualities, that is, with things as they really are. Common sense is concerned with things, with their primary qualities, and most of all with their secondary qualities, that is, mainly with things as they merely appear. On this showing, knowledge is science, and where common sense diverges from science, partly it is the darkness of ignorance and error, partly it is the twilight soon to be replaced by a scientific dawn. Naturally enough such exclusive pretensions were met by opposite pretensions equally exclusive, and the debate raged on a mistaken issue. Today, I think, we can be not only cooler but also wiser about the whole matter. As has been argued in the ~~sections on Image and~~ earlier chapters,

~~Truth and Description and Explanation~~, it is necessary to distinguish within knowledge between separate yet complementary domains. There is a comprehensive, universal, invariant, non-imaginable domain: its object is the thing-itself, with differences in kind defined by explanatory conjugates, and with differences in state defined by ideal frequencies. There is also an experiential, particular, relative, imaginable domain: its object is the thing-for-us, with differences in kind defined by experiential conjugates, and with differences in state defined by expectations of the normal. The former field of empirical science is to be reached only by abstracting from the empirical residue, ~~the individual, incidental, and non-systematically-divergent, the non-countable-multiplicity assignability of the continuum~~. The latter field includes the empirical residue; it views things in their individuality, their accidental determinations, their arbitrariness, their continuity.

The significance of this distinction appears in logic as the separation of two universes of discourse. To put the matter concretely, let us take illustrative propositions and consider the three cases of 1) ignoring the distinction of the domains, 2) denying the distinction of the domains, and 3) accepting the distinction of the domains. First, if one ignores the distinction of the domains, then one has the problem of choosing between the propositions:

The planets move in approximately elliptical orbits with the sun at their focus.

The earth is at rest, and the sun rises and sets. ~~Accordingly,~~

Secondly, if one denies the distinction of the domains, one is committed to the more rigorous choice between the propositions;

From every viewpoint, the planets move in elliptical orbits with the sun at their focus.

From every viewpoint, the earth is at rest and the sun rises and sets.

Thirdly, if one affirms the distinction of the domains, then one will reject all four of the preceding propositions to assert both of the following:

From the viewpoint of explanation, the planets move in approximately elliptical orbits with the sun at their focus.

From the viewpoint of ordinary description, the earth is at rest and the sun rises and sets.

On this third position there result two separate universes of discourse. All the affirmations of empirical science contain the qualifying reservation, "from the viewpoint of explanation". Similarly, all the affirmations of common sense contain the qualifying reservation,"from the viewpoint of ordinary description". Automatically, all logical conflict is eliminated, for the qualifying reservations prevent the propositions of one universe from contradicting the propositions of the other.

Underlying this logical separation, there will be more fundamental methodological differences. Both ordinary description and empirical science reach their conclusions through the self-correcting process of learning. Still they reach very different conclusions because though they use essentially the same process, they operate with different standards and criteria. What is a further, pertinent question for empirical science is not necessarily

a further, pertinent question for ordinary description. Inversely, what is a further, pertinent question for ordinary description is not necessarily a further, pertinent question for empirical science. It is this fundamental difference in the criterion of the relevance of further questions that marks the great divide between a scientific attitude and a common-sense attitude. Because he aims at ultimate explanation, the scientist has to keep asking "Why?", until ultimate explanation is reached. Because the layman aims at knowing things as related to us, as entering into the domain of human concerns, his questioning ceases as soon as further inquiry would lead to no immediate, appreciable difference in the daily life of man. Hence it is that the layman is attempting to impose his criteria on the scientist when he asks him what he is doing and follows that up with the further question:"What is the good of it?" For if the practical question can be put to engineers and technologists and medical doctors, its only effect upon pure science would be to eliminate all further progress. Inversely, the pure scientist is attempting to impose his criteria upon common sense, when he interprets a practical attitude as a lack of interest in truth; it is, indeed, a lack of interest in the truth that the scientist seeks, but that is not the sole domain in which truth is to be learned. Reflective understanding can reach the virtually unconditioned to pronounce correct judgments of concrete fact and to discern correct insights into concrete situations. Without those basic judgments, science has no starting-point and, ~~as well~~ equally, the

glorious achievemnets of applied science cannot be truly affirmed.

The difference of the domains appears not only in different criteria of the pertinence of further questions but also in the difference of the terms employed and in the possibilites they respectively offer for logical deduction. Because ordinary description is concerned with things-for-us, it derives its terms from everyday experience: because the elements of daily experience are constant, the terms of ordinary description are constant; visible shapes and the spectrum of colors, the volume, pitch, and tone of sounds, the hot and cold, wet and dry, hard and soft, slow and swift, now and then, here and there, do not shift in meaning with the successive revisions of scientific theories; the concrete unities that are men and animals and plants, the regularities of nature and the expectations of a normal course of events form a necessary and unchanged basis and context into which applied science introduces its improvements. Inversely, because science seeks knowledge of the things as related among themselves, because such relations lie outside our immediate experience, because the ultimates in such relations are to be reached only when ultimate explanation is reached, each great forward step of scientific knowledge involves a more or less profound revision of its fundamental terms. Again, because science is analytic and abstractive, its terms are exact; because its correlations purport to be generally valid, they must be determined with utmost precision; because its terms are exact and its correlations general, it must be ready to bear the weight of a vast superstructure of logical deductions in which each conclusion must be equally exact and valid

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generally. # On the other hand, as we have seen, ordinary description must be perpetually on its guard against analogies and generalizations: for though similars are similarly understood, still concrete situations rarely are similar, and the synthesis of an aggregate of concrete situations is not itself a concrete situation. Because things fall away from the Pole Star in the northern hemisphere, it does not follow that they will do so in the southern. Because within the range of human vision the earth is approximately flat, it does not follow that the integration of all such views will be a flat surface. The procedure of sound common sense is not to generalize nor to argue from analogy, but to retain the insights gained in former experience and to add the complementary insights needed in fresh situations. The collaboration of common sense aims, not at establishing general truths, but at building up a core of habitual understanding that is to be adjusted by further learning in each new situation that arises.

Common sense, then, has its own specialized field or domain. It has its own criteria on the relevance of further questions. It has its own specially constant vocabulary, its proper universe of discourse, and its own methodological precepts of keeping to the concrete, of speaking in human terms, of avoiding analogies and generalizations and deductions, of acknowledging that it does not know the abstract, the universal, the ultimate. Precisely because it is so confined, common sense cannot explicitly formulate its own nature, its own domain, its own logic and methodology. These it has to learn, if it would limit properly its pronouncements, but it has to learn them in its own shrewd fashion through instances

and examples, fables and lessons, paradigms and proverbs, that will function in future judgments not as premises for deductions but as possibly relevant rules of procedure. Finally, because common sense has to be acquired, it is not possessed equally by all. It has its adept pupils that make mistakes, indeed, but also learn by them. Within their familiar field they are masters, and (as) well they know that their mastery ends when they step beyond its limits. Above all they know that they must master their own hearts, that the pull of desire, the pain of fear, the deeper currents of passion are poor counsellors, for they rob a man of that full, untroubled, unhurried view denoted by sure and balanced judgment.

If the domains of science and common sense are distinct, so also they are complementary. If (2) one must recognize the differences in their objects, their criteria, their universes of discourse, their methodological precepts, one must also insist that they are the functionally related parts within a single knowledge of a single world. The intelligibility that science grasps comprehensively is the intelligibility of the concrete with which common sense deals effectively. To regard them as rivals or competitors is a mistake, for essentially they are partners and it is their successful cooperation that constitutes applied science and technology, that adds inventions to scientific discoveries, that supplements inventions with organizations, know-how, and specialized skills. # But if common sense itself, once it is supplied with its appropriate evidence, has little difficulty in recognizing this fact, theorists of science can hardly be credited with an equal perspicacity. Misled by a confusion between the heuristic and the

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representative functions of imagination, they assumed that the business of science was to paint a picture of the really real. If, as we have argued, such a picture is essentially unverifiable and gratuitous, it cannot coincide with the verifiable pictures of common sense. If from this conflict the theorists of science proceeded to conclude that common sense must be some brutish survival, that it was in need of being instructed in lofty tones on the far superior virtues and techniques of the scientist, one cannot be surprised that common sense retaliated with its jokes on the ineptitude of the theorists and professors and with its quietly imperious demand that, if they were to justify their existence, they had best continue to provide palpable evidence of their usefulness. But such opposition, I would contend, does justice neither to common sense nor to science: it has no better basis than a mistaken theory; and it had best be written off as an error incidental to an age of transition. During the past four centuries, empirical science has emerged and developed, to set us the twofold problem both of determining its nature and of working out the proper adjustment of the complementary functions of common sense. If such large problems cannot be solved in short order, one should not infer that they cannot be solved at all.

To conclude, common sense is one thing and common sense judgments are another. Common sense is common and specific. It is a specialized domain of knowledge with a proper universe of discourse, proper criteria on the pertinence of further questions, and proper methodological precepts. Operation within that domain is basically and fundamentally a communal collaboration in the self-

correcting process of learning. The fruit of that collaboration is a habitual core of accumulated insights into concrete situations and into the procedures needed to complement and adjust that core before one can pass judgment on further, concrete situations. Hence it is that common sense judgments are issued, not by some public authority named common sense, but only by individual judges in their own individual situations. Further, they can be known to be correct only by the individual judges in the individual situations, for no one else is in possession of the evidence as it is given and no one else is informed with the familiarity and mastery that result from the self-correcting process of learning within that situation. I can be certain that I am writing this, and you can be certain that you are reading it. But it is quite another matter for you to be certain that I am correct in affirming that I am writing, as it will be quite another matter for me to be certain that you are correct in affirming that you are reading. The common element in common sense is not some list of general truths about which all men can agree: it is not some list of particular truths about which all men can agree; but it is a collaboration in the erection of a basic structure by which, with appropriate adjustments, each individual is enabled to fill out his individual list of particular truths. Finally, each of those particular pronouncements occurs inasmuch as reflective understanding grasps the virtually unconditioned in the manner described in the subsections on concrete judgments of fact and on judgments on the correctness of insights into concrete situations.

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6. PROBABLE JUDGMENTS

When the virtually unconditioned is grasped by reflective understanding, we affirm or deny absolutely. When there is no preponderance of evidence in favor of either affirmation or denial, we can only acknowledge our ignorance. But between these extremes there is a series of intermediate positions, and probable judgments are their outcome.

This probability of judgment differs from the probability investigated in studying statistical method. As has been seen, the probable expectation answers a question for intelligence by assigning an ideal frequency from which actual events non-systematically diverge. But the probable judgment answers a question for reflection and, though it anticipates a divergence between the judgment and actual fact, still the ground of this anticipation lies, not in a non-systematic element in the facts, but in the incompleteness of our knowledge. Hence, judgments about things, about correlations, and about probability expectations, may be true or may be only probable.

Probable judgments differ from guesses. In both cases knowledge is incomplete. In both cases reflective understanding fails to reach the virtually unconditioned. But the guess is a non-rational venture beyond the evidence that resembles the non-systematic aspect of events. On the other hand, the probable judgment results from rational procedures. Though it rests on incomplete knowledge, still there has to be some approximation towards completeness. Though it fails to reach the virtually

unconditioned, still it has to be closing in upon that exigent norm. Thus, one may say that guesses are probably true only in the statistical sense of diverging non-systematically from true judgments: but probable judgments are probably true in the non-statistical sense of converging upon true judgments, of approaching them as a limit.

It is the nature of this approximation, approach, convergence, that constitutes the problem of the probable judgment. What precisely can be meant by such metaphors? If anything is meant, then how can it be known? No one surely makes a probable judgment when he can make a certain judgment; yet how can the probable be known to approach the certain, when the certain is unknown?

Fortunately, such paradox is not as acute as it may seem. We seek the truth because we do not know it. But, though we do not know it, still we can recognize it when we reach it. In like manner we also are able to recognize when we are getting near it. As we have seen, the self-correcting process of learning consists in a sequence of questions, insights, further questions, and further insights that moves towards a limit in which no further, pertinent questions arise. When we are well beyond that limit, judgments are obviously certain. When we are well short of that limit, judgments are at best probable. When we are on the borderline, the rash are completely certain and the indecisive full of doubts. In brief, because the self-correcting process of learning is an approach to a limit of no further, pertinent questions, there are probable judgments that are probably true in the sense that they approximate to a truth that as yet is not known.

Directly the foregoing analysis regards the probability of judgments on the correctness of insights into concrete situations. Indirectly, it can be extended to all other probable judgments. Thus, concrete judgments of fact involve some insight that links the level of presentation with the question for reflection, and so the probability of such concrete judgments may be reduced to the probability of the correctness of the insight they involve. Did something happen? Something did happen if the same set of things exhibits different data at different times. An insight is required to grasp the identity of the things, and such an identification may be certain or probable. But the data exhibited at different times either differ or do not differ. If no difference is detected, there is no ground whatever for asserting change. If any difference is detected, there are the grounds for asserting change. If you do not remember accurately the former data, then you just do not know whether or not there was a change. If you are inclined to think that the former data were different, then the issue shifts. What inclines you to think so? Any reason that can be offered will suppose some insight into the objective course of events or into the habits of your memory, and it is that insight that gives rise to probability. More complex cases call for a more complex analysis, but the general lines of the analysis will be the same.

This brings us to the probability of the empirical sciences. Two questions arise. Why are their conclusions no more than probable? In what sense are their conclusions an approximation to what is true and certain? Discussion of analytic propositions is

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deferred to the next ~~sub~~-section and so we have to consider the empirical sciences in their generalizations and in their particular judgments of fact.

Since similars cannot but be similarly understood, generalization itself offers no difficulty. If the particular case is understood correctly, then every similar case will be understood correctly. If the problem of induction arose because the rest of the particular cases were not inspected, then that problem would be insoluble because the rest of the particular cases never are inspected: were they, there would be no generalization. In fact, the problem of induction arises because the particular case may not be properly understood; and it is solved by seeking that correct understanding.

Still, seeking is one thing and finding another. Empirical science gets its start by hitting off significant correlations. The correlations implicitly define abstract correlatives. But precisely because they are abstract, the return to the concrete is greeted with further questions. The law of the lever is simplicity itself. But to have an independent measurement of weights, one needs the law of the spring. To test the law accurately, one needs the theorem on centers of gravity. To formulate the law, one needs the geometry of perpendiculars. ~~Automatically~~ Further one has embarked upon a vectorial representation of forces, an assumption of Euclidean geometry, a theory of the application of forces at a point, a parallel investigation of the tension of wires, and a certain amount of dabbling with gravitation. Automatically, further questions arise. Not only do they arise from the concrete

problems set by tension and gravitation, ~~which in turn give rise to further questions.~~ What is far more significant is the presence of the highly abstract theorems and procedures, ~~which give relevance to enormous ranges of further questions.~~ Can every force be represented by a vector? Are all forces applied at a point? Did Euclid have the last word? The initial abstraction allows one to return to the concrete only after the exploration of successively widening circles of inquiry. Statics is mastered only to raise the problems of kinetics. Kinetics is mastered only to reveal that thermal and electro-magnetic phenomena may be the antecedents or the consequents of local movements. One begins to get the lot in line and to feel that the future of physics is a matter of determining accurately a few more decimal points when along come a Planck and an Einstein with their further questions.

The generalization of classical laws, then, is no more than probable because the application of single laws raises further questions that head towards the systematization of a whole field. In turn, such systematization is no more than probable until the limit of no further, pertinent questions is reached. But that limit is not reached, first, if there may be further, unknown facts that would raise further questions to force a revision or, secondly, if there may be further, known facts whose capacity to raise such further questions is not grasped.

Similar considerations render the generalization of statistical laws no more than probable. For statistical laws presuppose some classification of events. One is not going to advance

quantum theory by investigating baseball averages. Hence definitive statistical laws suppose definitive classifications. The future discovery of new kinds or of new subdivisions of subatomic elements will invite a revision of the statistical laws. Similarly, more accurate investigations may lead to the discernment within the statistical law of a systematic element that can be abstracted in classical form to leave a new statistical residue.

If empirical generalizations are no more than probable, what about the particular facts that ground them? Here a distinction seems necessary. In so far as such facts are expressed in the terms of ordinary description, they fall under the criteria of the concrete judgment of fact. In so far as they are relevant to be the establishment of a scientific theory, they come under the control of empirical method. What has to be observed is, not the percept with its spontaneous integration into the processes of sensitive living, but the sheer datum that is stripped of non-scientific memories, associations, and anticipations. Again, measurements must conform to the best available rules and utilize the best available instruments. Finally, the observables have to be the terms defined by the theoretical structure, and as this structure is subject to revision, so also are its definitions. Hence, one may say that empirical science is solidly grounded in fact in virtue of its concrete judgments and, at the same time, one may add that technical developments and theoretical advance can render such facts more or less obsolescent.

But if empirical science is no more than probable, still it truly is probable. If it does not attain definitive truth, still

it converges upon truth. This convergence, this increasing approximation, is what is meant by the familiar phrase, the advance of science. Questions yield insights that are expressed in hypotheses: the testing of hypotheses raises further questions that generate complementary insights and more satisfactory hypotheses. For a while the process advances in widening circles; then the coherence of system begins to close in; investigation turns from fresh ventures in new fields to the labor of consolidation, of working out implications fully, of settling issues that leave the general view unchanged. The self-correcting process of learning is palpably approaching a limit, ~~so that, at the very time radical proposals for revision become again~~ possible, the open minds of scientists are becoming closed. As Max Planck put it: "A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar ~~with it." Scientific Autobiography and Other Papers, New York 1949.~~

An ulterior question may be raised. Is scientific ~~process~~ progress indefinite? Does the self-correcting process of learning reach one limit only to discover, sooner or later, that there are further developments to be effected? If I am unable to answer this question directly, still certain observations seem relevant.

First, the advance of science through increasing accuracy would seem to head towards a limit. A measurement is not a point but an interval, not simply a number but a number plus or minus some quantity determined by a theory of errors. Hence increasing accuracy has to result from the invention of new techniques and

instruments and, while such inventions may go well beyond our present anticipations, still we have no reason to expect an infinite series of them. Once such possibilities become exhausted, the canon of selection comes into play. Empirical method settles only the theoretical differences that imply sensible differences. If a second theory supplants a first by advancing from the second decimal place to the fourth, and a third supplants the second by advancing from the fourth decimal place to the sixth, it does not follow that there can be a mere nth theory established by advancing from 2n decimals to (n + 2), where n is as large a number as you please.

Secondly, as the advance of science has a lower limit in the field of presentations, so also it has an upper limit in the basic structure of the human mind. Theories can be revised if there is a revisor. But to talk about revising the revisers is to enter a field of empty speculation in which the name, revision, loses its determinate meaning. Moreover, theorists take advantage of this fact. Thus, the foundations of logic are placed in the inevitabilities of our processes of thought. Nor is logic a unique example. As we have already indicated, the theory of relativity in its basic postulate rests upon a structural feature of our cognitional process. Now if the invariants governing mental process imply invariants in our theoretical constructions, there will follow an upper limit to the variation of theoretical constructions and a possibility of mapping out in advance the alternatives between which theoretical effort has to choose. To this topic we return in investigating what will be named the elementary categories of the

range of proportionate being.

In conclusion, it may be noted that these considerations confirm the positive probability of the conclusions of empirical science. For those conclusions are probable inasmuch as the self-correcting process of learning is approaching a limit. Our argument was based upon the immanent tendency of the process itself to a limit, inasmuch as each great stage of scientific development heads for the closed coherence of system, and each successive system grips the facts with greater nuance and accuracy over wider expanses of data. Still this immanent tendency receives confirmation if there exist external limitations to the process itself. For they too, point to the possibility of some system, as yet unknown, that is increasingly determined inasmuch as it will have to meet the requirement of verification in a body of fact that is increasingly large and increasingly organized.

7. ANALYTIC PROPOSITIONS AND PRINCIPLES

A proposition is what is proposed either for consideration or for affirmation. An analysis of propositions is reached by distinguishing what is meant from acts of meaning and from sources of meaning. Any cognitional activity is a source of meaning. Conceiving, judging, and uttering are three quite different acts of meaning. Finally, as sources lead to acts of meaning, so acts refer to terms of meaning, to what is meant.

Terms of meaning may be divided in two ways. There is the basic distinction between what is meant when one affirms or denies and, on the other hand, what is meant when one merely considers, supposes, defines. Again, in utterances there is the obvious distinction between the incomplete meaning of a word and the complete meaning of a sentence. So one is led to distinguish 1) partial terms of meaning, 2) rules of meaning, 3) formal terms of meaning, and 4) full terms of meaning.

The full term of meaning is what is affirmed or denied.

The formal term of meaning is what could be affirmed or denied but, in fact, is merely supposed or considered.

The partial term of meaning is what is meant by a word or by a phrase.

Rules of meaning govern the coalescence of words and phrases into the complete sense that may be supposed or considered, affirmed or denied.

There results at once a particular case of the virtually unconditioned. A formal term of meaning provides the conditioned.

The definitions of its partial terms provide the fulfilling conditions. And the rules of meaning provide the link between the conditions and the conditioned. Such propositions are termed analytic.

Thus, if A is defined by a relation, R, to B, and B is defined by the converse relation, R', to A, then by the rules of meaning it follows that there cannot be an A without the relation, R, to B, and that there cannot be a B wit out the relation, R', to A. Such conclusions resting on definitions and rules of meaning are analytic propositions.

Fourthly, since the analytic proposition is an instance of the virtually unconditioned, reflective understanding will find in it its proper object and thereby ground a judgment. There then arises a further question. What precisely is the meaning or force or implication of such a judgment?

It would seem that its meaning is not assertoric but hypothetical. If there occur suppositions or judgments containing significant terms in the same sense as they are assigned in the analytic proposition, then such suppositions or judgments must be consistent with the analytic proposition; moreover, when that condition and other logical requirements are met, there follow valid inferences. On the other hand, the mere fact that a proposition is analytic offers no guarantee that its terms in their defined sense occur in any supposition or judgment apart from the affirmation of the analytic proposition.

It follows that analytic propositions remain in sterile isolation unless there accrues to them some form of validation. This will consist in the occurrence of the same terms in their defined

sense in some other supposition or judgment; and the precise nature of the validation will depend upon the nature of the added supposition or judgment.

There also follows the explanation of the fact that analytic propositions can be produced more or less at will and indefinitely. Partial terms of meaning are a vast multitude and further partial terms can be supplied by the art of definition. Rules of meaning provide a principle of selection of the partial terms that will coalesce into analytic propositions. And if this seems to require too much ingenuity, the task can be simplified by using symbols instead of words and by defining them by their relations in propositions. But significant increments of knowledge are not to be obtained by mere ingenuity and, in fact, the analytic proposition, by itself, is not a significant increment of knowledge; without the fulfilment of further conditions it remains in isolation and fails to enter fruitfully into the texture of knowing.

Hence, we are in substantial agreement with the contemporary view that mere analytic propositions are ~~mere~~ tautologies. The use of the term, tautology, would seem to be incorrect, but the general meaning of the statement is sound. However, it may not be out of place to add that the present point was made centuries ago. Aquinas advanced that conclusions depend upon principles and that principles depend upon their terms; but he was not ready to accept any terms whatever; he added that proper terms are selected by wisdom (I-II, 66, 5, 4m)* and by wisdom he meant an accumulation of insights that extends to the universe as common sense stands to the domain of the particular, incidental, relative, and imaginable.

*Sum. Theol. I-II, q. 66, a. 5, ad 4m.

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Let us now turn from analytic propositions to analytic principles.

By analytic principle is meant an analytic proposition of which the partial terms are existential; further, the partial terms of an analytic proposition are existential if they occur in their defined sense in judgments of fact, such as the concrete judgment of fact or the definitively established empirical generalization.

Further, since such analytic principles are hard to come by, we shall also speak of the mitigated cases.

The provisional analytic principle is an analytic proposition of which the terms are probably existential, that is, they occur in probable empirical generalizations.

The serial analytic principle is an analytic proposition of which the terms are serially existential; what is meant by the serially existential, will be clarified in our next section on mathematical judgments.

It may be remarked that the analytic principle also connotes in its terms not only an existential reference but also a basic, primitive character. I think this feature will be found to follow from the defined requirements for, as we shall proceed to argue, analytic principles lie pretty well outside the reach of common sense and empirical science.

They lie outside the reach of common sense because analytic principles are universal and common sense regards the particular. Common sense makes concrete judgments of fact and it passes judgment on the correctness of insights into concrete situations.

But in neither case does it employ terms in the sense assigned them by abstract definitions. As Socrates discovered, the average man does not define: he is suspicious of the search for definitions; and when that pursuit brings out the inference that he does not know what he is talking about, he is rather resentful.

The fact would seem to be that the structure of common sense meanings is much the same as the structure of common sense itself. There is a communal collaboration that yields a habitual core of understanding and, as well, a range of concepts and linguistic terms in ordinary use. But just as the common core of understanding has to be adjusted by complementary insights into the present, concrete situation before judgment occurs, so also common concepts and terms receive their ultimate complement of meaning from those complementary insights.

"This is a dog". "What do you mean by a 'dog'?" The question supposes that the term, "dog," has a precise meaning outside the series of statements in which it occurs. But in fact what comes first is the series of statements and what comes only later, and then only if one goes in for analysis, is the determination of the precise meaning of the single, partial term. What the average man means by a "dog" is 1) what he would with certainty pronouce to be a dog in any concrete situation with which he is familiar, 2) what he could learn to be to a "dog", and 3) what he would be willing to believe is a "dog". Hence, it is that a dictionary is constructed, not by the Socratic art of definition, but by the pedestrian, inductive process of listing sentences in which each word occurs in good usage.

It may be objected that one cannot make a brick house without first making bricks. But one is only arguing from a false analogy if one claims that the mind develops in the same fashion as the wall of a house is built. Prior to concepts there are insights. A single insight is expressed only by uttering several concepts. They are uttered in conjunction, and reflection pronounces whether the insight and so the conjunction is correct. The isolation and definition of concepts is a subsequent procedure and common sense does not undertake it.

Because we have denied that common sense reaches analytic principles, it is not to be inferred that the average man has no principles. Analytic principles suppose analysis; analysis supposes accurate conceptualization. But prior to analysis, to concepts, to judgments, there are the native endowments of intelligence and reasonableness and the inherent structures of cognitional process. These are the real principles on which the rest depends. Moreover, all understanding has its universal aspect, for similars are similarly understood. But it is one thing to exploit this universal aspect in a professional manner; it is another to exploit the intelligibility, which is by itself universal, by adding further intelligibilities until one comes to grips with concrete situations. The latter line of development we have named common sense so that, by definition, common sense deals with the particular. Again, the latter line of development is conspicuous in the average man. But what else the average man knows and how he knows it, are further questions. As has been remarked already, one cannot treat all issues at the same time.

Next, analytic principles lie outside the reach of empirical science. It is true of course, that every insight yields several concepts linked together through the insight; it also is true that the empirical scientist formulates definitions, postulates, and inferences; but the trouble is that the empirical scientist knows his insights not as certainly correct but only as probable. Hence his defined terms, in the sense they are defined, are as much subject to revision as the probable judgments of fact that contain them and validate them.

Thus, consider the assertions: 1) water probably is H_2O; 2) what I mean by water is H_2O; 3) this water contains impurities; 4) there are two kinds of water, heavy and ordinary.

The first is an empirical conclusion. The second is a definition. The third is a concrete judgment of fact; its meaning is that this sample is water in the sense of the empirical conclusion but it is not solely water in the sense of the definition. The fourth introduces a new basis of definition that has its ground in fresh experimental work. Now both the initial definition and the later definitions yield analytic propositions, namely, that what does not satisfy certain specifications is not pure water, or it is not pure water of molecular weight eighteen, or it is not pure heavy water. Moreover, none of these are merely analytic propositions; they are not the sort of thing that can be produced at will and indefinitely. On the other hand, they are not strictly analytic principles, for though their terms possess validating judgments of fact, still those judgments are subject to revision, and, indeed, the discovery of heavy water has already forced such a revision.

Generally one may say that the advance of empirical science is an instance of the advance of the self-correcting process of learning. But in this instance the previous insights yield correlations, definitions, and inferences. It is in terms of such formulations that are framed the further questions that will complement and modify the previous insights by later insights. In like manner the later insights receive their formulation which is presupposed by the further questions that lead to a still fuller understanding. Now in this process the successive formulations have three distinct aspects. First, they are the expression of insights that grasp the intelligible form of data; thus, they are probable empirical conclusions. Secondly, they are the presupposition of the further questions that lead to further insights; from this viewpoint they are provisional analytic principles. Thirdly, they are revised in the light of the further insights and so cease to be probable empirical conclusions and provisional analytic principles to pass into the limbo of the analytic propositions whose terms have no existential reference.

The reader interested in further illustrations of this process will find numerous examples in Arthur Pap's "The A Priori in Physical Theory", New York, 1946.

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8. MATHEMATICAL JUDGMENTS

In mathematical thought one may readily discern the difference between operations on the level of intelligence and operations on the level of reflection.

The level of intelligence is the level of discovery and invention, of definition and learning, of grasping problems and coming to grasp their solutions, of seeing the point made in each of a series of mathematical statements and then seeing how the successive points hang together.

The level of reflection is the complementary process of checking. One understands and now one wishes to know whether what is understood is also correct. One has grasped the point and one asks whether it is right. One has seen how the successive steps hang together and one is out to make sure that what hangs together is really cogent.

Now the process of checking can be developed into an elaborate technique. What is checked becomes a whole department of mathematics. Definitions are worked out. Postulates are added. From the definitions and postulates it is shown that all the conclusions of the department can be reached by the rigorous procedure of deductive inference.

But what is the goal of checking? Clearly, it is to marshal the evidence in the shape in which reflective understanding can grasp the virtually unconditioned and so ground rational judgment. In so far as the checking reduces conclusions to premises, there we have is the virtually unconditioned of the form of deductive inference.

In so far as the definitions and postulates coalesce into a self-justifying meaning, there is the virtually unconditioned of analytic propositions. Both of these types of the virtually unconditioned have already been considered and so, for us, the problem of mathematical judgment consists in determining what else is required for such judgment.

First of all, something else is required. For if the premises of mathematical thought are analytic propositions, still not all analytic propositions are mathematical premises. Analytic propositions can be produced at will and indefinitely. But the premises of mathematical thought are to be reached only through the discoveries of genius and the labor of learning what genius has grasped. Further, it does happen that abstruse regions of mathematics are occasionally pulled out of their cold and airy realms to become the tools of empirical hypotheses and theories and to share with such formulations the probable existential reference that they possess. But prior to a probable existential reference or isomorphism there is a possible existential reference or isomorphism; before a department of mathematics can be applied, it must possess an inherent possibility of being applied. What, then, is that inherent possibility? And what is its criterion?

Secondly, we have to undertake an examination of mathematics to determine what this further element is and what its criterion is. It is to say, then, that there is a mathematical series, that each term in the series is a department of mathematics, that each department consists 1) of rules governing and so defining operations and 2) of operations proceeding from some terms to

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9. SUMMARY

Prospective judgments are propositions 1) that are the content of an act of conceiving, thinking, defining, considering, or supposing, 2) that are subjected to the question for reflection, to the critical attitude of intelligence, and 3) that thereby are constituted as the conditioned.

There is sufficient evidence for a prospective judgment when it may be grasped by reflective understanding as virtually unconditioned. Hence sufficient evidence involves 1) a link of the conditioned to its conditions, and 2) the fulfilment of the conditions. Those two elements are supplied in different manners in different cases.

In formal inference the link is provided by the hypothetical premise: If the antecedent, then the consequent. The fulfilment is the minor premise.

In judgment on the correctness of insights, the link is that the insight is correct if there are no further, pertinent questions, and the fulfilment lies in the self-correcting process of learning reaching its limit in familiarity and mastery.

In judgments of fact the link is the correct insight or set of insights and the fulfilment lies in present and/or remembered data.

In generalizations the link is the conditional law that similars are similarly understood and the fulfilment lies in such similarity that further, pertinent questions no more arise in the general case than in the correctly understood particular case.

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In probable judgments the link is that insights are correct when there are no further pertinent questions and the fulfilment is some approximation of the self-correcting process of learning to its limit of familiarity and mastery.

In analytic propositions the link lies in rules of meaning that generate propositions out of partial terms of meaning and the fulfilment is supplied by the meanings or definitions of the terms.

Analytic propositions become analytic principles when their terms are existential; and terms are existential when they occur in definitive, factual judgments.

Provisional analytic principles are analytic propositions whose terms are probably existential.

~~Serial analytic principles are analytic propositions whose terms are a series, some of which, in some fashion, formally or virtually, actually or approximately, normally or under particular laws, are existential.~~

Serially analytic principles are the analytic propositions from which follow the ranges of systems some of which in some fashion exist.

Part II: Insight as Knowledge.

Chapter XI

SELF-AFFIRMATION OF THE KNOWER

It is time to turn from theory to practice. Judgment has been analyzed. Its grounds in reflective understanding have been explored. Clearly the next question is whether correct judgments occur, and the answer to it is the act of making one.

Since our study has been of cognitional process, the judgment we are best prepared to make is the self-affirmation of an instance of such a process as cognitional. By the "self" is meant a concrete and intelligible unity-identity-whole. By "self-affirmation" is meant that the self both affirms and is affirmed. By "self-affirmation of the knower" is meant that the self as affirmed is characterized by such occurrences as sensing, perceiving, imagining, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and affirming.

The affirmation to be made is a judgment of fact. It is not that I exist necessarily, but merely that in fact I do. It is not that I am of necessity a knower, but merely that, in fact, I am. It is not that an individual performing the listed acts really does know, but merely that I perform them and that by "knowing" I mean no more than such performance.

As all judgment, self-affirmation rests upon a grasp of the unconditioned. The unconditioned is the combination of 1) a conditioned, 2) a link between the conditioned and its conditions, and, 3) the fulfilment of the conditions. The relevant conditioned

is the statement, I am a knower. The link between the conditioned and its conditions may be cast in the proposition, I am a knower, if I am a concrete and intelligible unity-identity-whole, characterized by acts of sensing, perceiving, imagining, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and judging. The fulfilment of the conditions is given in consciousness.

The conditional offers no difficulty. It is merely the expression of what is to be affirmed. Similarly, the link offers no difficulty; the link itself is a statement of meaning; and the conditions which it lists have become familiar in the course of this investigation. The problematic element, then, lies in the fulfilment of the conditions and we proceed to indicate what is meant and not meant by consciousness and by the fulfilment of conditions.

1. The Notion of Consciousness

First, consciousness is not to be thought of as some sort of inward look. People are apt to think of knowing by imagining a man taking a look at something and, further, they are apt to think of consciousness by imagining themselves looking into themselves. Not merely do they indulge in such imaginative opinions but also they are likely to justify them by argument. Knowing, they will say, is knowing something: it is being confronted by an object: it is the strange, mysterious, irreducible presence of one thing to another. Hence, though knowing is not exclusively a matter of ocular vision, still it is radically that sort of thing. It is gazing, intuiting, contemplating. Whatever words you care to employ, consciousness is a knowing and so it is some sort of inward looking.

Now while consciousness is a factor in knowing, and while knowing is an activity to which a problem of objectivity is annexed, still it is one thing to give an account of the activity and it is something else to tackle the problem of objectivity. For the present we are concerned simply with an account of the activity, and so we have defined the knower, not by saying that he knows something, but solely by saying that he performs certain kinds of acts. In like manner, we have not asked whether the knower knows himself: we ask solely whether he can perform the act of self-affirmation. Hence, while some of our readers may possess the rather remarkable power of looking into themselves and intuiting things quite clearly and distinctly, we shall not base our case upon their success. For, after all, there may well exist other readers that, like the writer, find looking into themselves ~~is remote that, if not intrinsically wrong, is clearly wrong~~ rather unrewarding.

Secondly, by consciousness, we shall mean that there is an awareness immanent in cognitional acts. Already a distinction has been drawn between act and content, for instance, between seeing and color, hearing and sound, imagining and image, insight and idea. To affirm consciousness is to affirm that cognitional process is not merely a procession of contents but also a succession of acts. It is to affirm that the acts differ radically from such unconscious acts as the metabolism of one's cells, the maintonance of one's organs, the multitudinous biological processes that one learns about through the study of contemporary medical science. Both kinds of acts occur, but the biological occur outside consciousness, and the cognitional occur within consciousness. Seeing is not merely

a response to the stimulus of color and shape: it is a response that consists in becoming aware of color and shape. Hearing is not merely a response to the stimulus sound; it is a response that consists in becoming aware of sound. As color differs from sound, so seeing differs from hearing. Still seeing and hearing have a common feature, for in both occurrences there is not merely content but also conscious act.

By the conscious act is not meant a deliberate act; we are conscious of acts without debating whether we will perform them. By the conscious act is not meant an act to which one attends; consciousness can be heightened by shifting attention from the content to the act; but consciousness is not constituted by that shift of attention, for it is a quality immanent in acts of certain kinds, and without it the acts would be unconscious as the growth of one's beard. By the conscious act is not meant that the act is somehow isolated for inspection, nor that one grasps its function in cognitional process, nor that one can assign it a name, nor that one can distinguish it from other acts, nor that one is certain of its occurrence.

Does, then, "conscious act" mean no more than "cognitional act"? A distinction has to be drawn. First, I do not think that only cognitional acts are conscious. Secondly, there are those that would define "seeing" as "awareness of color" and then proceed to argue that in seeing one was aware of color but of nothing else whatever, that "awareness of color" occurs but that a concomitant "awareness of awareness" is a fiction. This, I think, does not accurately reflect the facts. If seeing is an awareness of nothing

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but color and hearing is an awareness of nothing but sound, why are both named "awareness"? Is it because there is some similarity between color and sound? Or is it that color and sound are disparate, yet with respect to both there are acts that are similar? In the latter case, what is the similarity? Is it that both acts are occurrences, as metabolism is an occurrence? Or is it that both acts are conscious? One may quarrel with the phrase, awareness of awareness, particularly if one imagines awareness to be a looking and finds it preposterous to talk about looking at a look. But one cannot deny that, within the cognitional act as it occurs, there is a factor or element or component over and above its content, and that this factor is what differentiates cognitional acts from unconscious occurrences.

2. Empirical, Intelligent, and Rational Consciousness.

Briefly, By consciousness is meant an awareness immanent in cognitional acts. But such acts differ in kind, and so the awareness differs in kind with the acts. There is an empirical consciousness characteristic of sensing, perceiving, imagining. As the content of these acts is merely presented or represented, so the awareness immanent in the acts is the mere givenness of the acts. But there is an intelligent consciousness characteristic of inquiry, insight, and formulation. On this level cognitional process not merely strives for and reaches the intelligible, but in doing so it exhibits its intelligence: it operates intelligently. The awareness is present but it is the awareness of intelligence, of what strives to understand, of what is satisfied by understanding, of what formulates the understood, not as a schoolboy repeating by rote a definition, but as one that defines because he grasps why

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that definition hits things off. Finally, on the third level of reflection, grasp of the unconditioned, and judgment, there is rational consciousness. It is the emergence and the effective operation of a single law of utmost generality, the law of sufficient reason, where the sufficient reason is the unconditioned. It emerges as a demand for the unconditioned and a refusal to assent unreservedly on any lesser ground. It advances to grasp of the unconditioned. It terminates in the rational compulsion by which grasp of the unconditioned ~~yields~~ commands assent.

Empirical consciousness needs, perhaps, no further comment, for by it we illustrated the difference between conscious and unconscious acts. Intelligent and rational consciousness, on the other hand, may be clarified by a contrast. In their different manners both common sense and positive science view the material world as subject to intelligible patterns and as governed by some law of causality. To confine our attention to what man knows best, namely, his own artefacts, there is discernible in them an intelligible design and their existence has its ground in the labor of production. But before the design is realised in things, it was invented by intelligence; before the sequence of productive operations was undertaken, it was affirmed as worth while for some sufficient or apparently sufficient reason. In the thing there is the intelligible design, but in the inventor there was not only the intelligibility on the side of the object but also intelligent consciousness on the side of the subject. In the thing there is the groundedness that consists in its existence being accounted for by a sequence of operations; but in the entrepreneur there was

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not only the groundedness of his judgment in the reasons that led to it but also the rational consciousness that required reasons to reach judgment. ¶ Intelligence and intelligibility are the obverse and reverse of the second level of knowing: intelligence looks for intelligible patterns in presentations and representations; it grasps such patterns in its moments of insight; it exploits such grasp in its formulations and in further operations equally guided by insights. In like manner, reasonableness and groundedness are the obverse and reverse of the third level of knowing. Reasonableness is reflection inasmuch as it seeks groundedness for objects of thought; reasonableness discovers groundedness in its reflective grasp of the unconditioned; reasonableness exploits groundedness when it affirms objects because they are grounded. In man's artefacts there are the reverse elements of the intelligibility and groundedness, but there are not the obverse elements of intelligence and reasonableness. The obverse elements pertain to cognitional process on its second and third levels; they do not pertain to the contents emergent on those levels, to the idea or concept, to the unconditioned or affirmed: on the contrary, they characterize the acts with which those contents are coupled and so they are specific differentiations of the awareness of consciousness. Clear and distinct conception not only reveals the intelligibility of the object but also manifests the intelligence of the subject. Exact and balanced judgment not only affirms things as they are but also testifies to the dominance of reasonableness in the subject.

Still, it may be asked, Am I really conscious of intelligence

and reasonableness? The question, I think, is misleading. It suggests that there is a type of knowing in which intelligence and reasonableness come up for inspection. But what is asserted is not that you can uncover intelligence by introspection, as you can point to Calcutta on a map. The assertion is that you have conscious states and conscious acts that are intelligent and reasonable. Intelligent and rational consciousness denote characters of cognitional process, and the characters they denote pertain not to the contents but to the proceeding. It is repugnant to me to place astrology and astronomy, alchemy and chemistry, legend and history, hypothesis and fact, on exactly the same footing. I am not content with theories, however brilliantly coherent, but insist on raising the further question, Are they true? What is that repugnance, that discontent, that insistance? They are just so many variations on the more basic expression that I am rationally conscious, that I demand sufficient reason, that I find it in the unconditioned, that I assent unreservedly to nothing less, that such demanding, finding, self-committing occur, not like the growth of my hair, but within a field of consciousness or awareness. Again, if at moments I can slip into a lotus land in which mere presentations and representations are juxtaposed or successive, still that is not my normal state. The Humean world of mere impressions comes to me as a puzzle to be pieced together. I want to understand, to grasp intelligible unities and relations, to know what's up and where I stand. Praise of the scientific spirit that inquires, that masters, that controls, is not without an echo, a deep resonance within me, for, in my more modest way, I too, inquire and catch on,

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see the thing to do and see that it is properly done. But what are these but variations on the more basic expression that I am intelligently conscious, that the awareness characteristic of cognitional acts on the second level is an active contributing to the intelligibility of its products? When I listen to the story of Archimedes and when I read the recital of a mystical experience, there is a marked difference. What a mystic experiences, I do not know. But, though I never enjoyed so reasonable an insight as Archimedes, still I do know what it is to miss the point and to get the point, not to have a clue and then to catch on, to see things in a new light, to grasp how they hang together, to come to know why, the reason, the explanation, the cause. After Archimedes shouted "I've got it", he might well be puzzled by the question whether he was conscious of an insight. Still there can be no doubt that he was conscious of an increment of knowledge, an increment that he had wanted very much. Did he want the king's favor? Did he want to enhance his reputation? Perhaps, but at a deeper and more spontaneous level, he wanted to know how to do something; he wanted to solve a problem; he wanted to understand; his consciousness was on the second level where it seeks the intelligible and follows up partial insights with further questions until there comes the final crowning insight that ends questioning and satisfies intelligent consciousness.

3. The Unity of Consciousness.

In the fourth place, there are unities of consciousness. Besides cognitional contents there are cognitional acts; different kinds of acts have different kinds of awareness, empirical, intelligent, rational. But the contents cumulate into unities: what

is perceived is what is inquired about; what is inquired about is what is understood; what is understood is what is formulated; what is formulated is what is reflected on; what is reflected on is what is grasped as unconditioned; what is grasped as unconditioned is what is affirmed. Now, just as there are unities on the side of the object, so there are unities on the side of the subject. Conscious acts are not so many isolated, random atoms of knowing, but many acts coalesce into a single knowing. Not only is there a similarity between my seeing and your hearing, inasmuch as both acts are conscious; there also is an identity involved when my seeing and my hearing or your seeing and your hearing are compared. Moreover, this identity extends all along the line. Not only is the percept inquired about, understood, formulated, reflected on, grasped as unconditioned, and affirmed, but also there is an identity involved in perceiving, inquiring, understanding, formulating, reflecting, grasping the unconditioned, and affirming. Indeed, consciousness is much more obviously of this unity in diverse acts than of the diverse acts, for it is within the unity that the acts are found and distinguished, and it is to the unity that we appeal when we talk about a single field of consciousness and draw a distinction between conscious acts occurring within the field and unconscious acts occurring outside it.

One might go further and argue that, were the unity of consciousness not given, then it would have to be postulated. For many contents on diverse levels cumulate into a single known. But how ~~can that occur~~? How can images be derived from sensations? How can inquiry ~~be~~ about percepts? How can insight ~~be~~ into images?

How can definition draw upon both images and the ideas grasped in insight? How can reflecting be about formulations? How can the grasp of the unconditioned be obtained by combining the conditioned that is thought and the fulfilment that is sensed? How can each judgment emerge in a context of other judgments that determine its meaning, complement it, qualify it, defend it, so that it is but a single increment within a far vaster knowing? I cannot inquire into your experience or reflect on your thoughts. But if there were no "I", how could there be a "my experience" with respect to which a "my inquiry " occurred, or "my thoughts" with respect to which "my reflection" occurred? If there were not one consciousness, at once empirical, intelligent, and rational, how could rational judgment proceed from an unconditioned grasped in the combination of thought and sensible experience?

4. The Unity As Given

Still, if the unity of consciousness would have to be postulated on the hypothesis that it were not given, it remains that it is given. By this, of course, I do not mean that it is the object of some inward look. What is meant is that a single agent is involved in many acts, that it is an abstraction to speak of the acts as conscious, that concretely, consciousness pertains to the acting agent. Seeing and hearing differ inasmuch as one is an awareness of color and the other an awareness of sound. Seeing and hearing are similar inasmuch as each is an awareness. But the similarity between my seeing and your hearing is an abstract indication of consciousness which, as it is given, is primarily an identity uniting my seeing and my hearing or your seeing and your hearing.

We have been engaged in determining what precisely is

meant by consciousness. We have contended that it is not some inward look but a quality of cognitional acts, a quality that differs on the different levels of cognitional process, a quality that concretely is the identity immanent in the diversity and the multiplicity of the process. However, one cannot insist too strongly that such an account of consciousness is not itself consciousness. The account supposes consciousness as its data for inquiry, for insight, for formulation, for reflection, for grasp of the unconditioned, for judgment. But giving the account is the formulating and the judging, while the account itself is what is formulated and affirmed. Consciousness as given is neither formulated nor affirmed. Consciousness is given independently of its being formulated or affirmed. To formulate it does not make one more conscious, for the effect of formulation is to add to one's concepts. To affirm it, does not make one more conscious, for the effect of affirmation is to add to one's judgments. Finally, as consciousness is not increased by affirming it, so it is not diminished by denying it, for the effect of denying it is to add to the list of one's judgments and not to subtract from the grounds on which judgments may be based.

By such experiential fulfilment, then, one does not mean the conditioned, nor the link between the conditioned and its conditions, nor the conditions as formulated, let alone as affirmed. One does mean that the conditions, which are formulated, also are to be found in a more rudimentary state within cognitional process. Just as inquiry brings about the advance from the perceived and not understood to the perceived and understood, so there is a reverse

shift by which one moves from the perceived and understood to the merely perceived. It is this reverse shift that commonly is meant by verification. If from a more general theory I obtain the formula, $PV = 64$, then I can infer that when P is 2, 4, 8, 16, 32, V will have theoretically the values 32, 16, 8, 4, 2. By setting up suitable apparatus and securing appropriate conditions defined by the theory, I can advance from theoretical inference to an experimental check. The results of the experiment may be expressed in a series of propositions, such as the statement that, when P was approximately 2, V was approximately 32, but such a series of statements, however accurate, is not what was given by the experiment. The statements represent judgments of fact; the judgments rest on grasping the unconditioned; the grasp rests on formulations and visual experiences. The experiment gives neither statements nor judgments nor reflective understanding nor formulations but only visual experiences. The experiment gives not visual experiences as described but visual experiences on the level of merely seeing. That P is 2 when the needle on a dial stands at a certain place, is a judgment. That V is 32 when certain dimensions of an object coincide with certain dimensions of a measuring rod is another judgment. All that is seen, is the needle in a position on the dial ~~and~~ the dimensions of an object standing in coincidence with numbered units on a rod. Nor is it this description that is seen, but only what is so described. In brief, verification is an appropriate pattern of acts of checking; acts of checking are reversals from formulations of what would be perceived to the corresponding but more rudimentary cognitional contents of acts of perceiving or

sensing. In the formulation there always are elements derived from inquiry, insight, conceiving. But in virtue of the checking one can say that the formulation is not pure theory, that it is not merely supposed or merely postulated or merely inferred, that its sensible component is given.

Now just as there is reversal to what is given sensibly, so there is reversal to what is given consciously. Just as the former reversal is away from the understood as understood, the formulated as formulated, the affirmed as affirmed, and to the merely sensed, so also the latter reversal is from the understood, formulated, affirmed as such, to the merely given. Hence, in the self-affirmation of the knower, the conditioned is the statement, I am a knower. The link between the conditioned and its conditions is cast in the proposition, I am a knower if I am a unity performing certain kinds of acts. The conditions as formulated are the unity-identity-whole to be grasped in data as individual and the kinds of acts to be grasped in data as similar. But the fulfilment of the conditions in consciousness is to be had by reverting from such formulations to the more rudimentary state of the formulated where there is no formulation but merely experience.

5. Self-Affirmation

From preliminary clarifications, we turn to the issue. Am I a knower? Each has to ask the question of himself. But anyone who asks it, is rationally conscious. For the question is a question for reflection, a question to be met with a "Yes" or "No"; and asking the question does not mean repeating the words but entering the dynamic state in which dissatisfaction with mere theory manifests itself in a demand for fact, for what is so. Further,

the question is not any question. If I ask it, I know what it means. What do I mean by I? The answer is difficult to formulate, but strangely, in some obscure fashion, I know very well what it means without formulation, and by that obscure yet familiar awareness, I find fault with various formulations of what is meant by "I". In other words, "I" has a rudimentary meaning from consciousness and it envisages neither the multiplicity nor the diversity of contents and conscious acts but rather the unity that goes along with them. But if "I" has some such rudimentary meaning from consciousness, then consciousness supplies the fulfilment of one element in the conditions for affirming that I am a knower. Does consciousness supply the fulfilment for the other conditions? Do I see, or am I blind? Do I hear, or am I deaf? Do I try to understand or is the distinction between intelligence and stupidity no more applicable to me than to a stone? Have I any experience of insight, or is the story of Archimedes as strange to me as the account of Plotinus' vision of the One? Do I conceive, think, consider, suppose, define, formulate, or is my talking like the talking of a parrot? I reflect, for I ask whether I am a knower. Do I grasp the unconditioned, if not in other instances, then in this one? If I grasped the unconditioned, would I not be under the rational compulsion of affirming that I am a knower and so, either affirm it, or else find some loophole, some weakness, some incoherence, in this account of the genesis of self-affirmation? As each has to ask these questions of himself, so too, he has to answer them for himself. But the fact of the asking and the possibility of

the answering are themselves the sufficient reason for the affirmative answer.

6. Self-affirmation as Immanent Law.

The foregoing account of self-affirmation stresses its positive aspect. It is a judgment of fact and so it rests heavily upon the experiential component in knowing. Still it is a singular type of judgment for it possesses a variety of overtones. I might not be, yet if I am, I am. I might be other than I am, yet, in fact, I am what I am. The contingent, if you suppose it as a fact, becomes conditionally necessary, and this piece of elementary logic places the merely factual self-affirmation in a context of necessity.

Am I a knower? The answer, Yes, is ≠ coherent, for if I am a knower, I can know that fact. But the answer, No, is incoherent, for if I am not a knower, how could the question be raised and answered by me? No less, the hedging answer, I do not know, is incoherent. For if I know that I do not know, then I am a knower; and if I do not know that I do not know, then I should not answer.

Am I a knower? If I am not, then I know nothing. My only course is silence. My only course is not the excused and explained silence of the skeptic, but the complete silence of the animal that offers neither excuse nor explanation for his complacent absorption in merely sensitive routines. For if I know nothing, I do not know excuses for not knowing. If I know nothing, then I cannot know the explanation of my ignorance.

It is this conditional necessity of contingent fact that involves the talking skeptic in contradiction. If enthusiasm for the achievement of Freud were to lead me to affirm that all thought

and affirmation is just a by-product of the libido, then since I have admitted no exceptions, this very assertion of mine would have to be mere assertion from a suspect source. If second thoughts lead me to acknowledge an exception, they lead me to acknowledge the necessary presuppositions of the exception. By the time that list has been drawn up and accepted, I am no longer a skeptic.

Still the Aristotelian prescription of getting the skeptic to talk derives its efficacy not only from the conditional necessity of contingent fact but also from the nature, the natural spontaneities and natural inevitabilities, that go with that fact. Why is it that the talking skeptic does not talk gibberish? Why is it that one can count on his being nonplussed by self-contradiction? It is because he is conscious, empirically, intelligently, and rationally. It is because he has no choice in the matter. It is because extreme ingenuity is needed for him not to betray his real nature. It is because, were his ingenuity successful, the only result would be that he had revealed himself an idiot and lost all claim to be heard.

This aspect of the matter deserves further attention. Cognitional process does not lie outside the realm of natural law. Not merely do I possess the power to elicit certain types of acts when certain conditions are fulfilled, but also with statistical regularity the conditions are fulfilled and the acts occur. I cannot escape sensations, percepts, images. All three keep occurring during my waking hours, and the images often continue during my sleep. No doubt, I can exercise a selective control over what I sense, perceive, imagine. But the choice I cannot make effective

is to sense nothing, perceive nothing, imagine nothing. Not only are the contents of these acts imposed upon me, but also consciousness in some degree is inseparable from the acts. Nor is that consciousness merely an aggregate of isolated atoms; it is a unity.

If I cannot escape presentations and representations, neither can I be content with them. Spontaneously I fall victim to the wonder that Aristotle named the beginning of all science and philosophy. I try to understand. I enter, without questioning, the dynamic state that is revealed in questions for intelligence. Theoretically there is a disjunction between "being intelligent" and "not being intelligent". But the theoretical disjunction is not a practical choice for me. I can deprecate intelligence; I can ridicule its aspirations; I can reduce its use to a minimum; but it does not follow that I can eliminate it. I can question everything else, but to question questioning is self-destructive. I might call upon intelligence for the conception of a plan to escape intelligence, but the effort to escape would only reveal my present involvement and, strangely enough, I would want to go about the business intelligently and I would want to claim that escaping was the intelligent thing to do.

As I cannot be content with the cinematographic flow of presentations and representations, so I can not be content with inquiry, understanding, and formulation. I may say I want not the quarry but the chase, but I am careful to restrict my chasing to fields where the quarry lies. If, above all, I want to understand, still I want to understand the facts. Inevitably, the achievement of understanding, however stupendous, only gives rise to the further question,

Is it so? Inevitably, the progress of understanding is interrupted by the check of judgment. Intelligence may be a thoroughbred exulting in the race; but there is a rider on its back; and, without the rider, the best of horses is a poor bet. The insistance that modern science envisages an indefinite future of repeated revisions does not imply an indifference to fact. On the contrary, it is fact that will force the revisions, that will toss into the waste-basket the brilliant theories of previous understanding, that will make each new theory better because it is closer to the facts. But what is fact? What is that clear, precise, definitive, irrevocable, dominant something that we name ineludable fact? The question is too large to be settled here. Each philosophy has its own view on what fact is and its consequent theory on the precise nature of our knowledge of fact. All that can be attempted now is to state what we happen to mean by knowing fact.

#New paragraph

Clearly, then, fact is concrete as sense or consciousness. Again, fact is intelligible: if it is independent of all doubtful theory, it is not independent of the correct insight and formulation necessary to give it its precision and its accuracy. Finally, fact is virtually unconditioned: it might not have been; it might have been other than it was; but as things stand, it possesses conditional necessity, and nothing can possibly alter it now. Fact, then, combines the concreteness of experience, the determinateness of accurate intelligence, and the absoluteness of rational judgment. It is the natural objective of human cognitional process. It is the anticipated unity to which sensation, perception, imagination, inquiry, insight, formulation, reflection,

grasp of the unconditioned, and judgment make their several, complementary contributions. When Newton knew that the water in his bucket was rotating, he knew a fact, though he thought he knew absolute space. When quantum mechanics and relativity posit the unimaginable in a four-dimensional manifold, they bring to light the not too surprising fact that scientific intelligence and verifying judgment go beyond the realm of imagination to the realm of fact. Just what that realm is, as has been said, is a difficult and complicated problem. Our present concern is that we are committed to it. We are committed, not by knowing what it is and that it is worth while, but by an inability to avoid experience, by the subtle conquest in us of the Eros that would understand, by the inevitable aftermath of that sweet adventure when a rationality identical with us demands the absolute, refuses unreserved assent to less than the unconditioned and, when that is attained, imposes upon us a commitment in which we bow to an immanent Anagke. #Confronted with the standard of the unconditioned, the skeptic despairs. Set before it, the products of human understanding are ashamed. Great are the achievements of modern science: by far are they to be preferred to earlier guesswork; yet rational consciousness finds that they approximate indeed to the unconditioned but do not attain it; and so it assigns them the modest status of probability. Still, if rational consciousness can criticize the achievement of science, it cannot criticize itself. The critical spirit can weigh all else in the balance, only on condition that it does not criticize itself. It is a self-assertive spontaneity that demands sufficient reason for all else

but offers no justification for its demanding. It arises, fact-like, to generate knowledge of fact, to push the cognitional process from the conditioned structures of intelligence to unreserved affirmation of the unconditioned. It occurs. It will recur whenever the conditions for reflection are fulfilled. With statistical regularity those conditions keep being fulfilled. Nor is that all, for I am involved, engaged, committed. The disjunction between rationality and non-rationality is an abstract alternative but not a concrete choice. Rationality is my very dignity, and so closely to it do I cling, that I would want the best of reasons for abandoning it. Indeed, I am so much one with my reasonableness that, when I lapse from its high standards, I am compelled either to repent my folly or to rationalize it.

Self-affirmation has been considered as a concrete judgment of fact. The contradiction of self-negation has been indicated. Behind that contradiction there have been discerned natural inevitabilities and spontaneities that constitute the possibility of knowing, not by demonstrating that one can know, but pragmatically by engaging one in the process. Nor in the last resort can one reach a deeper foundation than that pragmatic engagement. Even to seek it involves a vicious circle; for if one seeks such a foundation, one employs one's cognitional process; and the foundation to be reached will be no more secure or solid than the inquiry utilized to reach it. As I might not be, as I might be other than I am, so my knowing might not be and it might be other than it is. The ultimate basis of our knowing is not necessity but contingent fact, and the fact is established, not prior to our engagement in knowing, but simultaneously with it. The skeptic, then, is not in-

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volved in a conflict with absolute necessity. He might not be; he might not be a knower. Contradiction arises when he utilizes conditional process to deny it.

7 Description + Explanation

There is a further aspect to the matter. Is the self-affirmation that has been outlined descriptive of the thing-for-us or explanatory of the thing-itself? We have spoken of natural inevitabilities and spontaneities. But did we speak of these as they are themselves or as they are for us?

Unfortunately, there is a prior question. The distinction that was drawn, earlier, between description and explanation was couched in terms that sufficed to cover the difference in the fields of positive science. But human science contains an element not to be found in other departments. Both the study of man and the study of nature begin from inquiry and insight into sensible data. Both the study of man and the study of nature can advance from the descriptive relations of the object to the inquirer, to the explanatory relations that obtain immediately between objects. Just as the physicist measures, correlates measurements, and implicitly defines correlatives by the correlations, so too, the student of human nature can forsake the literary approach to determine economic, political, sociological, cultural, historical correlations. But the study of man also enjoys through consciousness an immediate access to man, and this access can be used in two manners. The initial use is descriptive. In this fashion we began from an account of an event named insight. We pointed out that it was satisfying, that it came unexpectedly, that its emergence was conditioned more by a dynamic inner state of inquiry than by external

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circumstance, that while the first emergence was difficult, repeated occurrence was easy and spontaneous, that single acts of insight accumulate into clusters bearing on a single topic, that such clusters may remain without exact formulation, or may be worked out into a systematic doctrine. Naturally enough, this general description of insight was presupposed and utilized when we came to examine it more closely; and this closer examination was in turn presupposed in our account of explanatory abstraction and explanatory system and in our study of empirical method. Moreover, since data, percepts, and images are prior to inquiry, insight, and formulation, and since all definition is subsequent to inquiry and insight, it was necessary to define data, percepts, and images as the materials presupposed and complemented by inquiry and insight and, further, it was necessary to distinguish between them by contrasting the formulations of empirical science with those of mathematics and the formulations of both of these with the formulations of common sense. Finally, the analysis of judgment and the account of reflective understanding consisted in relating these acts to each other, and to the formulations of understanding, and to the fulfilment provided by experience.

As the reader will discern, the initial procedure of description gradually yielded to definition by relation; and the defining relations obtained immediately between different kinds of cognitional state or act. But definition by this type of relation is explanatory, and so descriptive procedure was superseded by explanatory.

There are, then, two types of description and two types of

explanation. If ~~one~~ the inquirer starts from the data of sense, ~~he~~ he begins by describing but goes on to explain. Again, if ~~one~~ he starts from the data of consciousness, ~~he~~ he begins by describing and goes on to explain. Still, there is an important difference between the two types of explaining. For explanation on the basis of sense can reduce the element of hypothesis to a minimum but it cannot eliminate it entirely. For explanation on the basis of consciousness can escape entirely the merely supposed, the merely postulated, the merely inferred.

First, explanation on the basis of sense can reduce hypothesis to a minimum. This, of course, is the point of the principle of relevance. Galileo's law of falling bodies does not merely suppose or postulate distance or time or the measurements of either. It does not merely suppose or postulate the correlation between distance and time; for there is some relation between the two inasmuch as a falling body falls farther in a longer time; and the actual measurements ground a numerical determination of that relation. Moreover, what holds for the law of falling bodies, holds for the other laws of mechanics. If one pleases, one may contend that the use of inquiry, insight, formulation, and consequent generalization, is mere supposition or mere postulation; but at least it is not the type of mere supposition that the empirical scientist systematically avoids or that he seriously fears will be eliminated in some more intelligent method of inquiry to be devised and accepted in the future. To reach the element of mere supposition that makes any system of mechanics subject to future revision, one must shift attention from single laws to the set of primitive

terms and relations which the system employs in formulationg all its laws. In other words, one has to distinguish between, say, mass as defined by correlations between masses and, on the other hand, mass as enjoying the position of an ultimate mechanical concept. Any future system of mecnanics will have to satisfy the data that now are covered by the notion of mass. But it is not necessary that every future system of mechanics will have to satisfy the same data by employing our concept of mass. Further developments might lead to the introduction of a different set of ultimate concepts, to a consequent reformulation of all laws, and so to a dethronement of the notion of mass from its present position as an ultimate of mechanical system. Hence, while empirical method can reduce the hypothetical to a minimum, it cannot eliminate it entirely. Its concepts as concepts are not hypothetical, for they are defined implicitly by empirically established correlations. None the less, its concepts as systematically significant, as ultimate or derived, as preferred to other concepts that might be empirically reached, do involve an element of mere supposition. For the selection of certain concepts as ultimate occurs in the work of systematization, and that work is provisional. At any time, a system is accepted because it provides the simplest account of all the known facts. But at the same time it is acknowledged that there may be unknown yet relevant facts, that they might give rise to further questions that would lead to further insights, and that the further insights might involve a radical revision of the accepted system.

Secondly, explanation on the basis of consciousness can escape this limitation. I do not mean, of course, that such

explanation is not to be reached through the series of revisions involved in the self-correcting process of learning. Nor do I mean that, once explanation is reached, there remains no possibility of the minor revisions that leave basic lines intact but attain a greater exactitude, and a greater fullness of detail. Again, I am not contending here and now ~~donatheless~~, that human nature and so human knowledge are immutable, that there could not arise a new nature and a new knowledge to which present theory would not be applicable. What is excluded is the radical revision that involves a shift in the fundamental terms and relations of the explanatory account of the human knowledge underlying existing common sense, mathematics and empirical science.

8. The Impossibility of Revision

The impossibility of such revision appears from the very notion of revision. A revision appeals to data. It contends that previous theory does not satisfactorily account for all the data. It claims to have reached complementary insights that lead to more accurate statements. It shows that these new statements either are unconditioned or more closely approximate to the unconditioned than previous statements. Now, if in fact, revision is, as described, then it presupposes that cognitional process falls on the three levels of presentation, intelligence, and reflection; it presupposes that insights are cumulative and complementary; it presupposes that they head towards a limit described by the adjective, satisfactory; it presupposes a reflective grasp of the unconditioned or of what approximates to the unconditioned. Clearly, revision cannot revise its own presuppositions. A reviser cannot appeal to data to deny data, to his new insights to deny insights,

to his new formulation to deny formulation, to his reflective grasp to deny reflective grasp.

The same point may be put in another manner. Popular relativism is prone to argue that empirical science is the most reliable form of human knowledge: but empirical science is subject to indefinite revision: therefore, all human knowledge is equally subject to indefinite revision. Now such argument is necessarily fallacious. One must definitely know invariant features of human knowledge before one can assert that empirical science is subject to indefinite revision; and if one definitely knows invariant features of human knowledge, then one knows what is not subject to revision. Moreover, as is obvious, such knowledge surpasses empirical science at least in the respect that it is not subject to revision.

9. Self-Affirmation in the Possibility of Judgments of Fact.

The same conclusion may be reached by setting forth the a priori conditions of any possible judgment of fact. For any such judgment can be represented by a "Yes" or "No" in answer to a question, Is it so? The answer will be rational, that is, it will rest on known sufficient reason. Moreover, the answer will be absolute;"Yes" utterly excludes "No"; and "No" utterly excludes "Yes". Hence, since the known sufficient reason for an absolute answer must itself be absolute and known, the "Yes" or "No" must rest on some apprehension or grasp of the unconditioned. Now the judgment of fact is not to the effect that something must be so or could not be otherwise; it merely states that something is so; hence the unconditioned that grounds it will be not formally but only virtually unconditioned. The first condition, then, of any possible judgment of fact is the grasp of 1) a conditioned,

2) a link between the conditioned and its conditions, and 3) the fulfilment of the conditions. It is such a grasp that effects the transition from the question, Is it so? to a rational, absolute answer.

But this first requirement presupposes other requirements. The "it" of the judgment of fact is not a bare "it". On the contrary, it is the conditioned, known as conditioned, that through the fulfilment of its conditions is grasped as virtually unconditioned. Prior to the question for reflection, there must be a level of activity that yields the conditioned as conditioned, the conditioned as linked to its conditions. But this is a level of intelligence, of positing systematic unities and systematic relations. Moreover, it will be a freely developing level; for without free development questions of fact would not arise. The only instances of the conditioned that would be envisaged would be instances with the conditions fulfilled. In that case the answer would always be an automatic "Yes"; and if the answer were always an automatic "Yes", there would be no need to raise any questions of fact. Still, though there is free development of systematic unities and relations, such development cannot occur in some pure isolation from the fulfilling conditions. Were there such isolation, it would be impossible to tell whether or not conditions were fulfilled; and if that were impossible, then judgments of fact could not occur. This yields the second condition of judgment of fact. It is a level of intellectual activity that posits systematic unities and relations 1) with some independence of a field of fulfilling conditions and 2) with reference to such a field.

But this second requirement presupposes a third. There must be a field of fulfilling conditions. More exactly, since conditions are simultaneous with what they condition, there must be a prior field containing what can become fulfilling conditions. Of themselves, they will be neither conditioning nor conditioned: they will be merely given.

Finally, possibility is concrete. Logicians may say that a "mountain of gold" is possible if there is no intrinsic contradiction involved in supposing such a mountain. But, in fact, a mountain of gold is possible only if the means are available for acquiring enough gold to make a mountain, for transporting it to a single place, for heaping it up in the fashion of a mountain, and for keeping it there long enough for the golden mountain to exist for some minimum interval of time. Similarly, any possible judgment of fact would be some concrete judgment. The conditions of its possibility include the conditions of bringing together its diverse components. There must be, then, a concrete unity-identity-whole that experiences the given, that inquires about the given to generate the free development of systematic unities and relations, that reflects upon such developments and demands the virtually unconditioned as its ground for answering "Yes" or "No". It is this concrete unity that asks, "Is it so?" It is this concrete unity that initiates the free development by asking about the given, What is this? Why is it? How often does it exist or happen? It is this concrete unity that grasps and formulates the conditioned as conditioned and that appeals to the given to grasp the virtually unconditioned and to affirm it rationally and absolutely.

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~~9. Self-appropriation and the possibility of judgments of fact.~~

There remains a corollary. Judgments of fact may be not only possible. They may actually occur. But if any judgment of fact occurs, there must be as well the occurrence of its conditions. Hence, if there is any judgement of fact, no matter what its content, there also is a concrete unity-identity-whole that experiences some given, that inquires, understands, and formulates, that reflects, grasps the unconditioned, and so affirms or denies. Finally, such a concrete unity-identity-whole is a thing-itself, for it is defined by an internally related set of operations, and the relations may be experientially validated in the conscious and dynamic states: 1) of inquiry leading from the given to insight, 2) of insight leading to formulation, 3) of reflection leading from formulation to grasp of the unconditioned, and 4) of that grasp leading to affirmation or denial.

From the corollary there results our prior contention. There cannot occur a revision without the occurrence of some judgment of fact. But if there occurs any judgment of fact, there occur the dynamic states in which may be validated experientially the relations that define the conjugate terms by which the thing-itself that knows is differentiated.

What is the source of this peculiarity of cognitional theory? It is that other theory reaches its thing-itself by turning away from the thing as related to us by sense or by consciousness, but cognitional theory reaches its thing-itself by understanding itself and affirming itself as concrete unity in a process that is conscious empirically, intelligently, and rationally. Moreover, since every other known becomes known through this

process, no known could impugn the process without simultaneously impugning its own status as a known.

10. Contrast with Kantian Analysis.

We have performed something similar to what a Kantian would name a transcendental deduction. Accordingly, we shall be asked to explain the fact that our deduction yields different results from Kant's.

A first difference is that Kant asked the a priori conditions of the possibility of experience in the sense of knowing an object. We have distinguished two issues; there is the problem of objectivity, and from this we have carefully prescinded not only in the present section but also in all earlier sections; there also is the prior problem of determining just what activities are involved in knowing, and to this prior problem we have so far confined our efforts. Hence we asked, not for the conditions of knowing an object, but for the conditions of the possible occurrence of a judgment of fact. We have asked for the conditions of an absolute and rational "Yes" or "No" viewed simply as an act. We have not asked on what conditions there would be some fact that corresponded to the "Yes". We have not even noted what meaning such correspondence might have.

A second difference lies in the distinction between thing-for-us and thing-in-itself. Kant distinguished these as phenomenon and noumenon. Just what he meant is a matter of dispute but, at least, it is clear that the distinction pertained to his formulation of a theory of objectivity. Moreover, it seems to me to be probable enough that the historical origin of the Kantian distinction is to be sought in the Renaissance distinction of primary and

secondary qualities where the former pertained to the real and objective things themselves while the latter pertained to the subject's apprehension of them. In any case, our distinction is neither the Renaissance nor the Kantian distinction. It is simply a distinction between description and explanation, between the kind of cognitional activities that fix contents by indicating what they resemble and, on the other hand, the kind that fix contents by assigning their experientially validated relations. A thing is a concrete unity-identity-whole grasped in data as individual. Describe it, and it is a thing-for-us. Explain it, and it is a thing-itself. Is it real? Is it objective? Is it anything more than the immanent determination of the cognitional act? These are all quite reasonable questions. But as yet we answer neither "Yes" nor "No". For the most part, our answer is simply that objectivity is a highly complex issue and that we shall handle it satisfactorily only if we begin by determining what precisely cognitional process is. No doubt, there are objections that may be urged against this procedure; but the objections too will be handled satisfactorily only after the prior questions are answered.

A third difference regards universal and necessary judgments. They stand in the forefront of the Kantian critique which was largely engaged in the problem of transcending Hume's experiential atomism. But in our analysis they play a minor role. A universal and necessary judgment may be merely the affirmation of an analytic proposition, and such analytic propositions may be mere abstract possibilities without relevance to the central context of judgments that we name knowledge. Our emphasis falls on

the judgment of fact that itself is an increment of knowledge and, as well, contributes to the transition from the analytic proposition to the analytic principle, that is, to the universal and necessary judgment whose terms and relations are existential in the sense that they occur in judgments of fact.

A fourth difference regards the immediate ground of judgment. Kant formulated this ground by setting forth his schematism of the categories. There is a proper use of the category, Saul, if there occurs a filling of the empty form of Time. There is a proper use of the category, Gilbert, if there is a permanence of the soul in time. However, Kant's schematism is not regarded as one of his happiest inventions. What he was trying to get hold of was, perhaps, the reflective process of checking, of verifying, of bringing the merely conceived and the merely given into unity. In fact, that process is far more complicated and far more versatile than Kantian analysis would lead one to suspect. Verifying supposes a vast array of hypothetical propositions that state what would be experienced under precisely defined conditions. Verifying consists in having those experiences, all of them, and none but them, under the defined conditions. Moreover, what is verified, is what is conceived, formulated, supposed. It need have no imaginable counterpart, and so one can speak of verifying the theory of relativity or the affirmations of quantum mechanics. Indeed, as we have shown at length, there is a single formula that covers the immediate ground of all our judgments: it is the grasp of the virtually unconditioned. So far was Kant from positing the unconditioned as the immediate ground of every judgment, that he

described it as an Ideal of Pure Reason, as a notion that becomes operative in our knowing, not prior to judgment and as a condition of judgment, but subsequently inasmuch as each judgment rests on an infinite regress of prosyllogisms. As the reader familiar with Kant, I will note, our assertion of a demand for the unconditioned as a prior ground for judgment not merely implies that the Kantian analytic is seriously incomplete but also involves in utter ruin the Kantian dialectic. For the dialectic has but a single premise, namely, that since the demand for the unconditioned is not a necessary ground for judgment, therefore, it is a transcendental illusion; in other words, since the unconditioned is not constitutive of knowing an object in the sense of making a judgment, therefore, it has a purely regulative function in our knowing. On our showing, the unconditioned is prior and constitutive; to affirm a fact is to affirm an unconditioned.

A fifth difference has to do with consciousness. Kant acknowledged an inner sense that corresponds roughly to what we have termed consciousness, namely, the awareness that is immanent in acts of willing, perceiving, imagining, desiring, fearing, and the like. Besides this acknowledgement of inner sense, Kant deduced or postulated an original synthetic unity of apperception as the a priori condition of the "I think" accompanying all cognitional acts. On the other hand, Kantian theory has no room for a consciousness of the generative principles of the categories; the categories may be inferred from the judgments in which they occur; but it is impossible to reach behind the categories to their source. It is precisely this aspect of Kantian

thought that gives the categories their flexibility and their irreducible mysteriousness. It is the same aspect that provided Fichte and Hegel with their opportunity to march into the unoccupied territority of intelligent and rational consciousness. The dynamic stress named inquiry and reflection do occur. Inquiry is generative of all understanding, and understanding is generative of all concepts and systems. Reflection is generative of all reflective grasp of the unconditioned, and that grasp is generative of all judgment. If the Kantian proscribes consideration of inquiry and reflection, he lays himself open to the charge of obscurantism. If he admits such consideration, if he praises intelligent curiosity and the critical spirit, then he is on his way to acknowledge the generative principles both of the categories Kant knew and of the categories Kant did not know.

The foregoing list of differences account for the divergence between Kant's conception and our own. They are differences in the problem under consideration, in the viewpoint from which it is considered, in the method by which it is solved. More fundamentally there are differences about questions of fact, for our self-affirmation is, as we have insisted and may be pardoned for repeating, primarily and ultimately a judgment of fact. The orthodox Kantian would refer to our stand as mere psychologism, as an appeal to the empirical that can yield no more than a provisional probability. But our retort is simple enough. Without judgments of fact one cannot get beyond mere analytic propositions. Further, though self-affirmation is no more than a judgment of mere fact, still it is a privileged judgment. Self-negation is incoherent. One has only to inquire and reflect, to find

oneself caught in the spontaneities and inevitabilities that supply the evidence for self-affirmation. One has only to make a single judgment of fact, no matter what its content, to involve oneself in a necessary self-affirmation. Finally, cognitional theory differs from other theory: for other theory reaches explanation only by venturing into the merely supposed; but cognitional theory reaches explanation without any such venture: and since it contains no merely hypothetical element, it is not subject to radical revision.

11. Contrast with Relativist Analysis.

From Kantian to turn to relativist thought. The initial question in the present section was whether correct judgments occur. Our account of self-affirmation directly contradicts the relativist contention that correct judgments do not occur. Though the arguments for our position have been given, it will not be amiss to indicate where the relativist could disagree and why.

First, relativist thought is largely devoted to a refutation of empiricism. Correctly it insists that human knowing cannot be accounted for by the level of presentations alone. There is, as well, the level of intelligence, of grasping and formulating intelligible unities and systematic relations. Without this second level of activities, there is, indeed, (a given) but there is no possibility of saying what is given.

Secondly, just as the relativist insists on the level of intelligence against the empiricist, so we insist on the level of reflection against the relativist. Human knowing is not merely theory about the given: there are also facts; and the relativist has not and cannot establish that there are no facts, for the

absence of any other fact would itself be a fact.

Thirdly, just as the empiricist could have nothing to say if, in fact, he did not utilize operations on the level of intelligence, so also the relativist does not confine himself strictly to the levels of presentations and of intelligence. He is quite familiar with the notion of the unconditioned. He regards the unconditioned as the ideal towards which human knowing tends. But he supposes that this ideal is to be reached through understanding. If the universe in its every part and aspect were thoroughly understood, there could be no further questions; everything would be conceived as it ought to be; on every possible topic a man could say just what he meant and mean just what he said. On the other hand, short of this comprehensive coherence, there can be no sure footing. There is understanding, but it is partial; it is joined with incomprehension; it is open to revision when present incomprehension yields to future understanding; and so intimately are all things related that knowledge of anything can be definitive only when everything is known.

Fourthly, the relativist is able to follow up this general view by facing concrete issues. Is this a typewriter? Probably, Yes. For practical purposes, Yes. Absolutely? The relativist would prefer to be clear about the precise meaning of the name, typewriter: he would like to be told just what is meant by the demonstrative, this: he would be grateful for an explanation of the meaning of the copula, is. Your simple question is met by three further questions; and if you answer these three, your answers will give rise to many more. If you are quick and see that you are starting on an

infinite series, you may confront the relativist with a rounded system. But the relativist is also a smart fellow. He will point out that ordinary people, quite certain that this is a typewriter, know nothing of the system on which you base their knowledge. Nor is this all. For human knowledge is limited; systems have their weak points; and the relativist will pounce upon the very issues on which a defender of the system would prefer to profess ignorance.

Fifthly, not only will the relativist make it plain that there are further questions until everything is known, but also he will explain why this is so. A relation is named internal to an object when, without the relation, the object would differ radically. Thus, we have spoken of inquiry and insight. But by inquiry we have not meant some pure wonder; we have meant a wonder about something. Similarly, by insight we have not meant a pure understanding but an understanding of something. Inquiry and insight, then, are related internally to materials about which one inquires and into which one gains insight. Now, if one supposes that the whole universe is a pattern of internal relations, clearly it follows that no part and no aspect of the universe can be known in isolation from any other part or aspect; for every item is related internally to every other; and to prescind from such relations is to prescind from things as they are and to substitute in their place other imaginary objects that simply are not. If, then, one asks the relativist to explain why questions run off to infinity, he has a ready answer. The universe to be known by answering questions is a tissue of internal relations.

Sixthly, if the foregoing fairly represents the relativist position, it also reveals its oversights. Questions are of two kinds. There are questions for intelligence asking what this is, what that means, why this is so, how frequently it occurs or exists. There also are questions for reflection that ask whether answers to the former type of question are correct. Next, the unconditioned that is required for judgment is not the comprehensive coherence that is the ideal of understanding, that grounds answers to all questions of the first type. On the contrary, it is a virtually unconditioned that results from the combination of a conditioned with the fulfilment of its conditions. Further, a judgment is a limited commitment; so far from resting on knowledge of the universe, it is to the effect that, no matter what the rest of the universe may prove to be, at least this is so. I may not be able to settle border-line instances in which one might dispute whether the name, typewriter, would be appropriate. But, at least, I can settle definitely that this is a typewriter. I may not be able to clarify the meaning of is, but it is sufficient for present purposes to know the difference between is and is not, and that, I know. I am not very articulate when it comes to explaining the meaning of this; but if you prefer to use that, it will make no difference provided we both see what we are talking about. You warn me that I have made mistakes in the past. But your warning is meaningless, if I am making a further mistake in recognizing a past mistake as a mistake. And in any case, the sole present issue is whether or not I am mistaken in affirming this to be a typewriter. You explain to me that my notion of a typewriter would be very different, if I understood the chemistry of the materials, the mechanics of the construction, the psychology of the

typist's skill, the effect on sentence structure resulting from the use of a machine in composing, the economic and sociological reperoussions of the invention, its relation to commercial and political bureaucracy, and so forth. But may I not explain to you that all those further items, however interesting and significant, are to be known through further judgments, that such further judgments, so far from shifting me from my present conviction that this is a typewriter, will only confirm me in it, that to make those further judgments would be rather difficult if, at the start, I could not be certain whether or not this is a typewriter?

Seventhly, however, the questions that are answered by a pattern of internal relations are only questions that ask for explanatory system. But besides things-themselves and prior to them in our knowing, there are things-for-us, things as described. Moreover, the existents and occurrences, in which explanatory systems are verified, diverge non-systematically from the ideal frequencies that ideally would be deduced from the explanatory systems. Again, the activity of verifying involves the use of description as an intermediary between the system defined by internal relations and, on the other hand, the presentations of sense that are the fulfilling conditions. Finally, it would be a mistake to suppose that explanation is the one true knowledge; not only does its verification rest on description but also the relations of things to us are just as much objects of knowledge as are the relations of things among themselves.

Eighthly, the relativist invents for himself a universe that

consists merely of explanatory system because he conceives the unconditioned as the ideal of understanding, as the comprehensive coherence towards which understanding tends by asking what and why. But as we have seen, the criterion of judgment is the virtually unconditioned. Each judgment is a limited commitment. So far from pronouncing on the universe, it is content to affirm some single conditioned that has a finite number of conditions which, in fact, are fulfilled. No doubt, were the universe simply a vast explanatory system, knowledge of the conditions of any conditioned would be identical with knowledge of the universe. But, in fact, the universe is not simply explanatory system; its existents and its occurrences diverge non-systematically from pure intelligibility; it exhibits an empirical residue of the individual, the incidental, the continuous, the merely juxtaposed, and the merely successive; it is a universe of facts, and explanatory system has validity in the measure that it conforms to descriptive facts.

Ninthly, the relativist argument from unending further questions is more impressive than conclusive. Human knowing does not begin from previous knowing but from natural spontaneities and inevitabilities. Its basic terms are not defined for it in some knowing prior to knowing; they are fixed by the dynamic structure of cognitional process itself. The relativist asks what is meant by the copula, is, and the demonstrative, this. But neither he nor anyone else is given to confusing is with is not or this with not this; and that basic clarity is all that is relevant to the meaning of the affirmation, This is a typewriter. A cognitional theorist would be called upon to explain

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such elementary terms; he would do so by saying that is represents the Yes that occurs in judgment and that is anticipated by such questions as, Is it? What is it?. Similarly, a theorist would explain this as the return from the field of conception to the empirical residue in the field of presentations. But questions relevant to cognitional theory are not relevant to every instance of knowing. They are not universally relevant because, in fact, there is no operational obscurity about the meanings that cognitional theory elucidates. Again, they are not universally relevant, because such elementary meanings are fixed, in a manner that surpasses determination by definition, with the native immutability of the dynamic structures of cognitional process.

Tenthly, as human knowing begins from natural spontaneity, so its initial developments are inarticulate. As it asks what and why without being given the reason for its inquiry, so also it sets off on the self-correcting process of learning without the explicit formulations that rightly would be required in an explanatory system. Single insights are partial. Spontaneously they give rise to the further questions that elicit complementary insights. Were the universe purely an explanatory system, the minor clusters of insights reached by what is called common sense would not (tend to or towards head for) a limiting position of familiarity and mastery in which evidently it is silly to doubt whether or not this is a typewriter. But, in fact, the universe to be known by answering questions is not pure explanatory system. In fact, insights do

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lead to
X X (head for) limiting positions of familiarity and mastery. In fact, as everyone knows very well, it is silly to doubt whether or not this is a typewriter. The relativist would beg me to advert to the enormous difference in my notion of the typewriter were I to understand fully the chemistry of its materials, the mechanics of its construction, the psychology of the typist's skill, the effect given literary style by composing on a typewriter, the effect of its invention on the development of commercial and political bureaucracy, and so forth. But granted such an enrichment of my knowledge to be possible and desirable, none the less it is further knowledge to be obtained by further judgments; and since the enrichment is explanatory, since explanatory knowledge rests on descriptive knowledge, not only must I begin by knowing that this is a typewriter, not only must I advance by learning how similar other machines must be if they are to be called typewriters, but also I can attain valid explanation only in so far as my descriptions are exact.

Eventually, it is quite true that I can be mistaken. But that truth presupposes that I am not making a further mistake in acknowledging a past mistake as a mistake. More generally, judgments of fact are correct or incorrect, not of necessity, but merely in fact. If this is something, still it might be nothing at all. If it is a typewriter, still it might be something else. Similarly, if I am correct in affirming it to be a typewriter, it is not a pure necessity, but merely a fact that I am correct. To ask for the evidence that excludes the possibility of my being mistaken in affirming this to be a typewriter, is to ask too much. Such

evidence is not available, for if I am correct, that is merely fact. But if that evidence is not available, still less is there the evidence that will exclude the possibility of error in all judgments of fact. Errors are just as much facts as are correct judgments. But the relativist is in conflict with both categories of fact. For him nothing is simply true, for that is possible only when comprehensive coherence is reached; for him, nothing is simply wrong, for every statement involves some understanding and so some part of what he names truth. In the last analysis, just as the empiricist tries to banish intelligence, so the relativist tries to banish fact and, with it, what everyone else names truth.

Chapter XI: THE NOTION OF BEING

If the main lines of cognitional process have been set down, it remains that certain fundamental and pervasive notions should be clarified. Among them, in the first place, is the notion of being. It is a tricky topic and, perhaps, the most satisfactory procedure will be to begin from a definition.

1. A Definition.

Being, then, is the objective of the pure desire to know.

By the desire to know is meant the dynamic orientation manifested in questions for intelligence and for reflection. It is not the verbal utterance of questions. It is not the conceptual formulation of questions. It is not any insight or thought. It is not any reflective grasp or judgment. It is the prior and enveloping drive that carries cognitional process from sense and imagination to understanding, from understanding to judgment, from judgment to the complete context of correct judgments that is named knowledge. The desire to know, then, is simply the inquiring and critical spirit of man. By moving him to seek understanding, it prevents him from being content with the mere flow of outer and inner experience. By demanding adequate understanding, it involves man in the self-correcting process of learning in which further questions yield complementary insights. By moving man to reflect, to seek the unconditioned, to grant unqualified assent only to the unconditioned, it prevents him from being content with hearsay and legend, with unverified hypotheses and untested theories.

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Finally, by raising still further questions for intelligence and reflection, it excludes complacent inertia; for if the questions go unanswered, man cannot be complacent; and if answers are sought, man is not inert.

Because it differs radically from other desire, this desire has been named pure. It is to be known, not by the misleading analogy of other desire, but by giving free rein to intelligent and rational consciousness. It is, indeed, impalpable but also it is powerful. It pulls man out of the solid routine of perception and emotion, instinct and habit, doing and enjoying. It holds him with the fascination of problems. It engages him in the quest of solutions. It makes him aloof to what is not established. It compels assent to the unconditional. It is the cool shrewdness of common sense, the disinterestedness of science, the detachment of philosophy. It is the absorption of investigation, the joy of discovery, the assurance of judgment, the modesty of limited knowledge. It is the relentless serenity, the unhurried determination, the imperturbable drive of question following oppositely on question to the bounds of truth.

This pure desire has an objective. It is a desire to know. As mere desire, it is for the satisfaction of acts of knowing, for the satisfaction of understanding, of understanding fully, of understanding correctly. But as pure desire, as cool desire, disinterested, detached, it is not for cognitional acts, and the satisfaction they give their subject, but for cognitional contents, for what is to be known. The satisfaction of mistaken understanding,

provided one does not know it as mistaken, can equal the satisfaction of correct understanding. Yet the pure desire scorns the former and prizes the latter; it prizes it, then, as dissimilar to the former; it prizes it not because it yields satisfaction but because its content is correct.

The objective of the pure desire is the content of knowing rather than the act. Still, the desire is not itself a knowing, and so its range is not the same as the range of knowing. Initially in each individual, the pure desire is a dynamic orientation to a totally unknown. As knowledge develops, the objective becomes less and less unknown, more and more known. At any time the objective includes both all that is known and all that remains unknown, for it is the goal of the immanent dynamism of cognitional process and that dynamism both underlies actual attainment and heads beyond it with ever further questions.

What is this objective? Is it limited or unlimited? Is it one or many? Is it material or ideal? Is it phenomenal or real? Is it an immanent content or a transcendent object? Is it a realm of experience, or of thought, or of essences, or of existents? Answers to these and to any other questions have but a single source. They cannot be had without the functioning of the pure desire. They cannot be had from the pure desire alone. They are to be had inasmuch as the pure desire initiates and sustains cognitional process. Thus, if it is true that A is, that A is one, and that there is only A, then the objective of the pure desire is one. But if it is true that A is, that B is, that A is not B, then the objective is many. Which, you ask, is true? The fact that you ask, results from the pure desire. But to reach the

answer, desiring is not enough: answers come only from inquiring and reflecting.

Now our definition was that being is the objective of the pure desire to know. Being, then, is 1) all that is known and 2) all that remains to be known. Again, since a complete increment of knowing occurs only in judgment, being is what is to be known by the totality of true judgments. What, one may ask, is that totality? It is the complete set of answers to the complete set of questions. What the answers are, remains to be seen. What the questions are, awaits their emergence. Meaningless or incoherent or illegitimate questions may be possible, but how they are to be defined, is a further question. The affirmation in hand is that there exists a pure desire to know, an inquiring and critical spirit, that follows up questions with further questions, that "heads for" some objective which has been named being.

Our definition of being, then, is of the second order. Other definitions determine what is meant. But this definition is more remote for it assigns, not what is meant by being, but how that meaning is to be determined. It asserts that if you know, then you know being; it asserts that if you wish to know, then you wish to know being; but it does not settle whether you know or what you would know, whether your wish will be fulfilled or what you will know when it is fulfilled.

Still, though our definition is of the second order, it is not simply indeterminate. For neither the desire to know nor knowing itself are indeterminate. Inasmuch as knowing is determinate, we could say that being is what is to be known by true judgments.

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Inasmuch as the desire to know ever goes beyond actual knowledge, we could say that being is what is to be known by the totality of true judgments. Hence, being has, at least, one characteristic: it is all-inclusive. Apart from being there is nothing. Again, being is completely concrete and completely universal. It is completely concrete; over and above the being of any thing, there is nothing more of that thing. It is completely universal; apart from the reals of being, there is simply nothing.

2. An Unrestricted Notion.

One may wonder just how all-inclusive being is. That wonder may be formulated in a variety of manners. But no matter how it is formulated, no matter whether it can be formulated, it can serve only to show how all-inclusive being is. For the wonder is inquiry. It is the desire to know. Anything it can discover or invent, by that very fact is included in the notion of being. Hence, the effort to establish that being is not all-inclusive must be self-defeating; for at the root of all that can be affirmed, at the root of all that can be conceived, is the pure desire to know: and it is the pure desire, underlying all judgment and formulation, underlying all questioning and all desire to question, that defines its all-inclusive objective.

None the less, it may not be amiss to illustrate this principle concretely. It will be said that there is much we do not know. No doubt, our ignorance is great, but we know that fact by raising questions that we do not answer; and being is defined not only by the answers we give but also by the questions we ask. Next, it will be said that there is much it would be futile for us to try to learn. No doubt, the proximately fruitful field of

inquiry is restricted. But we know that fact by distinguishing between the questions we can hope soon to answer and those that, as yet, we are not prepared to cope with tackle; and being is defined, not only by the questions we can hope to answer, but also by the questions whose answer we have to postpone.

Thirdly, it will be objected by many that they have no desire to know everything about everything. But how do they know that they do not already know everything about everything? It is because so many questions can be asked. Why do they not effectively will to know everything about everything? Because it is so troublesome to search even a few answers that they are completely disheartened by the prospect of answering all the questions they could ask.

The attack may be made from the opposite flank. The trouble is that the definition of being is too inclusive. Questions can be meaningless, illusory, incoherent, illegitimate. Trying to answer them does not lead to knowledge of anything. How, no doubt, there are mistaken questions that lead nowhere. But mistaken questions are formulated questions. Being has been defined, not as the objective of formulated questions, but as the objective of the pure desire to know. Just as that desire is prior to any answer and it itself is not the answer, so too, it is prior to any formulated question and it itself is not a formulation. Moreover, just as the pure desire is the intelligent and rational basis from which we discern between correct and incorrect answers, so also it is the intelligent and rational basis from which we discern between valid and mistaken questions. In brief, the pure desire to know, whose

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objective is being, is the source not only of answers but also of their criteria, and not only of questions but also of the grounds on which they are screened. For it is intelligent inquiry and reasonable reflection that just as much yield the right questions as the right answers.

More fundamental misgivings may arise. If one pleases, one may define being as what is to be known through the totality of true judgments. But is being really that? Might it not be something entirely different? The questions arise. They may be valid or mistaken. If they are mistaken, they are to be ignored. If they are valid, then our misgivings are without foundation. For the being that might be totally different, turns out to be exactly what we are talking about. For we ask whether it might be; and the being we are talking about, is the being we ask about.

Again, might there not be an unknowable? If the question is invalid, it is to be ignored. If the question is valid, the answer may be "Yes" or "No". But the answer, "Yes", would be incoherent, for then one would be knowing that the unknowable is; and the answer, "No", would leave everything knowable and within the range of being.

Other doubts may arise, but instead of chasing after them one by one, it will be better to revert to our initial theorem. Every doubt that the pure desire is unrestricted serves only to prove that it is unrestricted. If you ask whether X might not lie beyond its range, the fact that you ask proves that X lies within its range. Or else, if the question is meaningless, incoherent, illusory, illegitimate, then X turns out to be the more

nothing that results from aberration in cognitional process.

Not only, then, is judgment absolute, not only does it rest upon a grasp of the unconditioned, not only does reflection set the dichotomy, Is it or is it not? But at the root of cognitional process there is a cool, detached, disinterested desire to know and its range is unrestricted. Being is the anything and everything that is the objective of that desire.

3. A Spontaneous Notion

If we have explained what we mean by being, we must now ask what the notion of being is.

In the first place, a distinction has to be drawn between the spontaneously operative notion and, on the other hand, theoretical accounts of its genesis and content. The spontaneously operative notion is invariant; it is common to all men; it functions in the same manner no matter what theoretical account of it a man may come to accept. On the other hand, theoretical accounts of the content and genesis of the notion are numerous; they vary with philosophic contexts, with the completeness of a thinker's observations, with the thoroughness of his analysis. First, we shall give our account of the spontaneously operative notion, and then we shall add a few notes on other theoretical accounts of it.

On the supposition of our analysis of cognitional process, it is easy enough to conclude that the spontaneously operative notion of being has to be placed in the pure desire to know. For, first of all, men are apt to agree that things are, whether or not we know them and, moreover, that there are many things that are known only incompletely or even not at all. The notion of being, then, extends beyond the known. Secondly, being is known

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in judgment. It is in judgment that we affirm or deny and, until we are ready to affirm or deny, we do not yet know whether or not any X happens to be. Still, though being is known only in judging, the notion of being is prior to judging. For prior to any judgment there is reflection, and reflection is formulated in the question, Is it? That question supposes some notion of being and, strangely enough, it is prior to each instance of our knowing being. Not only does the notion of being extend beyond the known but also then, it is prior to the final component of knowing when being is actually known. Thirdly, there are objects of thought. I can think of a horse and, no less, I can think of a centaur. I can think of the best available scientific opinion on any subject and, no less, I can think of all the previous opinions that in their day were the best available on the same subject. In one sense, they are all equivalent, for as long as one is merely thinking, merely considering, merely supposing, one deals merely with the conditioned and it makes no difference whether or not its conditions are fulfilled. Thinking, then, prescinds from existing. But if it prescinds from existing, does it not prescind from being? and if it prescinds from being, is not all thinking about nothing? The trouble with this argument is that thinking also prescinds from not existing. If I think of a centaur or of phlogiston, I prescind from the fact that they do not exist; hence, if prescinding from existing is prescinding from being, prescinding from non-existence is prescinding from not being; if prescinding from being proves that I am thinking of nothing, then prescinding from not being proves that I am thinking of something. Now this type of consideration has led many thinkers

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to suppose that being is one thing and existing is another, that horses and centaurs, electrons and phlogiston, equally are, but horses and electrons exist while centaurs and phlogiston do not exist. Still that conclusion does not satisfy the facts, for apart from the oddity of asserting that the non-existent is, there is (the (oversight of) the dynamism of cognitional process. In a sense, thinking prescinds from existing and not existing, for it is not thinking but judging that determines whether or not anything exists. In another sense, thinking does not prescind from existing and not existing, for thinking is purposive: we think to get our concepts straight: we wish to get our concepts straight that we may be able to judge: so far from prescinding from existing and not existing, thinking is for the purpose of determining whether or not what is thought does exist. # It follows that the notion of being goes beyond the merely thought, for we ask whether or not the merely thought exists. No less, it follows that the notion of being is prior to thinking, for were it not, then thinking could not be for the purpose of judging, for the purpose of determining whether or not the merely thought exists. The notion of being, then, is prior to conception and goes beyond it: and it is prior to judgment and goes beyond it. That notion must be the immanent, dynamic orientation of cognitional process. It must be the detached and unrestricted desire to know as operative in cognitional process. Desiring to know is desiring to know being; but it is merely the desire and not yet knowing. Thinking is thinking being; it is not thinking nothing; but thinking being is not yet knowing it. Judging is a complete increment in knowing: if correct, it is a knowing of

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being; but it is not yet knowing being, for that is attained only through the totality of correct judgments.

Still, how can an orientation or a desire be named a notion. A foetal eye is orientated towards seeing; but a foetal eye does not see and it has no notion of seeing; a notion arises only in so far as understanding discovers future function in present structure. Hunger is orientated towards food and eating; it is a desire; it lies within empirical consciousness; but a notion arises only in so far as the orientation of hunger is understood. Purposive human action is orientated to ends-one and or product; cognitional elements provide the rule and guide of such action; but the cognitional elements are prior to the action; they are constituted, not by the action itself, but by the planning that precedes it.

It remains that none of these instances is exactly parallel to the relation between the desire to know and cognitional process. For the desire to know is not unconscious, as is the foetal eye, nor spiritually conscious, as is hunger, nor a consequence of intellectual knowledge, as are deliberation and choice. The desire to know is conscious intelligently and rationally; it is inquiring intelligence and reflecting reasonableness. Simply as desire, it is orientation without, as yet, involving any cognitional content or notion. Still intelligence, as obverse, looks for the intelligible, as reverse. Reasonableness, as obverse, looks for the grounded, as reverse. More fundamentally, the looking for, the desiring, the inquiring-and-reflecting is an obverse that intelligently and rationally heads for an unrestricted objective named being. Were that heading unconscious, there would be an

orientation towards being but there would be no desire to know being and no notion of being. Were that knowing empirically conscious, there would be an orientation towards being and a felt desire to know being, but there would be no notion of being. In fact, this knowing is intelligent and rational, and so there is not only an orientation towards being, not only a pure desire to know being, but also a notion of being.

Let us try to catch this notion, this intention of being, in the act. We call it abstraction, and commonly we mean a direction of attention to some aspects of the given with a concomitant neglect of other aspects. The geometer considers the circle as a plane figure obeying a certain rule: he disregards the size, the color, the inexactitude of the figure he draws or imagines; still more does he disregard other and more loosely connected aspects of the given. But that is not all. He disregards all other questions in geometry, all other departments of mathematics, all other fields of science, all other human occupations to which he could turn his hand. He considers only the circle. He abstracts from everything else. He does so intelligently, for though the objective of his desire is unrestricted, still he can move to seize it only by concentrating on one element at a time. Again, as intelligence abstracts, so reflection presupposes. If I am to judge whether or not this is a theorem, I have to prescind from all that is not relevant to the question. I have to know all that is relevant. If I were a relativist, I would have to know the universe to know all that is relevant to that single judgment. Even though I am not a relativist, even though I find that many conditioned proposi-

tions become virtually unconditioned on the fulfilment of a manageable number of conditions, still this restriction of the relevant is accompanied by an acknowledgement of a universe of irrelevancies. Finally, as intelligence concentrates on the significant to abstract from all else, as reflection concentrates on the relevant to prescind from all else, so further questions and further issues arise neither as a surprise nor as a new beginning. The abstracting and the prescinding were provisional: they were only moments in a larger process. Nor is that larger process merely the object of introspective analysis. Immanent within it and operative of it lies an intelligent and rational consciousness that unrestrictedly intends a correspondingly unrestricted objective named being, or the all, or everything about everything, or the concrete universe. Just as the notion of the intelligible is involved in the actual functioning of intelligence, just as the notion of the grounded is involved in the actual functioning of reasonableness, so the notion of being is involved in the unrestricted drive of inquiring intelligence and reflecting reasonableness.

4. An All-pervasive Notion

Hence it is that the notion of being is all-pervasive. It underpins all cognitional contents. It penetrates them all. It constitutes them as cognitional.

// It underpins all cognitional contents. Without the pure desire to know, sensitive living would remain in its routine of perception and conation, instinct and habit, emotion and action. What breaks that circuit and releases intellectual activity is the wonder Aristotle described as the beginning of all science and philosophy. But that wonder is intelligent inquiry. It selects

data for insight and by selecting it, underpins even the empirical component in our knowing. Still more obviously all ideas and all concepts are responses to the desire to understand, and all judgments are responses to the demand for the unconditioned.

Secondly, the notion of being penetrates all cognitional contents. It is the supreme heuristic notion. Prior to every content, it is the notion of the to-be-known through that content. As each content emerges, the "to-be-known through the content" passes without residue into the "known through that content". Some blank in universal anticipation is filled in, not merely to end that element of anticipation, but also to make the filler a part of the anticipated. Hence, prior to all answers, the notion of being is the notion of the totality to be known through all answers. But, once all answers are reached, the notion of being becomes the notion of the totality known through all answers.

Thirdly, the notion of being constitutes all contents as cognitional. Experiencing is only the first level of knowing: it presents the matter to be known. Understanding is only the second level of knowing: it defines the matter to be known. Knowing reaches a complete increment only with judgment, only when the merely experienced has been thought and the merely thought has been affirmed. But the increment of knowing is always completed in the same fashion. Experience is a kaleidoscopic flow. Objects of thought are as various as the inventiveness of human intelligence. But the contribution of judgment to our knowing is ever a mere "Yes" or "No", a mere "is" or "is not". Experience is for inquiring into being. Intelligence is for thinking out being.

But by judgment being is known, and in judgment what is known is known as being. Hence knowing is knowing being, yet the known is never mere being, just as judgment is never a mere "Yes" apart from any question that "Yes" answers.

5. The Core of Meaning.

As the notion of being underpins all contents, and penetrates them, and constitutes them as cognitional, so also it is the core of meaning.

Distinguish 1) sources of meaning, 2) acts of meaning, 3) terms of meaning, and 4) the core of meaning.

Any element of knowledge may serve as a source of meaning. Hence, sources of meaning include data and images, ideas and concepts, the grasp of the unconditioned and judgment and, no less, the detached and unrestricted desire to know.

Acts of meaning are of three kinds. They are 1) formal, 2) full, 3) instrumental. The formal act of meaning is an act of conceiving, thinking, considering, defining, supposing, formulating. The full act of meaning is an act of judging. The instrumental act of meaning is the implementation of a formal or of a full act by the use of words or symbols in a spoken, written, or merely imagined utterance.

Terms of meaning are what is meant. They are formal or full. Formal terms of meaning are what is conceived, thought, considered, defined, supposed, formulated. Full terms of meaning are what is affirmed or denied.

Now the all-inclusive term of meaning is being, for apart from being there is nothing. Inversely, the core of all acts of meaning is the intention of being.

Thus, any given judgment pertains to a context of judgments, and it is from the context that the meaning of the given judgment is determined. But why is the meaning of the given judgment a function of a context of other judgments? Because any judgment is but an increment in a whole named knowledge; because the meaning of the judgment is but an element in the determination of the universal intention of being.

Again, judgments may be true or false. The true judgment affirms what is and denies what is not. In the true judgment there is harmony between what is intended and what is meant. But in the false judgment there is conflict between intention and meaning. The false judgment as a judgment intends being; it intends to affirm what is and to deny what is not. But the false judgment as false is a failure to carry out its intention as a judgment. It affirms what is not and denies what is. It means not what is but only what would be, were it not false but true: again, in its negative form, it means, not what is not, but what would not be, were it not false but true.

Perhaps it is this internal conflict that has led some to the conclusion that a false judgment is meaningless. But such a conclusion seems astoundingly false. Were the false judgment meaningless, there would be nothing to be false. The false judgment is false precisely because it means a state of affairs that is the opposite of the state one intends to affirm, namely, the state that truly is.

On the level of conception there is a similar but less conspicuous contrast between meaning and its core, which is the

intention of being. Horses and unicorns, electrons and phlogiston, may be equally valid as formal terms of meaning. One can suppose them, or consider them, or define them, and that is all that is required of the formal term of meaning. Still, horses and electrons seem preferable as formal terms to unicorns and phlogiston. Absolutely, one can think of the latter, but there is something idle, something superfluous, something futile about such thinking. The reason for this is that thinking is a moment in the unfolding of the pure desire to know; though the thought as thought is merely a formal term of meaning, though the unicorn is just as valid a formal term as is the horse, still we do not merely think. Our thinking is purposive. It is a tentative determination of the all-inclusive notion of being. It not merely thinks the object of thought but also anticipates the object of judgment. It not merely means the formal term of meaning but also looks ahead to the full term. Because the unicorn and phlogiston are known to be unsuccessful determinations of being, they are formal terms in which the core of meaning, the intention of being, has become uninterested.

Finally, in view of the prevalence of empiricist theories of meaning, a few words may be added on instrumental acts. Ordinary instrumental acts, such as spoken or written words or symbols, offer no special interest. But the empiricist emphasizes ostensive acts, such as demonstrative pronouns and adjectives and, of course, gestures. The reason for this emphasis may be readily grasped if one distinguishes between the function of gestures in any theory of meaning and the function gestures acquire in virtue

of empiricist affirmations. In any theory of meaning an ostensive act is an instrumental act of meaning: it presupposes formal or full acts of meaning, inasmuch as one knows what one means; and it refers to formal or full terms of meaning, inasmuch as all meaning refers to a meant. Again, in any theory of meaning the ostensive act is operative inasmuch as it succeeds in drawing another's attention to a sensible source of meaning, so that by drawing on that source, by understanding, and by reflecting he may reach the appropriate formal or full term of meaning that is meant. But in empiricist opinion the ostensive act has a third function; for the empiricist identifies the valid field of full terms of meaning (i.e., the universe of being) with the range of sensible presentations; hence, for the empiricist, the ostensive act not merely indicates a source of meaning but also a full term of meaning. Whether or not this empiricist modification of the general theory of meaning is correct, will depend on the question whether or not the set of propositions that enunciate empiricism are to be pronounced true or false.

6. A Puzzling Notion.

Before going on to consider other accounts of the notion of being, it will be well to deal with a series of puzzles that seem to have a common root. Just as other concepts, the notion of being is represented by instrumental acts that are the name, being, and the verb, to be. By mistaken analogy it is inferred that the notion of being resembles concepts in their other aspects. But, in fact, the notion of being is unique; for it is the core of all acts of meaning; and it underpins, penetrates, and goes beyond all other

cognitional contents. Hence, it is idle to characterize the notion of being by appealing to the ordinary rules or laws of conception. What has to grasped, is its divergence from such rules and laws and, to descend to details, a series of questions will be briefly considered.

First, does the notion of being result from the expression or formulation of an act of understanding?

Other concepts result from some insight either into the use of their names, or into things-for-us, or into things-themselves. The notion of being penetrates all other contents, and so it is present in the formulation of every concept. ~~But the notion of being has quite a different origin.~~ It cannot result from an insight into being, for such an insight would be an understanding of everything about everything, and such understanding we have not attained. It is, as has been said, the orientation of intelligent and rational consciousness towards an unrestricted objective.

Secondly, has the notion of being an essence, or is it an essence?

As other concepts result from acts of understanding, as acts of understanding consist in grasping what, from some viewpoint, is essential, other concepts are essences. Moreover, as other concepts are complete prior to the question for reflection that asks whether or not any such essence is, other concepts are merely essences and prescind from existence or actuality. But the notion of being does not result from an understanding of being; it does not rest on the grasp of what from some viewpoint is essential; and so the notion of being is not the notion of some essence.

Further, the notion of being remains incomplete on the level of intelligence; it moves conception forward to questions for reflection; it moves beyond single judgments to the totality of correct judgments; and so it does not prescind from existence and actuality.

Thirdly, can the notion of being be defined?

It cannot be defined in any ordinary manner, for it underpins and penetrates and goes beyond the content of every definition. However, it does possess certain definite characteristics. F For it regards the unrestricted objective of our knowing, the concrete universe, the totality of all that is. Moreover, it is determinate inasmuch as the structure of our knowing is determinate, and so it can be defined, at a second remove, by saying that it refers to all that can be known by intelligent grasp and reasonable affirmation. On the other hand, such definition does not settle which questions are appropriate to our knowing or which answers are correct. It leaves the materialist free to claim that to be is to be material. Equally, it allows the empiricist to claim that to be is to be experienced, the idealist to insist that to be is to be thought, the phenomenalist to explain that to be is to appear, and so forth.

Fourthly, how can one notion have such diverse meanings?

Because it is determinate only at a second remove. The notion of being is the notion of what is to be determined by correct judgments. If the strategic correct judgments are that matter exists and nothing but matter exists, then the materialist is right. If the strategic correct judgments are that there is

appearance and nothing but appearance, then the phenomenalist is right. Similarly, if the propositions enunciating other positions are correct, then being is as such positions declare. The notion of being does not determine which position is correct; it merely determines that the intelligently grasped and reasonably affirmed is being.

Fifthly, has the notion of being any presuppositions or properties?

Other concepts are determinate essences and so they have presuppositions and implications. If X is not an animal, then X is not a man. If X is a man, then X is mortal. But the notion of being is not the notion of some essence. It becomes determined only as correct judgments are made, and it reaches its full determination only when the totality of correct judgments (are to be made). However, the making of judgments is a determinate process, and one does not have to make all judgments to grasp the nature of that process. It is this fact that makes cognitional theory a base of operations for (the determination of) the general structure of the concrete universe.

Sixthly, is the notion of being univocal or analogous?

Concepts are said to be univocal when they have the same meaning in all applications, and they are said to be analogous when their meaning varies systematically as one moves from one field of application to another. The notion of being may be named univocal inasmuch as it underpins all other contents; for in that respect it is the one desire to know and it regards one unres-

tricted objective that is the concrete universe. Again, the notion of being may be named analogous inasmuch as it penetrates all other contents: in this fashion it is said that esse viventium est vivere; the being of living things is being alive. Finally, the notion of being may be said to be neither univocal nor analogous, for this distinction regards concepts, while the notion of being both underpins and goes beyond other contents. It may be noted, however, that what frequently enough is meant by the analogy of being is precisely what we mean by saying that the notion of being underpins, penetrates, and goes beyond other contents.

Seventhly, is the notion of being abstract?

For a notion to be abstract it must possess a determinate content and abstract from other contents. The notion of being abstracts from nothing whatever. It is all-inclusive. Its content is determined by the totality of correct judgments.

However, there is a still larger totality of possible judgments; within it there are strategic sets that serve to define the general character of the concrete universe in accord with the varying viewpoints of different philosophies. Such strategic sets have already been illustrated, e.g., there is matter and nothing but matter, or there is appearance and nothing but appearance, or there is thought and nothing but thought, or the structure of our knowing is determinate and so the structure of being proportionate to our knowing is determinate.

Now in virtue of such strategic sets of judgments it is possible to distinguish between the general character of the concrete universe, and, on the other hand, the concrete universe

in all its details. Closely enough, a determination of the general character of the concrete universe is an abstract view of being, for it considers not the whole of being as a whole but the whole of being as fixed by some strategic part or aspect.

In this fashion one reaches a general meaning for the phrase, being as being. But to determine what being as being is in any particular philosophy, one has to examine the strategic judgments of that philosophy: and to determine what is the correct meaning of being as being, one has to examine the strategic judgments of the correct philosophy.

Eighthly, is the notion of being a genus or species or difference?

Inasmuch as the notion of being is prior to all other cognitional contents, it is like a genus awaiting division by the addition of differences. But inasmuch as the notion of being anticipates, penetrates, and includes all other contents, it differs from the genus, which is a determinate content quite distinct from the content of its differences. Thus, being can be divided into red, green, and blue beings; and color can be divided into red, green and blue colors. But the concept of red has a content or element of content absent in the concept of color, and so it differentiates the genus by adding to it from without. On the other hand, the concept of red has no content and no element of content absent in the notion of being; it cannot differentiate being by adding to it from without for, without being, apart from being, there is simply nothing. Finally, the notion

of being not only underpins and penetrates all other contents but also complements them inasmuch as the "Yes" of judgment constitutes them as actually unconditioned, and so endows them with an actual objective reference.

Ninthly, when one thinks without as yet judging, either one is thinking of being or of nothing. If one is thinking of being, then one does not need to judge in order to know being. If one is thinking of nothing, then all thought must be identical, for it always deals with the same nothing.

When one thinks, conceives, considers, supposes, or defines, one looks to with respect to being. Hence we accept the first alternative. What one thinks of, is being. Still, to think of being is one thing; to know being is another. To think of being is to operate on the second level of cognitional process: it is to be on the way towards a complete increment of knowing; but it is not to have reached anything more than a partial increment that can be completed only by judging.

Tenthly, the notion of being is the notion of the concrete universe. But universal propositions are abstract and, none the less, they may be affirmed in judgment. Either, then, judgment is not about being, or else being is not concrete.

The notion of being is the notion of the concrete in the same manner as it is of the universe. It is of the universe because questions end only when there is nothing more to be asked. It is of the concrete, because until the concrete is reached, there remain further questions. Hence, it is not the single judgment but the totality of correct judgments that equates with the con-

crete universe that is being.

The problem of the universal proposition may be met by distinguishing between the formal and the material aspects of the analytic proposition. Formally an analytic proposition is 1) a conditioned, 2) linked to its conditions by the laws governing the coalescence of the partial instrumental meanings of words into the complete instrumental meaning of the sentence, and 3) having its conditions fulfilled by the meanings or definitions of the words it employs. Materially analytic propositions differ inasmuch as the terms and relations employed 1) may be known to occur in concrete judgments of fact, 2) may not be known to occur in concrete judgments of fact, or 3) may be known not to occur in concrete judgments of fact.

Formally every analytic proposition regards the concrete universe inasmuch as syntactical laws are factual aspects of the coalescence of partial into complete instrumental meanings. Materially some analytic propositions regard the concrete universe either in fact, as in the first case, or tentatively, as in the second.

7. Theories of the notion of Being

A distinction has been drawn between the spontaneously operative notion of being, common to all men, and theoretical accounts of that notion, ~~that~~ which differ from one philosophy to another. Our own theoretical account has been given. It remains that further clarifications be sought by contrasting it with some of the views that have been proposed by others.

For Parmenides, Being was one, without origin or end, homogeneous and indivisible, immovable and unchangeable, full and

spherical. See F.M. Cornford, Plato and Parmenides, London, 1939, pp. 23 ff.

The genesis of this position would seem to be as follows. Parmenides eliminated the alternative of blank negation, and so was left with the alternative of affirming. Affirmation may be reasonably grounded, and then it is the Way of Truth, or it may lack reasonable grounds, and then it is the Way of Seeming. Parmenides arrived at his notion of being by following the Way of Truth.

What does the choice of reasonable affirmation imply being to be? If one accepts any affirmation, one has also to accept the correct statement of the meaning, suppositions, and consequences of that affirmation. Every judgment stands in need of a context, and without affirming the context the affirmation of the initial judgment loses its meaning. Thus, reasonable affirmation has to be the affirmation of a set of judgments, which form a single whole, and so the affirmed is a corresponding single whole.

What is this single whole that is affirmed to be? The proper answer is to get to work inquiring and reflecting with respect to the whole of experience. The whole to be known corresponds to the totality of correct judgments. But Parmenides took a shorter route. He did not advert to the fact that being admits no more than a definition of the second order. He treated the notion of being as though it were a concept like "man" or "circle". He supposed that it was a determinate essence with determinate suppositions and determinate consequences. Because being is, it cannot be not-being, nor becoming, nor ceasing to be. Inversely, neither not-being nor becoming nor ceasing to be are being, and so they

must be nothing. Again, being cannot be differentiated; what differs from being, is not being; and what is not being, is nothing. Again, since there are no differences within being, there can be no motion or change within being. Finally, emptiness, the void, is nothing; being is not nothing, and so it cannot be emptiness; therefore, it is full. Etc.....

Plato's Forms were projections into a poetic heaven of what transcends ordinary, sensitive experience. The Forms, then, are the ideal objectives of 1) aesthetic experience, 2) the insights of the mathematician and physicist, 3) the unconditioned of reflective understanding, 4) moral conscience, and 5) intelligently and reasonably purposive living. They are a confused bag and, as it seems, the Parmenides marks the turning point in which the necessity of drawing distinctions and setting up a more comprehensive theory becomes evident.

In the Sophistes the philosopher is described as heading through rational discourse for the Idea of Being (254 a). It is acknowledged that the isolation of each Form from all the others would eliminate the possibility of discourse which lies in the conjunction of distinct Forms or categories (259 e). There is, then, a commingling or participation among the Forms (259 a) and there is a Form of Not-being just as much as of the Great or the Fair (258 c).

The inadequacy of this position lies in its failure to distinguish between the level of intelligence and the level of reflection. Without that distinction, the unconditioned of judgment is surreptitiously attributed to mere objects of thought to transform them into eternal Forms and, inversely, the "is" and "is not"

by which judgment posits the unconditioned can have a meaning only if they too are supposed to be Forms. There results an aggregate of Forms, each radically and eternally distinct from all the others. Still they are to be reached only through rational discourse, and if discourse is to refer to them, then there must be a commingling on their part to correspond to the synthetic elements in discourse. What is this commingling of distinct Forms? It would seem better, before trying to answer so difficult a question, to determine whether or not the question really arises. In fact, we would argue, it does not. Until judgment is reached, the increment of knowing is incomplete. Before judgment is reached, the synthetic element is already present in knowing. All that judgment adds to the question for reflection is the "Yes" or "No", the "is" or "is not". What is affirmed or denied may be a single proposition or the whole set of propositions constitutive of a hypothesis, for either may be regarded as conditioned and either may be grasped as virtually unconditioned. Judgment, then, is not a synthesis of terms but the unconditioned positing of such a synthesis. Corresponding to judgment there is not a synthesis of Forms but the absolute of fact. Platonism is magnificient in its devotion to the pure desire to know. But its failure to grasp the nature of judgment resulted in a deviation from the concrete universe of fact to an ideal heaven.

Aristotle clung to the Platonist definition of judgment as a synthesis (Sophistes 263; De Anima III,6,430a 26). Still, he distinguished sharply between questions for intelligence (What is

it? Why is it so?) and questions for reflection (Is it? Is it so?) [Post. Anal., II,1,89b 23ff] with the result that he had a sane and clear-headed respect for fact without reaching its exact implications. He would not have agreed with the empiricist that who places fact, not in the virtually unconditioned, but in the sensible fulfilment through which the conditioned becomes grasped as unconditioned. But you could put him a question he had not adequately considered, if you asked him whether the virtually unconditioned was a third component in our knowing or, on the other hand, merely a rubber-stamp of approval attached to the conceptual unification of the sensible and intelligible components.

This unresolved ambiguity appears both in his methodology and in his metaphysics. For him the supreme question was the question of existence. Still it was a question that was already answered in descriptive knowing; that answer had to be presupposed in the search for explanation; and the function of explanation was simply to determine what things are and why they have the properties they possess. The intrinsically hypothetical character of explanation and its need of a further, verifying judgment of existence were overlooked. Again, Aristotle asks, what being is. That question expresses the demand for understanding, for knowledge of the cause. Quite naturally, Aristotle answers that the cause of being is its immanent form (Met. Z,17). Primarily, being is what is constituted by a substantial form or, on second thoughts, by the combination of substantial form and matter. Secondarily, being is what is constituted by accidental forms; "white","heat", "strength" are not nothing though they are not simply what is

meant by being. Again, being is the collection of existing substances with their properties and incidental modifications; but though being denotes the factually existent, still existing is no more than the reality of substantial forms along with their mainly immanent superpositions and consequences. (See S. Mansion, Le jugement d'existence chez Aristote, Louvain-Paris 1946; J. Owens, The Doctrine of Being in Aristotle's Metaphysics, Toronto Pontifical Institute of Mediaeval Studies, 1951.)

Quite plainly this position is going to give rise to a problem of the unity of the notion of being. Aristotle broke with his Parmenidean and Platonist antecedents by identifying being with the concrete universe as, in fact, it is known to be. But Aristotle did not break with their supposition that the notion of being was a conceptual content. He asked what being is. In other words, he supposed that being is some conceptual content and he demanded what act of understanding occurred prior to the formulation of that content. But, as we have seen, being can be defined by us only indirectly, and so Aristotle was unable to assign any specific act of understanding that resulted in the conceptual content of being. However, the conspicuous type of acts of understanding is the insight that grasps intelligible form emergent in sensible data; and so Aristotle assigned the ontological principle, form, as the ground of being in things and the cognitional act of grasping form as the insight from which originates the conceptual content, being.

In this fashion, mediaeval Scholasticism inherited a pro-problem. Is the notion of being one or is it many? If it is one,

is its unity the unity of a single content or is it the unity of a function of variable contents?

Henry of Ghent seems to have held that the unity of being is merely the unity of a name. God is, and I am. In both cases, being is affirmed. But the realities affirmed are simply disparate.

Duns Scotus contended that, besides the unity of the name, there is also a unity of content. If no part or aspect of you is by identity a part or aspect of me, still neither of us is nothing. There is, then, some minimal conceptual content that positively constitutes what is expressed negatively by the negation of nothing. What it is, cannot be declared by appealing to other positive contents, for it is one of the ultimate atoms of thought; it is simply simple. Still one can approach it by noting that Socrates supposes man, man supposes animal, animal supposes living material substance, and substance supposes a something that is even less determinate and less exclusive. The concept of being is the concept with least connotation and greatest denotation. Moreover, it is essentially abstract. What it denotes, is never just being, but either the infinite or some finite mode of being, where the mode is to be viewed not as some further and distinct content but rather as an intrinsic variation of basic, indeterminate content. (See A.B. Wolter, The Transcendentals and their Function in the Metaphysics of Duns Scotus, Washington: CUA* 1946; A . Mare, L'Idée de l'être chez saint Thomas et dans la scholastique postérieure, Arch. de Phil.,X, 1933, 31-49).

The as to Vio Caietanus was no more satisfied with the Scotist view, than Scotus himself had been satisfied with that of Henry of Ghent. If a single name without a single meaning will not

*CUA = Catholic University of America.

do, neither will a single meaning that as single seems restricted to the order of thought. Accordingly, Cajetan worked out his theory of the unity of a function of variable contents. Just as "double" denotes indifferently the relation of 2 to 1, 4 to 2, 6 to 3, and so forth, so "being" denotes indifferently the proportion of essence to existence or, as we might say, the proportion between what is formulated by thought and what is added to it by judgment. On this position the notion of being always includes some conceptual content but it may include any; again, being in act will never be known without some affirmative judgment, but the affirmation is never mere affirmation nor the affirmation of an indeterminate content: it is always the affirmation of some determinate content, and any affirmable, determinate content will do. In brief, Cajetan can grant that atomic conceptual contents are many and disparate; he can deny the Scotist view that there is some common factor, some positive counterpart of "not nothing", of absolutely universal denotation; and yet by his theory of the unity of a function of variable contents, he can possess not only a single name, being, and a single notion of being, but also a single notion that is applicable to anything that in fact, is known to exist. (A. Marc., Op. cit. 50-66).

It is to be noted that, if Scotus stands for the Parmenidean and Platonist suppositions from which Aristotle did not free himself, Cajetan stands for the main orientation of Aristotelian thought but succeeds in doing so only by going beyond it. If conceptual contents are products of acts of understanding that grasp forms emergent in sensible presentations, one may well expect such contents to be a disparate multiplicity. Hence,

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Aristotle answered the question, What is being?, not by assigning a conceptual content but by assigning the ground of being in the general object of understanding, form. Since forms are many, it follows that the ground of being is a variable; further, it follows that if the notion of being is to be one, then its unity will have to be the unity of a function of variable contents. What, then, are the variables within the single function? One of them is form. At first sight, the obvious candidate for the other is matter. Still, if it were selected, it would follow that Aristotle's immaterial substance would not belong to the universe of being. To maintain the Aristotelian position in its integrity, it was necessary to make the second variable the virtually unconditioned grasped by reflective understanding and affirmed in judgment; this in the general case is existence, actuality, fact, that combines with pure form or the compound of form and matter to constitute a being in act.

Brilliant as it is, Cajetan's position has its shortcomings. It envisages an aggregate of concrete beings each of which is constituted by essence and existence. It offers as the unity of the notion of being the relation or proportion of what is conceived to its being affirmed. But it does not elucidate how that relation emerges in our knowledge as a single notion; and it gives no clue to account for the fact that by "being", we mean, not only this and that being, but everything, totality, the universe. In brief, Cajetan seems to have been more interested in explaining the unity of the notion of being than the notion itself.

To complete Cajetan's position, it is necessary to go back to his master, St. Thomas Aquinas. For Aquinas, as for Aristotle,

human intellect is a potential omnipotence, a potens omnia facere et fieri. But Aquinas could exploit that affirmation in a manner that would have startled Aristotle.

First, he recognized an unrestricted desire to know. As soon as we learn of God's existence, we wish to understand His nature. To achieve such understanding is beyond the power of our natural capacity, yet in such achievement lies our spontaneously desired beatitude. (I,12,1,ff; I-II,3,8; 5,5).

Secondly, the unrestrictedness native to intellect grounds the affirmation that the object of intellect has to be being. Because intellect is potens omnia fieri, its object is ens. (I,79, 7,c). Being and everything are equivalent notions.

Thirdly, for the same reason, an intellect fully in act must be infinite and uncreated act. Any created intellect must in some manner be potential, and our intellects start from a zero of potentiality. (1,79,2 c. CG. II,98).

Fourthly, none the less, being is per se and naturally known to us (CG. II,83,#31), and it cannot be unknown to us. (De Ver. 11,1,3m). Avicenna had interpreted Aristotle's agent intellect as some separate immaterial substance. Aquinas found it immanent within us; the light of intelligence, which is in us, performs the functions Aristotle attributed to agent intellect, and, moreover, Aristotle compared agent intellect to a light. (CG. II,77,#5). Augustine had advanced that our knowledge of truth originated, not without but within us, yet not simply within us, but in some illumination in which we consulted the eternal

grounds and norms of things. Aquinas explained that we consult the eternal ground and norms, not by taking a look at them, but by having within us a light of intelligence that is a created participation of the eternal and uncreated light. (I,84,5,c.)

Fifthly, though being is naturally known, though our intellects are created participations of uncreated light, still, there is no valid ontological argument for the existence of God (I,2,1c.). God's knowledge of being is a priori; He is the act of understanding that grasps everything about everything; but we advance towards knowledge by asking the explanatory question, Quid sit? and the factual question, An sit?

In such positions it is easy to discern not only the justification of Cajetan's theory of analogy but also the elements which that theory tends to overlook. Prior to conception and to judgment, there is the dynamic orientation of intelligent and rational consciousness with its unrestricted objective. This orientation is man's capacity to raise questions and thereby to generate knowledge. Immanent within man, it is a spark of the divine. Cognate to God, still it is knowing, not in act but in sheer potency. As it is the common root of intelligent grasp and reasonable judgment, so also it is the root of the relation or proportion between the conceived essence and the affirmed existence. As its objective is unrestricted, so it regards not only single compounds of essence and existence but also the universe, totality, infinity.

It has been noted how Cajetan saves the main orientation of Aristotelian thought by going beyond it and, though this involves still more metaphysics, it may be added how Aquinas does so.

Aristotle asked what being is. But "What?" is just a disguised "Why?" What the question really asks for is the ground of being, and so Aristotle answered by indicating substantial form as the immanent cause of each being. But since his substantial form was not some unique and separate Platonic Idea, his answer gave rise to the problem of the unity of the notion of being. Now if Aquinas were to ask the same question, his answer would be that God is the ground of being; God's own being is self-explanatory and necessary; by the Aristotelian theorem of the identity of knower and known, God's being is identical with God's understanding; by that single act of understanding, God understands himself, and so he understands his own power, and so he understands all that by that power could be produced. God, then, is the act of understanding that grasps everything about everything. The content of the divine act of intellect is the idea of being and so, precisely because our intellects are potential, they can define being only at a second remove as whatever is to be known by intelligent grasp and reasonable affirmation.

Again, both the position of Cajetan and the position of Scotus stand within the field accessible to the logician. By going behind that field to its dynamic basis, one can find the ground not only of Cajetan's proposition but also of Scotus' minimal content. What is it that is common to every conceptual content? It is that all are underpinned and penetrated by the pure desire's intention of its unrestricted objective. The Scotist notion of being is reached by distinguishing between the penetrating intention of being and the penetrated conceptual content; from instance to

instance the conceptual content differs; but in every instance, there is the anticipating, enveloping, penetrating intention, and that is what the Scotist alleges to be a common factor in all contents.

Still if the intention of being is a common factor in all conceptual contents, it is also a dynamic factor that goes beyond them. To set aside this dynamism is to nullify not only what lies beyond the conceptual contents but also the intention of being itself. In a famous little treatise, Aquinas had remarked, "Essentia dicitur secundum quod per eam et in ea ens habet esse." It is in and through essences that being has existence. Hence, being apart from essence is being apart from the possibility of existence; it is being that cannot exist; but what cannot exist is nothing, and so the notion of being apart from essence is the notion of nothing.

It will be worth grasping why Scotus felt he could escape this conclusion while Hegel felt that he could not avoid it. Scotus felt he could avoid it because he conceived knowing, not as process that reaches a complete increment in judgment, but as taking a look. When Scotus separated his notion of being from other conceptual contents, he also separated that notion from the possibility of judgment. Still that separation did not imply for Scotus a separation from the possibility of knowing, for he viewed knowing, not as ultimately constituted by judging, but as essentially a matter of looking. He would grant that there was no look in which the seen was solely the common content that he named being. But he would insist that that common content was included in the

object of every intellectual intuition, and still more would insist that a look at nothing, an intuition of nothing, was absurd. In brief, for the Scotist, being is an aspect of the real at which intellect looks; the theory of modes and the distinction between quidditative and tendinative being are efforts to blow this aspect up to the dimensions of the whole. For the Thomist, on the other hand, being is the whole of what intelligence anticipates; it is the objective of an unrestricted, dynamic orientation; it is whatever intelligent grasp and reasonable affirmation will determine; and so the notion of being is open to all the incomplete and partial moments from which cognitional process suffers without ever renouncing its all-inclusive goal.

Four Five hundred years separate Hegel from Scotus. As will appear from our discussion of the method of metaphysics, that notable interval of time was largely devoted to working out in a variety of manners the possibilities of the assumption that knowing consists in taking a look. The ultimate conclusion was that it did not and could not. If the reader does not himself accept that conclusion as definitive, certainly Hegel did and so Hegel could not take advantage of the Scotist escape from the identification of the notion of being with the notion of nothing. But Hegel was bored on the other side as well. He effectively acknowledged a pure desire with an unrestricted objective. But he could not identify that objective with a universe of being, with a realm of factual existents and occurrences. For being as fact can be reached only in so far as the virtually unconditioned is reached; and as Kant had ignored that constitutive component of judgment,

so Hegel neither rediscovered nor re-established it. The only objective Hegel can offer the pure desire is a universe of all-inclusive concreteness that is devoid of the existential, the factual, the virtually unconditioned. There is no ~~rhyme or~~ reason why such an objective should be named being. It is, as Hegel named it, an Absolute Idea. It is the all-inclusive summit of the pure desire's immanent dialectical process from position through opposition to sublation that yields a new position to recommence the triadic process until the Absolute Idea is reached.

Now if the intention that is the pure desire has neither a Scotist reality on which it can look back, nor a Thomist universe of existents, to which it can look forward, none the less, in psychological fact it underpins and penetrates all conceptual contents. It constitutes, then, a common factor in all conceptual contents; it can be distinguished from them, for it is identical with none of them; yet, as distinguished from them, it becomes indistinguishable from the notion of nothing; for the only ground of the latter distinction would be that it looked back or forward to something.

It is interesting to note that, if the foregoing succeeds in fixing fundamental features of Hegel's thought, by that very fact it shows that on Hegelian criteria, Hegelianism is mistaken. Hegel's System is not afraid of facts; it explains any fact alleged against it by showing it to be a manifestation of an incomplete viewpoint included within the System. Hegel's System is not afraid of contradictions; it explains any contradiction alleged against it by revealing that opposed and incomplete viewpoints, accounted

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for by the System, yield the alleged contradictory terms. The only thing the System has to fear is that it itself should be no more than some incomplete viewpoint and, in fact, that is what it is. Hegel aimed at rehabilitating the speculative reason that Kant had dethroned. But the basis of the Kantian attack was that the unconditioned is not a constitutive component of judgment. A complete rehabilitation of human rational consciousness will show that the unconditioned is a constitutive component of judgment. This, Hegel did not do. His viewpoint is essentially the viewpoint of a thinker who does not and cannot regard the factual as unconditioned, who cannot acknowledge any factually fixed points of reference, who cannot advance by distinguishing the definitively certain, the more or less probable, and the unknown. Hegel's range of vision is enormous; indeed, it is unrestricted in extent. But it is always restricted in content, for it views everything as it would be if there were no facts. It is a restricted viewpoint that can topple outwards into the factualness of Marx or inwards into the factualness of Kierkegaard. It is a viewpoint that is transcended automatically by anyone that, in any instance, grasps the virtually unconditioned and affirms it.

For this reason, we placed the discussion of Self-affirmation prior to the discussion of the Notion of Being. Self-affirmation is the affirmation of the knower, conscious empirically, intelligently, rationally. The pure desire to know is a constituent element both of the affirming and of the self that is affirmed. But the pure desire to know is the notion of being as it is spontaneously operative in cognitional process and being itself is the to-be-known towards which that process heads.

human intellect is a potential omnipotence, a potens omnia facere et fieri. But Aquinas could exploit that affirmation in a manner that would have startled Aristotle.

First, he recognized an unrestricted desire to know. As soon as we learn of God's existence, we wish to understand His nature. To achieve such understanding is beyond the power of our natural capacity, yet in such achievement lies our spontaneously desired beatitude. (I, 12, 1, ff.; I-II, 3, 8; 5,5).

Secondly, the unrestrictedness native to intellect grounds the affirmation that the object of intellect has to be being. Because intellect is potens omnia fieri, its object is ens. (I, 79, 7, c). Being and everything are equivalent notions.

Thirdly, for the same reason, an intellect fully in act must be infinite and uncreated act. Any created intellect must in some manner be potential, and our intellects start from a zero of potentiality. (1, 79, 2 c. CG. II, 98).

Fourthly, none the less, being is per se and naturally known to us (CG. II, 83, #31) and it cannot be unknown to us. (De Ver. 11, 1, 3m). Avicenna had interpreted Aristotle's agent intellect as some separate immaterial substance. Aquinas found it immanent within us; the light of intelligence, which is in us, performs the functions Aristotle attributed to agent intellect, and, moreover, Aristotle compared agent intellect to a light. (CG.II, 77, #5). Augustine had advanced that our knowledge of truth originated, not without but within us, yet not simply within us, but in some illumination in which we consulted the eternal

Chapter XIII : THE NOTION OF OBJECTIVITY

Human knowing is cyclic and cumulative. It is cyclic inasmuch as cognitional process advances from experience through inquiry and reflection to judgment, only to revert to experience and recommence its ascent to another judgment. It is cumulative, not only in memory's store of experiences and understanding's clustering of insights, but also in the coalescence of judgments into the context named knowledge or mentality.

This complexity of our knowing involves a parallel complexity in our notion of objectivity. Principally the notion of objectivity is contained within a patterned context of judgments which serve as implicit definitions of the terms, object, subject. But besides this principal and complete notion, there also are partial aspects or components emergent within cognitional process. Thus, there is an experiential aspect of objectivity proper to sense and empirical consciousness. There is a normative aspect that is contained in the contrast between the detached and unrestricted desire to know and, on the other hand, merely subjective desires and fears. Finally, there is an absolute aspect that is contained in single judgments considered by themselves inasmuch as each rests on a grasp of the unconditioned and is posited without reservation.

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1. The Principal Notion

Principally, the notion of objectivity is contained in a patterned context of judgments. For one may define as object any A, B, C, D,..... where, in turn, A,B, C, D,..... are defined by the correctness of the set of judgments:

A is; B is; C is; D is; .....
A is ~~not~~ neither B nor C nor D nor ....
B is ~~not~~ neither C nor D nor .....
C is ~~not~~ neither D nor .. ..

Again, one may define a subject as any object, say A, where it is true that A affirms himself as a knower in the sense explained in the ~~section~~ chapter on self-affirmation.

The bare essentials of this notion of objectivity are reached if we add to the judgments already discussed, viz., I am a knower, This is a typewriter, the further judgment that I am not this typewriter. An indefinite number of further objects may be added by making the additional appropriate positive and negative judgments. Finally, in so far as one can intelligently grasp and reasonably affirm the existence of other knowers besides oneself, one can add to the list of the objects that are also subjects.

The properties of the principal notion of objectivity have now to be noted. First, as has already been remarked, the notion resides in a context of judgments; without a plurality of judgments that satisfy a definite pattern, the notion does not emerge. Secondly, there follows an immediate corollary: the principal notion of objectivity, as defined, is not contained in any single judgment and, still less, in any experiential or normative factor

that occurs in cognitional process prior to judgment. Thirdly, the validity of the principal notion of objectivity is the same as the validity of the set of judgments that contain it; if the judgments are correct, then it is correct that there are objects and subjects in the sense defined, for the sense defined is simply the correctness of the appropriate pattern of judgments.

Fourthly, to turn to certain broader aspects of the principal notion, judgments in the appropriate pattern commonly are made and commonly are regarded as correct. It follows that commonly people will know objects and subjects and that commonly they will be surprised that any doubt should be entertained about the matter. On the other hand, it does not follow that people will commonly be able to give a lucid account of their knowledge of objects and subjects. For the lucid account employs the somewhat recondite art of implicit definition and, at the same time, people are apt to jump to the conclusion that so evident a matter as the existence of objects and subjects must rest on something as obvious and conspicuous as the experiential aspect of objectivity. Hence, on the one hand, they will say that the typewriter is an object because they see it or feel it; on the other hand, however, they will admit that would not consider the typewriter an object if they knew it to be true either that there was no typewriter at all or that what they named a typewriter was identical with everything else.

Fifthly, the principal notion of objectivity is closely related to the notion of being. Being is what is to be known through the totality of correct judgments. Objectivity in its principal

sense is what is known through any set of judgments satisfying a determinate pattern. In brief, there is objectivity if there are distinct beings, some of which both know themselves and know others as others. Moreover, the notion of being explains why objectivity in its principal sense is to be reached only through a pattern of judgments. For the notion of being becomes determinate only in so far as judgments are made; prior to judgment, one can think of being but one cannot know it; and any single judgment is but a minute increment in the process towards knowing it. Again, being is divided from within; apart from being there is nothing; it follows that there cannot be a subject that stands outside being and looks at it: the subject has to be before he can look; and, once he is, then he is not outside being but either the whole of it or some part. If he is the whole of it, then he is the sole object. If he is only a part, then he has to begin by knowing a multiplicity of parts (A is; B is; A is not B) and add that one part knows others (I am A).

Sixthly, the principal notion of objectivity solves the problem of transcendence. How does the knower get beyond himself to a known? The question is, we suggest, misleading. It supposes the knower to know himself and asks how he can know anything else. Our answer involves two elements. On the one hand, we contend that, while the knower may experience himself or think about himself without judging, still he cannot know himself until he makes the correct affirmation, I am, and then we contend that other judgments are equally possible and reasonable, so that through experience, inquiry, and reflection there arises knowledge of other

objects both as beings and as being other than the knower. Hence, we place transcendence, not in going beyond a known knower, but in heading for being within which there are positive differences and, among such differences, the difference between object and subject. Inasmuch as such judgments occur, there is in fact, objectivity and transcendence; and whether or not such judgments are correct, is a distinct question to be resolved along the lines reached in the analysis of judgment.

2. Absolute objectivity

Besides the principal notion of objectivity, there also are the partial aspects of experiental, normative, and absolute objectivity. It will be convenient to begin from the last of the three.

The ground of absolute objectivity is the virtually unconditioned that is grasped by reflective understanding and posited in judgment. As formally unconditioned, which has no conditions at all, stands outside the interlocked field of conditioning and conditioned; it is intrinsically absolute. The virtually unconditioned stands within that field; it has conditions; it itself is among the conditions of other instances of the conditioned; still its conditions are fulfilled; it is a de facto absolute.

Because the content of the judgment is an absolute, it is withdrawn from relativity to the subject that utters it, the place in which he utters it, the time at which he utters it. Caesar's crossing of the Rubicon was a contingent event occurring ~~in~~ at a particular place and time. But a true affirmation of that event is an eternal, immutable, definitive validity. For if it is true that he did cross, then no one whatever at any place or time can

truly deny that he did.

Hence, it is in virtue of absolute objectivity that our knowing acquires what has been named its publicity. For the same reason that the unconditioned is withdrawn from relativity to its source, it also is accessible not only to the knower that utters it but also to any other knower.

Again, it is the absolute objectivity of the unconditioned that is formulated in the logical principles of identity and contradiction. The principle of identity is the immutable and definitive validity of the true. The principle of contradiction is the exclusiveness of that validity. It is, and what is opposed to it, is not.

Further, absolute objectivity pertains to single judgments as single. As has been argued, the principal notion of objectivity is constituted only by a suitable constellation of judgments. But each judgment in such a constellation is an absolute and, moreover, it is an absolute in virtue of its own affirmation of the unconditioned. The validity of the principal notion is a derived validity resting on the set of absolutes it involves. But the absolute aspect of objectivity has its ground in the single judgment to which it pertains. It is quite compatible with the affirmation that there is but one being, that there is no object except the affirming subject; accordingly, the absolute aspect of objectivity does not imply any subject-object relation; it constitutes the entry of our knowing into the realm of being but, by itself, it does not suffice to posit, distinguish, and relate beings. However, this insufficiency arises, not from some defect of absolute

objectivity, nor because the posited beings, their distinction, and their relations are not all unconditioned, but because several judgments are needed to posit, to distinguish, and to relate.

It is important not to confuse the absolute objectivity of any correct judgment with the invariance proper to the expression of universal judgments. Both universal and particular judgments, if correct, are absolutely objective. But the former are expressed invariantly because the expression is independent of variations in spatio-temporal reference frames, while the latter are expressed relatively because their expression does not enjoy such independence. However, the variation of the expression presupposes and reveals the absolute objectivity of what is expressed. Because "I am here now" has absolute objectivity, ~~you can repeat the same~~ there is an identical truth to be repeated only by employing the different words, "he was there then".

Again, absolute objectivity has no implications of an absolute space or of an absolute time. If it is true that space is, then what is absolute is the truth and not the space. Whether the space is absolute or relative, is a further question. If it is true that space consists of an infinite set of immovable and empty places, then space is absolute. If it is true that space is not such a set, then space is relative. Which is correct? At least, the issue cannot be settled by appealing to the fact that a true judgment posits an unconditioned.

Further, as Zeno argued, to affirm that something or other is, does not imply that it is within space. If it did, one could ask whether or not the space (within which it is) is. If not, that space is nothing and to affirm things within nothing is meaningless. If, however, it is, then since "to be" is "to be within space", the

question recurs; if "X is" means "X is within space", it would seem to follow that "space is" means that "space is within space"; the second space cannot be identical with the first, else it would not contain it; and if it is distinct, then it can be only by being within a further space, and so on indefinitely.

The same argument holds for being within time. If "to be" is "to be at some time", then either there is time or there is not. If there is not, then "to be at some time" is really a mere "to be". If there is time, then it has to be at some time, and that at some time, and so forth to infinity.

Interpretations of being or of absolute objectivity in terms of space and time are mere intrusions of imagination. Absolute objectivity is simply a property of the unconditioned; and the unconditioned, as such, says nothing about space or time. If one's imagination makes the use of the preposition "within" imperative, then one may say that every judgment is within a context of other judgments and that every unconditioned is within a universe of being. Then "space is" by being within the universe of being, and "time is" by being within the universe of being, where to "be within the universe of being" is to "be unconditioned along with other instances of the unconditioned".

3. Normative objectivity

The second of the partial aspects of objectivity is the normative. It is objectivity as opposed to the subjectivity of wishful thinking, of rash or excessively cautious judgments, of allowing joy or sadness, hope or fear, love or detestation, to interfere with the proper march of cognitional process.

The ground of normative objectivity lies in the unfolding of the unrestricted, detached, disinterested desire to know.

Because it is unrestricted, it opposes the obscurantism that hides truth or blocks access to it, in whole or in part. Because it is detached, it is opposed to the inhibitions of cognitional process that arise from other human desires and drives. Because it is disinterested, it is opposed to the well-meaning but disastrous reinforcement that other desires lend cognitional process only to twist its orientation into the narrow confines of their limited range.

Normative objectivity is constituted by the immanent exigence of the pure desire in the pursuit of its unrestricted objective. A dynamic orientation defines its objective. No less, it defines the means towards attaining its objective. Not only does the pure desire head for the universe of being, but also it does so by desiring to understand and by desiring to grasp the understood as unconditioned. Hence, to be objective, in the normative sense of the term, is to give free rein to the pure desire, to its questions for intelligence, and to its questions for reflection. Further, it is to distinguish between questions for intelligence that admit proximate solutions and other questions of the same type that, at present, cannot be solved. Similarly, it is to distinguish between sound questions and, on the other hand, meaningless questions, or incoherent or illegitimate questions. For the pure desire not only desires; it desires intelligently and reasonably; it desires to understand because it is intelligent and it desires to grasp the unconditioned because it desires to be reasonable.

Upon the normative exigences of the pure desire rests the validity of all logics and all methods. A logic or method is not

an ultimate that can be established only by a hullabaloo of starry-eyed praise for Medieval Philosophy or for Modern Science, along with an insecure resentment of everything else. Logic and method are intelligent and rational; their grounds are not belief nor propaganda nor the pragmatic utility of atom-bombs and nylon stockings; their grounds are the inner exigence of the pure desire to know. They are to be accepted in so far as they succeed in formulating that dynamic exigence; and they are to be revised in so far as they fail.

In various manners this dependence has already been noted. Thus, the logical principles of identity and contradiction result from the unconditioned and the compulsion it exercises upon our reasonableness. The principle of excluded middle possesses ultimate but not immediate validity; it possesses ultimate validity because, if a judgment occurs, it must be either an affirmation or a denial; it does not possess immediate validity, for with respect to each proposition, rational consciousness is presented with the three alternatives of affirmation, of negation, and of seeking a better understanding and so a more adequate formulation of the issue. Again, the procedures of empirical method in its classical and statistical phases have been accounted for by the pure desire's movement towards understanding, towards an understanding that regards not only things as related to us by our senses but also things as related functionally among themselves, towards an understanding that presupposes data to admit systematization in the classical phase and, in other respects, to be non-systematic and so necessitate a statistical phase. Finally, precepts regarding judgment can

be derived from the general requirement of the unconditioned and from the special circumstances of different kinds of judgments which may be primitive or derived, theoretical or concrete, descriptive or explanatory, certain or probable.

4. Experiential objectivity

The third partial aspect of objectivity is the experiential. It is the given as given. It is the field of materials about which one inquires, in which one finds the fulfilment of conditions for the unconditioned, to which conditional process repeatedly returns to generate the series of inquiries and reflections that yield the contextual manifold of judgments.

Further, the given is unquestionable and indubitable. What is constituted by answering questions, can be upset by other questions. But the given is constituted apart from questioning; it remains the same no matter what the result of questioning may be: it is unquestionable in the sense that it lies outside the conditional levels constituted by questioning and answering. In the same fashion the given is indubitable. What can be doubted is the answer to a question for reflection; it is a "Yes" or a "No". But the given is not the answer to any question; it is prior to questioning and independent of any answers.

Again, the given is residual and, of itself, diffuse. It is possible to select elements in the given and to indicate them clearly and precisely. But the selection and indication are the work of insight and formulation, and the given is the residue that remains when one subtracts from the indicated 1) the instrumental act of meaning by which one indicates, 2) the concepts expressed

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by that instrumental act, 3) the insights on which the concepts rest. Hence, since the given is just the residue, since it can be selected and indicated only through intellectual activities, of itself it is diffuse; the field of the given contains differences, but in so far as they simply lie in the field, the differences are unassigned.

Again, the field of the given is equally valid in all its parts but differently significant in different parts.

It is equally valid in all its parts in the sense that there is no screening prior to inquiry. Screening is the fruit of inquiry. It takes place once inquiry has begun.

It is differently significant in different parts in the sense that some parts are significant for some departments of knowledge and other parts for other departments. The physicist has to disregard what he merely imagines, merely dreams, merely derives from his personal equation. The psychologist has to explain imagination, dreaming, and personal equations. Hence, once inquiry begins, the first step is the screening that selects the relevant field of the given.

~~Hence~~ We are employing the name, "given", in an extremely broad sense. It includes not only the veridical deliverances of outer sense but also images, dreams, illusions, hallucinations, personal equations, subjective bias, and so forth. No doubt, a more restricted use of the term would be desirable, if we were speaking from the limited viewpoint of natural science. But we are working at a general theory of objectivity and so we have to acknowledge as given not only the materials into which natural

science inquires but also the materials into which the psychologist or methodologist or cultural historian inquires.

There is a profounder reason. Our account of the given is extrinsic. It involves no description of the stream of sensitive consciousness. It involves no theory of that stream. It discusses neither the contribution of the empirically conscious subject nor the contribution of other "outside" agents. It simply notes that reflection and judgment presuppose understanding, that inquiry and understanding presuppose materials for inquiry and something to be understood. Such presupposed materials will be unquestionable and indubitable, for they are not constituted by answering questions. They will be residual and diffuse, for they are what is left over once the fruits of inquiry and reflection are subtracted from cognitional contents.

No such unquestionable and indubitable, residual and diffuse materials for inquiry and reflection must be regarded as equally valid in all their parts. Were they all invalid, there could be neither inquiry nor reflection, and so no reasonable pronouncement that they are invalid. Were some valid and others invalid, there would have to be a reasonably affirmed principle of selection; but such a principle can be grasped and reasonably affirmed only after inquiry has begun. Prior to inquiry there can be no intelligent discrimination and no reasonable rejection.

There is still a deeper reason. Why is the given to be defined extrinsically? Because all objectivity rests upon the unrestricted, detached, disinterested desire to know. It is that desire that sets up the canons of normative objectivity. It is

~~that desire that gives rise to the absolute objectivity. It is~~ that desire that gives rise to the absolute objectivity implicit in judgment. It is that desire that yields the constellation of judgments that implicitly define the principal notion of distinct objects in the universe of being, some of which know others. Experiential objectivity has to rest on the same basis, and so the given is defined, not by appealing to sensitive process, but by the pure desire regarding the flow of empirical consciousness as the materials for its operation.

5. Characteristics of the notion

An account has been given of a principal notion of objectivity and of its three partial aspects, the experiential, the normative, and the absolute. However, there also exists subjectivity, and the reader may be inclined to find in the present section a full confirmation of a suspicion that he has for some time entertained, namely, that we have failed to place our finger on what is objective, that we are confusing with the objective either in part or in whole what really is subjective. To deal with this problem will call for a further and rather complex investigation but, before we go on to it, let us note the more general characteristics of the notion of objectivity that has just been outlined.

First of all, despite its complexity, it can be the notion of objectivity that common sense presupposes and utilizes. The principal notion is implicit within a suitable pattern of judgments; it arises automatically when the judgments that happen to be made fall within such a pattern. The absolute aspect is implicit in judgment for, as we have argued at length, judgment affirms the unconditioned that reflective understanding grasps. The

normative aspect is not any set of rules that has to be invented; it results from the intelligent inquiry and the reflective reasonableness that are the unfolding of the pure desire to know. Finally, the experiential aspect, while it may appear to do violence to common sense expectations, is fully in accord with scientific practice which claims to be an extension and refinement of common sense.

Secondly, the notion of objectivity that has been outlined is a minimal notion. There arises the question, What is objectivity? If the answer is to be intelligent and reasonable, then the pure desire and its normative exigences must be respected. Moreover, there must be materials into which intelligence inquires and on which reasonableness reflects. Further, if there is a definitive answer, the unconditioned and so the absolute will be attained. Finally, if the question and answer have a point, there will be other judgments which, if they occur in an appropriate pattern, will yield the principal notion.

Thirdly, our notion of objectivity begs no questions. Just as our notion of being does not decide between empiricism and rationalism, positivism and idealism, existentialism and realism, but leaves that decision to the content of correct judgments that are made, so also our notion of objectivity is equally open. If judgments occur in the appropriate pattern, then it involves a plurality of knowing subjects and known objects. If in effect, there is only one true judgemnt, say, the affirmation of the Hegelian Absolute Idea, our notion of objectivity undergoes no formal modification. If true judgments are never reached, there arises

Chapter XV : Elements of Metaphysics. Foot-note to p. 704.

The relation of potency, form, and act, as defined, to Scholastic potentia, forma, actus, may be bracketed under the three headings of technique, principle, and method.

My definitions are systematic. In contrast, the normative influence exercised by the Scholastic disputation set a premium on definitions that were nominal, that prescinded from systematic views, that stated what would be meant by the members of any school.

Secondly, the principles on which my definitions are based would be rejected by the conceptualist wing of Scholastic thinkers. Because conceptualists deny insight, they eliminate what I mean by form. Because they conceive abstraction as impoverishing, they eliminate the distinction I draw between potency and form and deny its implication that matter is a principle of individuation. Because they consider judgment to be an adhaesio mentis that does not augment the content of knowledge, they eliminate the distinction I draw between form and act and deny its implication of a real distinction between essence and contingent existence.

Thirdly, the Scholastics that employ systematic definitions and hold principles similar to my own follow quite a different method. While the present metaphysics is epistemologically constructed in terms of the causa cognoscendi, theirs is ontologically constructed in terms of the causa essendi; and while my starting-point is restricted to proportionate being, theirs contains an explicit reference to transcendent being in a theorem on the intrinsic illimitation of act and the limiting roles of form and potency. Accordingly, it is only at the end of Chapter XIX that the isomorphism between Thomism and the present metaphysics can begin to appear.