Chapter V. Space and Time.

For a variety of reasons attention is now directed to the notions of space and time. Not only are these notions puzzling and so interesting, but they throw considerable light on the precise nature of abstraction, they provide a concrete and familiar context for the foregoing analyses of empirical science, and they form a natural bridge over which we may advance from our examination of science to an examination of common sense.

The present chapter falls into ~~four~~ five sections. First of all, there is set forth a problem that is peculiar to physics as distinct from other natural sciences such as chemistry and biology. Secondly, there is worked out a descriptive account of space and time. Thirdly, an attempt is made to formulate their abstract intelligibility. Fourthly, there follows a discussion of rods and clocks. Finally, the concrete intelligibility of space and time is indicated.

1. The Problem Peculiar to Physics.

1.1 Invariant and Relative Expressions

To formulate this problem, distinctions must be drawn 1) between propositions and expressions and 2) between invariant and relative expressions.

For present purposes the distinction between propositions and expressions will be indicated sufficiently by such illustrative statements as the following.

"It is cold" and "Il fait froid" are two expressions of the same proposition.

Again, "2 + 2 = 4" and "10 + 10 = 100" are respectively the decimal and binary expressions of the same proposition.

Now just as different expressions may stand for the same proposition, so the same expression under different circumstances may stand for different propositions. This fact leads to a distinction between invariant and relative expressions.

Expressions are named invariant if, when employed in any place or at any time, they stand for the same proposition.

Expressions are named relative if, when employed in different places or at different times, they stand for different propositions.

Thus, "2 + 2 = 4" stands for the same proposition no matter where or when it is uttered. It is invariant. On the other hand, "John is here now" stands for as many different propositions as there are places in which it is uttered and times at which it is uttered. It is relative.

~~The reason why some expressions is invariant is easily discerned.~~

Their Ground in Abstraction.

1.2 It is not difficult to discern the reason why some expressions are invariant and others are relative. For if an expression stands for an abstract proposition, it contains no reference to any particular place or time; if it contains no reference to particular places or times, it contains no element that might vary with variations of the place or time of the speaker. Inversely, if an expression stands for a concrete proposition, it will contain a reference to a particular place or time and so it will include an element that can vary with variations of the speaker's position and time.

The point may be illustrated by contrasting the use of the copula, "is," in the two expressions, "John is here," and "Pure water is H_2O." In the first expression, which stands for a concrete proposition, the copula is relative to the time of utterance; the grammatical present tense of the verb, <u>to be</u>, has its proper force; and saying that John is here has no implication that John was or was not here, or that John will or will not be here. On the other hand, to say that pure water is H_2O is to utter an abstract law of nature; grammatically, the copula occurs in the present tense, but it is not intended to confine the force of the expression to the present time. For if really it is true that pure water is H_2O, then necessarily pure water was H_2O even before oxygen was discovered and pure water will remain H_2O even after an atom-bomb has eliminated anyone interested in chemistry. In brief, the copula, "is," in abstract expressions occurs not in the ordinary present tense but rather in an invariant tense that abstracts from particular times.

Abstraction in Physics.

1.3 Now if the invariance or relativity of expressions follows from the abstractness or concreteness of the propositions for which they stand, then, since all mathematical principles and all natural laws of the classical type are abstract, it follows that their appropriate expression must be invariant.

In fact, such invariance of expression is secured automatically in mathematics, in chemistry, and in biology. There never arose any tendency to write out the multiplication table or to state the binomial theorem differently in Germany and France, in the nineteenth or twentieth centuries. In like manner it would be impossible to find relative expressions for the hundreds of thousands of formulae for chemical compounds. Such statements simply contain no reference to space or time, and so cannot vary with variations of the speaker's position or epoch.

However, the science of physics does not enjoy the same immunity. It investigates local movements, and it cannot state their laws without some reference to places and times. Since the laws contain a reference to places and times, they include an element that can vary with variations of the speaker's position and time. Accordingly, there arises a problem peculiar to physics. Just as ordinary language develops an invariant copula to express general truths, so too the physicist has to find spatio-temporal invariants, if he is to employ the appropriate invariant expressions in stating laws of local motion.

2. The Description of Space and Time.

Before tackling the problem peculiar to physics, it will be well to review the materials or data that are involved. Such a review is a task for description and, as we have seen, descriptions are cast in terms of experiential conjugates. Accordingly, we shall begin from elementary experiences, work out the resultant notions of space and time, and show how they necessarily involve the use of frames of reference and of transformations.

Extensions and Durations.

2.1 ~~Thrs~~ There exist certain elementary and familiar experiences of looking, moving about, grasping, etc.

The experiences themselves have a duration. They occur, not all at once, but over time. Moreover, correlative to the duration of the looking, there is the duration of what is looked at. Correlative to the duration of the moving, there is the duration of what is moved through or over. Correlative to the duration of the grasping, there is the duration of what is grasped. Descriptively, then, a duration is either an immanent aspect or quality of an experience or a correlative aspect or quality of what is experienced.

While duration is commonly attributed both to the experience and to the experienced, extension is attributed only to the latter. The colors I see, the surfaces I grasp, the volumes through which I move, all have extension. But it would seem paradoxical to speak of the extension of the experience of seeing, of the experience of grasping, of the experience of moving. Descriptively, then, extensions are correlative to certain elementary and familiar experiences, but they are in the experienced and not in the experiencing.

Descriptive Definitions.

2.2 Let us now define Space as the ordered totality of concrete extensions, and Time as the ordered totality of concrete durations. Further, let us give notice that henceforth, when Space and Time are written with capital letters, the words will be employed in accord with the foregoing definitions.

For besides the totalities of concrete extensions and concrete durations, there also are merely imaginary totalities. What a man experiences, he also can imagine. As he experiences extension, he also imagines extension. As he experiences duration, he also imagines duration. Our concern is, not with imaginary extensions or imaginary durations, but with the concrete extensions and durations correlative to experience.

Immediately, however, there arises an obvious difficulty. For neither the totality of concrete extensions nor the totality of concrete durations falls within the experience of the human race, let alone the human individual. For this reason the definition refers, not to any totalities, but to ordered totalities. It is true enough that only a fragment of concrete extension and of concrete duration fall within human experience. Still, one can take that fragment as origin. Beyond the extension that is experienced, there is further ~~extension~~ extension; and since it is continuous with the extension of experience, it is not merely imagined. Similarly, beyond the duration of experience, there is further duration, and since it is continuous with the duration of experience, it is not merely imagined.

There follows a simple criterion for distinguishing between the notion of concrete Space or Time and, on the other hand, merely imaginary space or time. Within concrete Space there is some extension that is correlative to experience; all other extension in Space is related to that concrete extension; and in virtue of that relation all ~~otherxext~~ other extension in ~~Space is concrete. Similarly, concrete Time is constructed~~ Space is concrete. Similarly, a notion of concrete Time is constructed about a nucleus of experienced duration. On the other hand, merely imaginary space or time contain no part that is correlative to actual experience.

From the criterion there follows a ~~corol~~ corollary. Imaginary space or time may or may not be ~~structed~~ structured about an origin. ~~B~~ But notions of concrete Space or Time must be structured about an origin. For only fragments of concrete Space or Time enter into human experience, and so it is only by a relational structure to given extensions or durations that totalities of extensions or durations can be concrete. In other words, frames of reference are essential to the notions of Space and of Time.

~~2.31 Frames of reference are ordering structures~~

Frames of Reference

2.3 Frames of ~~refrene~~ reference are structures of relations employed to order totalities of extensions and/or durations. They fall into three main classes: the personal, the public, and the special.

First, everyone has his personal reference frame. It moves when he moves, turns when he turns, and keeps its "now" synchronized with his psychological present. The existence of this personal reference ~~x~~ frame is witnessed by the correlation between the place and time of the speaker and, on the other hand, the meaning of such words as <u>here</u>, <u>there</u>, <u>near</u>, <u>far</u>, <u>right</u>, <u>left</u>, <u>above</u>, <u>below</u>, <u>in front</u>, <u>behind</u>, <u>now</u>, <u>then</u>, <u>soon</u>, <u>recently</u>, <u>long ago</u>, etc.

Secondly, there are public reference frames. Thus, men become familiar with the plans of buildings, the net-work of streets in which they move, the maps of their cities, countries, continents. Similarly, they are familiar with the alternation of night and day, with the succession of weeks and months, with the ~~us~~ use of clocks and calendars. Now such relational schemes knit together extensions and durations. But they are not personal reference frames that shift about with an individual's movements. On the contrary, they are public, common to many individuals, and employed to translate the <u>here</u> and <u>now</u> of the personal reference frame into generally intelligible locations and dates. Finally, the difference between personal and public ~~ref~~ reference frames comes out clearly in the occurrence of such questions as, Where am I? What time is it? What is the date? Everyone is always aware that he is ~~hes~~ here and now. But further knowledge is required to correlate one's <u>here</u> with a place on a map and one's <u>now</u> with the reading of a clock or a calendar.

Thirdly, there are special reference frames. A basic position, direction, and instant are selected. Coordinate axes are drawn. Divisions on the axes are specified, and so any point ~~of~~ at any// instant can be denoted univocally as an (x, y, z, t).

Special reference frames may be mathematical ~~and~~ or physical. They are mathematical if they order an imaginary space and time. They are physical if they order concrete Space and Time. The distinction is brought to light by selecting any (x, y, z, t) and asking where and when it is. For if the frame is physical, the answer will be to indicate some precise point in Space and some precise instant in Time. But if the frame is mathematical, the answer will be that any point-instant whatever will do.

2.4 Transformations

There can be as many distinct reference frames of any kind, as there are possible origins and orientations.

From this multiplicity there follows the problem of transposing from statements relative to one reference frame to statements relative to another.

Solutions may be particular, and then they are obtained by inspection and insight. Thus, when two men face each other, one may observe that the region of Space to the right of one man is to the left of the other, and so one concludes that under such circumstances what for one is "right" for the other is "left." In like manner, maps of different countries may be correlated by turning to the map of the continent that includes both countries, and clocks in different positions may be synchronized by appealing to the earth's spin.

Special reference frames admit a more general solution. Let the point (x, y, z) in the frame, K, be identical with the point specified as (x', y', z') in the frame, K'. From geometrical considerations it will be possible to find three equations relating x, y, and z, respectively to x', y', and z' and, further, to show that these equations hold for any point (x, y, z). In this fashion there are obtained transformation equations and by the simple process of substitution ~~ax~~ any statement in terms of x, y, z can be transformed into a statement in terms of x', y', z'.

For example, the wave-~~form~~ front of a light signal emitted from the origin of a frame, K, might be the sphere

$$x^2 + y^2 + z^2 = c^2t^2$$

The equations for transforming from the frame, K, to a frame, K', might be

$$x = x' - vt'; \quad y = y'; \quad z = z'; \quad t = t'.$$

On substituting, one would obtain the equation of the wave-front in the frame, K', namely,

$$(x' - vt')^2 + y'^2 + z'^2 = c^2t'^2$$

2.5 Generalized Geometry.

In the foregoing consideration of transformations, the procedure was based in the special case upon geometrical considerations. It is worth noting that the inverse procedure is possible, that is, that from a consideration of transformations one can work out the general theory of geometries.

Consider any function of <u>n</u> variables, say,

$$F(x_1, x_2, \ldots) = 0 \tag{1}$$

and any <u>n</u> arbitrary transformation equations, say,

$$\begin{aligned} x_1 &= X_1(x'_1, x'_2, \ldots) \\ x_2 &= X_2(x'_1, x'_2, \ldots) \end{aligned} \tag{2}$$

which on substitution yield ~~in~~ the new function, say,

$$G(x'_1, x'_2, \ldots) = 0 \tag{3}$$

Let these mathematical expressions have a geometrical interpretation, so that the initial variables in x_i refer to ~~distances~~ positions along the axes of a coordinate system, K, and the subsequent variables in x'_i refer to positions ~~distances~~ along the axes of another coordinate system, K', and the transformation equations represent a shift from the reference frame, K, to the frame, K'.

Now mathematical expressions have the same meaning, stand for the same ~~proposition~~ propositions, and require the same geometrical interpretation, if they have the same symbolic form. For the meaning of a mathematical expression resides, not in the material symbols employed, but in the form of their combination to indicate operations of adding, multiplying, and so forth.

Accordingly, when the symbolic form of a mathematical expression is unchanged by a transformation, the meaning of the expression is unchanged. But a transformation is a shift from one spatio-temporal stand-point to another and, when expressions do not change their meaning under such shifts, then, as we have seen above, the expressions are invariant and the ground of that invariance is that the expressions stand for abstract and generally valid propositions.

Now the principles and laws of a geometry are abstract and generally valid propositions. It follows that the mathematical expression of the principles and laws of a geometry will be invariant under the ~~xxx~~ permissible transformations of that geometry.

Such is the general principle, and it admits at least two applications. In the first application, one specifies successive sets of transformation equations, ~~xxx~~ determines the mathematical expressions invariant under those transformations, and concludes that the successive sets of invariants ~~xxxxxxx~~ represent the principles and laws of successive geometries. In this fashion one may differentiate Euclidean, affine, projective, and topological geometries. See, for instance, the summary outline offered by V, Lenzen in his Nature of Physical Theory, New York 1931, pp. 59 ff.

A second, slightly different application of the general principle occurs in the theory of Riemannian manifolds. The one basic law governing all such manifolds is given by the equation for the infinitesimal interval, namely,

$$ds^2 = \sum g_{ij}dx_i dx_j \qquad [i, j = 1, 2, \ldots n]$$

where $dx_1, dx_2, \ldots$ are differentials of the coordinates, where the coefficients, g_{ij}, are functions of the coordinates, and where in general there are n^2 products under the summation. Since this equation defines the infinitesimal interval, it must be invariant under all permissible transformations. However, instead of working out successive sets of transformations, one considers any transformations to be permissible and effects the differentiation of different manifolds by imposing restrictions upon the coefficients. This is done by appealing to the tensor calculus. For tensors are defined by their transformation properties and it can be shown that, in the present case, ~~instance~~, if the coefficients, g_{ij}, are any instance of a covariant tensor of the second degree, then the expression for the infinitesimal interval will be invariant under arbitrary transformations. It follows that there are as many instances of the Riemannian manifold and so as many distinct geometries, as there are instances of covariant tensors of the second degree employed to specify the coefficients, g_{ij}. Thus, in the familiar Euclidean instance, g_{ij} is unity when *i* equals *j*; it is zero when *i* does not equal *j*; and there are three dimensions. In Minkowski space, the g_{ij} is unity or zero as before, but there are four dimensions, and x_4 equals *ict*. In the General Theory of Relativity, the coefficients are symmetrical, so that g_{ij} equals g_{ji}; and in the Generalized Theory of Gravitation, the coefficients are anti-symmetrical.

2.6 A Logical Note.

It is to be observed that transformation equations, operations of transforming, the definition of tensors by their transformation properties, and the whole foregoing account of the differentiation of geometrical manifolds belong to higher-order statements.

For distinct reference frames assign different specifications to the same points and instants and they assign the same specifications (numbers) to different points and instants. Accordingly, they must belong to different universes of logical discourse, else endless ambiguities would result. Now the relations between different universes of discourse can be stated only in a further, higher-order universe of discourse; in other words, the relations between different universes of discourse regard, not the things specified in those universes, but the specifications employed to denote the things. Thus, a transformation equation does not relate points or instants, but it does relate different ways of specifying the same points and instants. Similarly, such a property as invariance is a property, not of a geometrical entity, but of an expression regarding geometrical or other entities.

3. The Abstract Intelligibility of Space and Time.

The argument began from a problem peculiar to physics. Because that science deals with objects in their spatial and temporal relations, the expression of its principles and laws does not automatically attain the invariance proper to such abstract propositions. However, as was shown in Chapter II, this difficulty can be turned to profit, inasmuch as the physicist can posit a postulate of invariance and then employ that postulate as a heuristic norm in determining which expressions can represent physical principles and laws.

The second strand of the argument consisted in an outline of the descriptive notions of Space and Time. It began from experiences of concrete extensions and durations and it showed that we can form notions of all concrete extensions and of all concrete durations if, and only if, these totalities are ordered by frames of reference. Essentially, then, the descriptive notion of Space is of Space-for-us and the descriptive notion of Time is of Time-for-us. Again, one might say that these notions necessarily contain, on the one hand, an empirical or material element and, on the other hand, an intelligible or formal element. The empirical or material element consists of concrete extensions and of concrete durations. The intelligible or formal element orders these materials into singular totalities. Moreover, without this intervention of ordering intelligence, the notion of Space cannot be both concrete and all-embracing, and similarly the notion of Time cannot regard the totality of concrete durations.

Still, these descriptive notions of Space and Time cannot contain the intelligibility that is explanatory of Space and of Time. It is true that they contain an intelligible or formal component. But that component is the order of a reference frame, and reference frames are an infinity. They can be the intelligibility of Space-for-us and of Time-for-us, that is, they can be the manners in which we intelligently order extensions and durations in accord with the convenience of the moment. But they cannot be the immanent intelligibility that is explanatory of Space nor the immanent intelligibility that is explanatory of Time, for reference frames are infinite, but ~~the~~ correct explanations are unique.

However, this gives rise to a further problem. On the one hand, if we retain reference frames, we are dealing with infinities of formally different notions of Space and Time. On the other hand, if we drop reference frames, then our inquiry is confined either to merely imaginary space and time or else to the relatively few extensions and durations that fall within our experience. It is this dilemma that reveals the significance of transformations and invariance under transformations. For, while such considerations belong to a higher-order universe of discourse, which directly regards not objects but expressions referring to objects, still they can serve to point the way to grasping the intelligibilities immanent in Space and in Time. Inasmuch as we say what we think, ~~x~~ the properties of our expressions reflect the properties of our thoughts. Inasmuch as we think intelligently, the properties of our thoughts ~~refle~~ reflect the properties of our insights. In this fashion, the invariance of expression has already been traced to the abstractness of what is thought or meant and, at an earlier stage of the inquiry, the abstractness of classical laws was grounded on the enriching contribution of insight. Accordingly, we shall not be venturing into a new line of thought, if we argue that the set of insights, by which we grasp the intelligibility immanent in Space and Time, will be the set that is formulated in spatial and temporal principles and laws invariant under transformations of reference frames.

Clearly enough, this conclusion gives no more than a generic answer to our question. It amounts to saying that the immanent intelligibility of Space and of Time will be formulated in one of the geometries that fall under the generalized notion of geometry. There remains the task of assigning the specific geometry that governs concrete extensions and concrete durations. Still, one has only to mention this task to be reminded that there is a problem peculiar to the empirical science of physics, that this problem arises in physics inasmuch as it is involved in spatial and temporal relations, and that the general form of its solution is to postulate the invariance of physical principles and laws.

3.1 The Theorem. It is time to turn from talk about what we propose to do and settle down to the work of doing it.

~~The abstract intelligibility immanent in~~

The abstract formulation of the intelligibility immanent in Space and in Time will be one of the possible sets of definitions, postulates, ~~x~~ and inferences that systematically unify the relations of ~~concrete~~ extensions and of ~~concrete~~ durations. All such possible sets of definitions, postulates, and inferences are ~~may be named~~ geometries. Therefore, the abstract formulation of the ~~intali~~ intelligibility immanent in Space and in Time will be a geometry.

The expression of the principles and laws of any geometry will be invariant. For principles and laws are independent of particular places and times, and so their proper expression cannot vary with variations of spatio-temporal stand-points.

~~Moreover, the expression of the principles and laws of any geometry may be expected to be algebraic or, at least, to admit an algebraic formulation. The Algebraic formulation seems to admit geometrical interpretation only through the use of reference frames and, in any case, unless rel?~~

~~Moreover, a geometry applicalbe to applicable~~

~~Moreover, if a geometry is to be applied to Space or Time, the application will occur through the use of a reference frame. For without any reference frame~~

Moreover, a geometry cannot refer to Space or to Time except through a reference frame. Accordingly, the invariance proper to the expression of geometrical principles and laws is an invariance under transformations of reference frames.

There follows at once the generic solution. The abstract formulation of the intelligibility of Space and Time consists in a set of invariants under transformations of reference frames. However, there is a range of such sets of invariants, and so there remains the task of ~~daterming~~ determining the specific solution.

We note, accordingly, that the relevant intelligibility is immanent in concrete extensions and in concrete durations. It is an intelligibility that belongs not to the imagined but to the experienced. Now the empirical canon of complete explanation has already assigned to natural science the duty of doing for experienced/~~extensions~~ extensions/ and durations exactly what is done for experienced colors, experienced sounds, experienced heat, experienced electro-magnetic phenomena. Further, physics is the natural science on which this duty falls, as appears from its peculiar problem of invariance. Again, if the physicist solves his peculiar problem and arrives at an invariant expression of his principles and laws under transformations of reference frames, he cannot avoid reaching the specific solution which we are seeking. For the specific solution we are seeking is the set of invariants under transformations that is verifiable in experienced extensions and durations.

The abstract formulation, then, of the intelligibility immanent in Space and in Time is, generically, a set of invariants under transformations of reference frames and, specifically, the set verified by physicists in establishing the invariant formulation of their abstract principles and laws.

A corollary may be added. The intelligibility immanent in Space and in Time is identical with the intelligibility reached by physicists investigating objects ~~as extended and as enduring instances,~~ as involved in spatial and temporal relations. Hence, to eliminate the concrete objects of physics would be to eliminate the intelligibility of Space and of Time. Again, inasmuch as physical objects are involved differently in spatial and temporal relations, there result different intelligibilities of Space and of Time. This conclusion may be illustrated by the possibility of different types of tensors being employed to secure the covariance of different sorts ~~types~~ of physical principles and laws.

~~There is a further corollary. The validity of Euclidean geometry does not become a question for physical investigation in virtue of an interpretation of geometrical coordinates in terms of physical rods and clocks. Euclidean geometry, as a geometry, is valid independently of physics. But no unverified geometry is the intelligibility immanent in concrete extensions and concrete durations. And the task of determining the verifiable geometry falls to physics, not as an inquiry into rods and clocks, but in virtue of the empirical canon of complete explanation and the involvement of physica-l principles and laws in spatial and temporal relations.~~ in virtue of

Euclidean Geometry

3.2 While the foregoing argument of itself says nothing for or against the verifiability of Euclidean geometry, still it supposes that Euclidean geometry is not the one and only true geometry, and it admits the possibility of other geometries being verifiable.

The supposition is, of course, far more fundamental than the admission. It is difficult not to find the~~ins~~ inspiration of rationalism, which deduces everything else from alleged self-evident principles, in the notion that Euclid formulated the one and only true geometry. After all, the supreme rationalist wrote on his title page, Ethica ordine geometrico demonstrata. Still, these high matters lie beyond the range of present considerations though, in due course, we hope to meet this issue with a distinction between analytic propositions, which are not far from tautologies, and analytic principles, whose terms and relations are verifiable in the ~~exist~~ existent.

At any rate, present concern has to be confined to meeting claims that Euclidean geometry obviously is verified in concrete extensions and that ordinary notions of simultaneity obviously are verified in concrete durations.

~~Clearly~~ Clearly, there is a sense in which these claims are true. It has been seen that one cannot form a notion of Space without invoking a frame of reference. It is plain that men form notions of Space and, no less, that the frames of reference they construct satisfy Euclidean requirements. Similarly, one cannot form a notion of Time without introducing a frame of reference, and the frame ordinarily introduced is necessarily in complete accord with ordinary notions on simul- ~~xxxxxxxxxxxxxx~~ taneity. Not for a moment would I dispute the contention that Euclidean geometry and the common view of simultaneity are both verifiable and verified in the descriptive notions men form of Space and Time.

However, after granting all that is obvious, we must now add that it is quite beside the point. The analysis of descriptive notions of Space and Time has its significance, but that significance is anthropological. It reveals how men commonly proceed from the extensions and durations of experience to the totalities named Space and Time. On the other hand, when we admit that Euclidean geometry might not be verifiable, we are ~~speaking~~ speaking of a verification, not in human notions, but in concrete extensions and durations. We are not asking how men find it convenient to conceive Space and Time; ~~but~~/we are asking how/ scientists may correctly explain Space and Time. Were the scientists in question the psychologists, one might appeal ~~against their conclusion~~ to what is obvious in the mentality of Western man. But the scientists in question happen to be physicists, and the data of consciousness, however clear, are not among the data proper to ~~physi~~ physics.

So much, then, for the sweeping claim that our conclusion must be wrong because its error is obvious. It remains that objections may be less sweeping, and these must now be met.

Absolute Space.

3.3 The absolute space and the absolute time of Newtonian thought possess the twofold merit of exhibiting an "obvious" view and of inviting criticism that goes to the root of the matter.

Suppose a penny to fall to the floor of a moving train, and ask for an account of the trajectory of the fall. Unfortunately, there are many accounts. Relatively to the floor, the trajectory is a vertical straight line. Relatively to the earth, it is a parabola. Relatively to axes fixed in the sun, it is a more complicated curve that takes into account the spin and orbit of the earth's movements. Relatively to the receding nebulae, it contains still further components. ~~Still,~~ But there is only one penny in question, and there is only one fall. Which, really, is the trajectory?

Newton would answer by ~~distinguished~~ distinguishing between true and apparent motion. Both are relative. But, while apparent motion is relative to other bodies, such as the train, the earth, the sun, the nebulae, true motion is relative to an eternal set of immutable places named absolute space. If one thinks of apparent motion, one can say that the penny moves relatively to the train, the train relatively to the earth, the earth relatively to the sun, and the sun relatively to the nebulae. But if one thinks of true motion, one can say that, perhaps, ~~both~~ the penny, the train, the earth, the sun, and the nebulae have a common velocity relatively to a set of eternal and immutable places.

Moreover, if Newton named his absolute space mathematical, he also considered it real. He admitted the difficulty of determining when there was a true motion. But he was far from acknowledging such a conclusion as impossible. On the contrary, he performed his famous bucket experiment to show that true motion relative to absolute space could be detected. A bucket of water was suspended from a twisted rope. The bucket span and, for a while, the surface of the water remained flat. The surface then hollowed out into a paraboloid. Eventually, the bucket ceased to spin, but the surface remained hollow. Finally, the surface became flat again. Now the hollowing of the surface of the water ~~is~~ was due to the rotation of the water and, as this hollowing occurred both while the bucket was spinning and while the bucket was not spinning, it could not be merely an apparent motion relative to the bucket. Therefore, it was a true motion relative to absolute space.

Let us now turn to criticism.

~~First of all, the existence of the problem should cause surprise to every follower of Galileo. For, after all, Galileo had distinguished between the real and objective and, on the other hand, the merely apparent, and his criterion was the primary qualities~~

~~After all, the problem of distinguishing the merely apparent from the real and objective had already been undertaken by Galileo, and his solution was that colors, sounds, heat, and the like were merely apparent secondary qualities, while the mathematical dimensions of matter in motion were real and objective primary qualities. Now, however, it ~~as~~ would seem that the eminently mathematical trajectory of a falling penny happens to be a multiplicity, and so there arises the question of distinguishing between the one real trajectory and the many merely apparent trajectories. Evidently, Galileo had~~

Let us now turn to criticism.

First of all, the bucket experiment does not establish the existence of an absolute space. From the experiment one might conclude that really and truly the water was rotating; for in the hollowing of the surface one might verify a centrifugal acceleration; and if there is a verified centrifugal acceleration, there is a verified motion. However, true motion in the sense of verified motion is one thing; and true motion in the sense of motion relative to absolute space is quite another. The bucket experiment does not establish true motion in this second sense. Indeed, the sole link between the experiment and absolute space lies in an equivocal use of the term, true.

Secondly, the Newtonian distinction between true and apparent motion involves the use of an extra-scientific category. There are the data of experience. There are inquiries, insights, and formulations. There are verifications of formulations. But just as Galileo impugned given colors, sounds, heat, and the like as merely apparent, so Newton impugned as apparent the observable changes of ~~p~~ relative position of observable bodies. Just as Galileo affirmed as real and objective the primary qualities ~~of~~ that are mathematical dimensions of matter in motion, so Newton, after eliminating experienced motions as apparent, acknowledged as true the motions relative to ~~an absolute space~~ a non-experienced absolute space. What is this truth of true motion? Though Newton confused it with the truth of experiment and verification, it has to be something else; otherwise, there would be no confusion. What, then, is it?

A fuller account will be attempted when we treat the notion of objectivity. ~~in Chapters 12 and 13.~~ For the present, it will suffice to recall that the Galilean assertion of the reality and objectivity of primary qualities was not in accord with the canon of parsimony but, as we have seen, extra-scientific (see Chapter III, §5.). In simpler terms, Galileo's real and objective was the residue left in the popular category of the "really out there," after colors, sounds, heat, etc., had been eliminated. By parallel reasoning, Newton's absolute space was the "really out there" but emptied not only of Galileo's secondary qualities but also of his own apparent motions. From this position to Kant's, it is an easy step. For Kant, as for his scientific predecessors, all sensible presentations were phenomenal. But, while Newton secured a ~~metaphysica~~ metaphysical status for his absolute space by naming it the <u>divine sensorium</u> (see E. A. Burtt, The Metaphysical Foundations of Modern Science, London and New York, 1925, pp. 257 ff.), Kant gave this empty "really out there" a critical status by making it an <u>a priori</u> form of human sensibility.

(...) a footnote.

Thirdly, Galileo, Newton, and Kant were looking for some sort of absolute, but they were looking in the wrong places. They sought the ~~re~~ real as opposed to the ~~apparent~~ apparent, only to end up with everything apparent, the notion of the real included. Let us follow a different tack. Then every content of experience will be equally valid, for all are equally given, and all equally are to be explained. Next, explanations result from enriching abstraction, and so they are abstract, and their proper expression must be invariant. Thirdly, not every explanation is equally correct; some can be verified, and some cannot. There follows at once the conclusion that the real, objective, true consists of what is known by formulating and verifying invariant principles and laws. Our account of Space is simply a particular case of that conclusion.

Fourthly, ~~ixxm~~ let us attempt to meet the problem of the trajectory of the penny. As we have seen, possible frames of reference are infinite; but in any determinate frame of reference, there is only ~~but~~ one correct trajectory for the penny. Next, while some possible frames of reference are more convenient than others, still all are equally valid, and so there are many correct trajectories for the penny. Further, this involves no contradiction; just as what is to my right can be to your left, so the one fall of the one penny can be a straight line in one frame of reference and a parabola in another frame of reference; there would be a contradiction only if the same fall were both a straight line and a parabola in the same frame of reference. Finally, this position is not unsatisfactory. As long as we are speaking of particular things at particular times in particular places, we cannot avoid employing relative expressions; for it is through our senses that we know the particular; and our senses are in particular places at particular times. On the other hand, invariant expression, which is independent of the spatio-temporal stand-point of particular thinkers, is a property of abstract propositions; it can be demanded only of the principles and laws of a science; and the trajectory of the fall of a particular penny is not a principle or a law in any science.

3.4 Simultaneity

The common view of simultaneity possesses, perhaps, a larger and more ~~xx~~ resolute following than Newton's absolute space. If two events are at the same time for any observer, then, we shall be told, they must be at the same time for every observer.

The first line of defence will be, no doubt, the principle of contradiction. The same events cannot be both at the same time and not at the same time. Therefore, to say that the same events are ~~xhx~~ at the same time for one observer and not at the same time for another, is simply to violate the principle of contradiction.

Still, this first line can be turned. What is "now" for me writing is not "now" for you reading. If the same event can be both now (for me) and not now (for you), it may be true that "at the same time" belongs to the same class of relative terms as does "now"; and if it does, then there is no more a contradiction in saying that events, simultaneous for one observer, are not simultaneous for another, as there is ~~ix~~ in saying that events of the present for one observer will be events of the past for another.

The issue is not the principle of contradiction. The issue is simply whether or not "at the same time" is to be listed along with such relative terms as "now" and "soon," "here" and "there," "right" and "left." ~~Moreover, the only way to settle this issue is to determine what exactly one means when one uses the expression "at the same time."~~

The simplest approach to the issue is to analyze elementary apprehensions of simultaneity. Already we have remarked that we experience duration ~~inxth~~ both in the sense that the experiencing is over time and in the sense that the experienced endures through time. Now we have to add that these two aspects of the experience of duration stand in a certain order. Thus, when I watch a man crossing a street, I look out and ~~inskp~~ inspect the distance that he traverses, but I cannot look out and inspect in the same manner the time ~~he takes to cross. What comes first is an awareness of the duration of my own watching, and only in and through that duration do I apprehend the duration of his crossing he takes to cross. The reason for this is simple enough. For, while the whole distance traversed traversed is all at once there to be inspected, the duration of traversing is there to be inspected, and not all at once, but only in successive bits. Nor is this all. If one supposed the possibility of a timeless inspection, one would infer~~ he takes to cross. Nor is this surprising. The whole distance traversed is there to be inspected all at once, but the duration of the traversing is ~~thre~~ there to be inspected, not all at once, but only in successive bits. Moreover, what is true of the traversing is also true of the inspecting; it too is, not all at once, but over time. If one supposed the possibility of a timeless inspecting, one might infer the inspection of a four-dimensional continuum in which both distances and durations were presented in exactly the same fashion. But when inspecting takes time, then the time of the inspecting runs concurrently with the time of the inspected.

Such remarks on the apprehension of durations seem relevant to ~~the~~ an account of the apprehension of simultaneous durations. Instead of watching one man cross a street, I might watch two men crossing a street at the same time. Since it would be perfectly obvious that they were crossing at the same time, it should be equally obvious that there is some time that is one and the same. What time, then, obviously is the same? It must be the time of the watching. For, in the first place, the watching has a duration, for it is not all at once. In the second place, the duration of the watching runs concurrently with the duration of what is watched. In the third place, ~~efxthe~~ when two movements are the object of one and the same watching, there are, in all, three durations, namely, one in each movement and one in the watching; but it is the duration of the watching that is apprehended as running concurrently both with the duration of one movement and with the duration of the other; and so it is the duration of the watching that is the one and same time at which both the movements are occurring.

This analysis is confirmed by a consideration of apprehensions of "apparent" simultaneity. If you stand beside a man swinging a hammer, then the sight and the sound of the blow are at the same time. If you stand off at a distance of a few hundred feet, the sight of the blow is prior to the sound. In the first case, the sight and sound are at the same time. In the second ~~caxmxs~~ case, the sight and the sound are not at the same time. Still, the blow is always a simultaneous source of both light-waves and sound-waves. The reason why there are different, "apparent" simultaneities must be that the "appearance" of simultaneity has its ground in the duration immanent in the flow of consciousness.

Such seem to be the facts and, like the facts of relative motion, they give rise to a problem. Is one to follow Galileo and Newton and insist that, beyond the multiplicity of merely apparent simultaneities, there is a real, objective, and true simultaneity that is unique? If so, one can omit further mention of the observer, and one will end up with an absolute time that flows equably everywhere at once. It will not be the time of clocks, which run fast or slow. It will not be the time of the spinning earth, for under the action of the tides and the ~~rec~~ receding moon, that spin is decelerating. It will be an exact, constant velocity that at every point in perpetually/ the universe/separates the present from the past and the future in precisely the same manner.

Still, this absolute time will not be what we have defined as Time. For Time, as we defined it, is an ordered totality of concrete durations. It includes the concrete durations both of our experiencing and of what we experience. Through an ordering structure or reference frame it reaches out to embrace in a single totality all the other concrete durations which we do not experience yet we do relate to the concrete durations that are experienced. In contrast with this Time, absolute time simply lies outside experience. It meets the requirements of ~~an absol~~ a mathematical ideal and, stangely enough, unlike other mathematical ideals, it is said to be "really out there." Rather, it once was thought to be really out there. For the Newtonian rejection of experienced durations as apparent time in favor of a non-experienced absolute time promptly ~~trans~~ was ~~transformed~~ followed by Kant's transformation of absolute time into an a priori form of human sensibility.

Nor is this the only complaint against the Newtonian procedure. As absolute space, so absolute time is a result of looking for the absolute where the absolute does ~~not exist. A statement of simultaneity is a statement about particular events at a particular time. There is no reason to expect such statements to be, not relative, but invariant. For invariance of expression rests upon the abstractness of what is expressed; and no abstract proposition refers to particular events at a particular time. Still, the demand for an objective, real, true simultaneity thati is unique is, in more elementary terms, a demand that statements of simultaneity be invariant particular events at a particular time. If it were true that events, simultaneous for one observer, must be simultaneous for all observers, then expressions of simultaneity would be invariant. They would fall in the same class as the expression, Twice two are four. They would not fall in the same class as the expression, John is here now.~~
not exist. If it were true that events, simultaneous for one observer, must be simultaneous for every other observer, then it would be true that expressions of simultaneity are invariant. But there is no reason to expect invariant expressions of simultaneity, for invariance results from abstractness, and no statement regarding the particular times of particular events is abstract. From the very structure of our cognitional apparatus, particulars are known through our senses, and our senses operate under spatio-temporal conditions. They cannot escape relativity and so, if an absolute is wanted, it must be sought on the level of intelligence which by abstraction from particulars provides a ground for invariant expressions.

Motion and Time.

3.5 We have been speaking of the elementary durations and simultaneities of the personal reference frame. But, besides personal reference frames, there are public and special reference frames, and they call for a few remarks.

Aristotle defined time as the number and measure of local motion derived from successively traversed distances. Such is the time of the spinning earth and of clocks. "Two o'clock" is a number and "two hours" is a measure. Both are reached from the local motion of the hands over the face of a dial.

However, there are many local motions, and every one successively traverses a series of distances. It follows that, though all do not yield numbers and measures indicating time, still all could do so. Objectively, then, and fundamentally there are many times.

This implication of the Aristotelian position was noted by Aquinas. However, it seemed to him, not an important truth, but rather an objection to be answered. Time must be ~~one, and so he appealed to the primum mobile, that grounded all other movements in the sky and on earth. The local movement of the primum mo~~ one, and so he appealed to the primum mobile, the outermost ~~celestial~~ celestial ~~sphe~~ sphere. There was only one such sphere, and it had only one local motion. Moreover, as it grounded all other local motions both in the sky and on the earth, the time of its movement must be the ground of all other times. (See S. Thomas Aquinas, In IV lib. Phys. Arist., lect. 17, ed. Leon. Rome 1884, vol. 2, p. 202, §§3, 4).

One will be inclined, I think, to agree that as long as Aristotle's primum mobile was supposed to exist, our universe was supplied with a single, standard time. On the other hand, once Copernicus eliminated the Ptolemaic system, that standard time no longer was possible and, in its place, there arose the problem of synchronization, of making many movements yield a single time for public and special reference frames.

Suppose, then, an aggregate of clocks scattered about the universe. Let their relative positions be/ constant, and/ let them be// known in terms of some reference frame, K. Let light signals be sent from the origin of coordinates to the clocks and reflected from the clocks back to the origin. Then, a synchronization of clocks might be effected by laying down the rule,

$$2t = t' + t''$$

where t is the reading of the distant clock when the light signal is received and reflected, and where t' and t'' are the readings of the clock at the origin when the light signal is emitted and when it returns.

However, synchronization by this rule would be successful, only if the outward and the return journeys of the light signal took the same length of time. To satisfy this requirement, one might distinguish between basic and derived synchronizations and demand that the basic synchronization take place with clocks that are at rest with respect to the ether and in a reference frame that similarly is at rest. Then, synchonization in moving frames would be the synchonization of their clocks with the clocks of the basic frame, and there would follow for all point-instants an observable time that conformed to the properties of Newton's absolute time.

There is, however, one difficulty to this solution. One can in principle suppose any number of reference frames exhibiting as many varieties of relative motion as one pleases. One can supply each frame with clocks that, relatively to the frame, are at rest. But a difficulty arises when one attempts to select the frame that absolutely is at rest and, if one cannot determine the basic synchronization, much less can one reach the derived synchronizations.

Still there is an alternative. Instead of seeking the absolute in the field of particular reference frames, one can seek it in the field of abstract propositions and invariant expressions. Accordingly, one may postulate that the mathematical expression of physical principles and laws be invariant under inertial transformations, and one may note that from the postulate it follows that in all reference frames moving with a relative uniform motion the velocity of light will be the same. For the consequent derivation of the Einstein-Lorentz transformation and of Minkowski space, the reader may be referred to Lindsay and Margenau, pp. 333 ff.

3.6 Still, the theory of special relativity has its paradoxes. If the distance between two points in one reference frame is measured by some number, S, then the distance between the same points in a relatively moving frame will be some other number, S'. Similarly, if the time interval between two instants in one frame is measured by some number, T, then the interval between the same instants in a relatively moving frame will be some other number, T'. Finally, there will be a similar difference between the numbers measuring in different frames the velocity of the same moving body. See Lindsay and Margenau, pp. 337 ff.

In so far as this paradox generates a problem in the conception of measurement, it provides the topic of our next section on Rods and Clocks. For the moment, however, we wish to restrict attention to the origin of the paradox in the relativity of simultaneity.

When we speak of the distance between two moving objects, we mean the distance between their simultaneous positions. Thus, the distance between two planes flying in the sky is, not the distance between the present position of one and a past position of the other, but the distance between the two at the same time. But, if simultaneity is relative, then different observers will select different positions as simultaneous and so arrive at different distances between the two moving objects. Accorindingly, relativity of simultaneity involves similarly, relativity of distances; relativity of distances involves relativity of velocities; and relativity of velocities involves relativity of measurements of time intervals. Such is the general origin, then, of the paradox. But, the special theory of relativity supposes a relativity of simultaneity; it deals with reference frames which are systems of moving points; and it incorporates in the specifications of the points the consequences of the relative simultaneity that it supposes. Not merely does it effect synchronization in every reference frame by the formula, $2t = t' + t''$, but also it imposes on the coordinates of every distant clock the condition that

~~that the infinitesimal interval, ds, be invariant under transformations, where~~

$$\cancel{ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2}$$

~~where (x, y, z, t) are the coordinates and c is the invariant velocity of light.~~

3. The Principle at Issue. Before closing this section, it will be well to set forth briefly the principles that have guided us in determining the abstract intelligibility of Space and Time and, no less ~~at the same time~~, to indicate the grounds that lead to different views.

Our position follows from our account of abstraction. Because the principle or law is abstract, its expression cannot vary with variations of spatio-temporal stand-point. On the other hand, because we know particulars through spatio-temporally conditioned senses, we know them from some point and instant within Space and Time. It follows that concrete places and times are apprehended only as relative to an observer, that their totalities can be embraced only through the device of reference frames, that reference frames will be many, and that transformations of reference frames can involve changes in the relativity of places and times to observers. Accordingly, it would be a mistake to look for the fixed or absolute on the level of particular places and times; the only absolute relevant to Space and Time resides in the abstract propositions whose expression remains invariant under permissible transformations of reference frame.

On the other hand, opposed positions take their stand on the premise that something fixed or absolute is to be acknowledged on the level of sense. In the Aristotelian world view, this was supplied by the outermost celestial sphere which bounded effective Space and, for Aquinas at least, provided the universe with a standard time. Newton's absolute space and absolute time were in the first instance imaginary mathematical constructions; but they were objectified through a confusion of the truth of verification and the truth, ~~of~~ prior to intelligence and thought, that resides in a "really out there"; finally, they were given a metaphysical status by being connected with the omnipresence and the eternity of God. Kant simplified this position by making Newton's empty space and time into _a priori_ forms of the sensibility.

4. Rods and Clocks.

On Galilean and Newtonian suppositions, measurements of distance and of duration are invariant, so that if a measurement is correct in any frame of reference, the same measurement must be correct in all frames of reference that are permissible.

On the Special Theory of Relativity the invariant is the four-dimensional interval, <u>ds</u>, where

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2$$

Hence, if the value of <u>ds</u> is correct in any reference frame, the same value must be correct in all permissible frames. On the other hand, the values of the spatial components, <u>dx</u>, <u>dy</u>, <u>dz</u>, and the value of the temporal component, <u>dt</u>, can be correct in one reference frame without therefore being correct in other permissible frames. As is clear from the above equation, the spatial and temporal components can assume any number of values compatible with the constancy of the interval, <u>ds</u>.

Clearly enough, ~~the theory~~ this theory necessitates some revision of earlier notions on measurable magnitudes, standard units, measuring, and measurement. For on the earlier view a measurement of a distance or 1 duration is some single number valid in all reference frames. On the new view a measurement of a distance or a duration 1 ~~is~~ a series of numbers in correspondence with a series of reference frames.

seems to be 1

Such a revision is not easy. Ordinarily people form their notions of measurements at a time when they take Newtonian presuppositions for granted. Later, when they are confronted with relativity, they are apt to be content to make obvious alterations without thinking things through to a fully coherent position. There results a piecemeal and inadequate revision of basic concepts and this manifests itself in a parade of alleged Einsteinian paradoxes.

Our proposal is to attempt a ~~thes~~ thorough revision. First, we shall examine the elementary paradox that the measuring rods of one reference frame are both shorter and longer than those of another, and that the clocks of one frame run both slower and faster than those of another (for an exposition, see Lindsay and Margenau, pp. 339 ff). Secondly, we shall work out a generic notion of measurement that is independent of differences between Galileo and Einstein. Thirdly, we shall show how the same generic notion admits differentiation into the two different specific views.

Here follow pp 261-264.

4.1 The Elementary Paradox.

Consider the pair of point-instants, P and Q, which in a frame of reference, K, have the coordinates, (x_1, t_1) and (x_2, t_2), and in a frame, K', moving with a relative constant velocity, u, have the coordinates, (x'_1, t'_1) and (x'_2, t'_2). Then by the Lorentz-Einstein transformation, writing

$$H = 1/(1 - u^2/c^2)^{1/2}$$

one easily obtains the equations

$$x'_2 - x'_1 = (x_2 - x_1)H - (t_2 - t_1)uH \qquad (1)$$

$$t'_2 - t'_1 = (t_2 - t_1)H - (x_2 - x_1)uH/c^2 \qquad (2)$$

It is to be noted that if either of the equations, (1) and (2), can be obtained then both can be obtained. Moreover, by transforming in the opposite direction from K' to K, there are two other equations, similar to (1) and (2), to be obtained.

Now these equations admit both a spatial and a temporal application, and to each application threes interpretations can be given. The spatial application is to suppose that P and Q are the simultaneous end positions of a standard rod of unit length in K so that

$$x_2 - x_1 = 1 \qquad (3)$$

$$t_2 - t_1 = 0 \qquad (4)$$

whence by equations (1) and (2)

$$x'_2 - x'_1 = H \qquad (5)$$

$$t'_2 - t'_1 = -uH/c^2 \qquad (6)$$

The temporal application is to suppose that P and Q are readings at successive seconds on a stationary standard clock in K so that

$$x_2 - x_1 = 0 \qquad (7)$$

$$t_2 - t_1 = 1 \qquad (8)$$

whence by equations (1) and (2)

$$x'_2 - x'_1 = -uH \qquad (9)$$

$$t'_2 - t'_1 = H \qquad (10)$$

Accordingly, inasmuch as standard units of distance and of time are expected to transform invariantly, a problem of interpretation arises and three answers may be given.

A first interpretation seems inspired by the Fitzgerald contraction. Since H is greater than unity, it is concluded from equations (3) and (5) that the standard rod in K' is shorter than the standard rod in K. Similarly, it is concluded from equations (8) and (10) that the unit of time in K' is shorter than the unit of time in K. Moreover, the opposite conclusions are reached from the equations obtained by transforming from K' to K. But quite apart from its paradox, this interpretation has the defect of saying very little about equations (4) and (6), (7) and (9).

A second interpretation begins by noting that in Special Relativity clocks are synchronized in each frame of reference by assuming, not that simultaneity is identical, but that the velocity of light is the same constant in all frames of reference. Accordingly, on this interpretation equations (5) and (6) are taken together, and at once it is apparent that a distance between simultaneous positions in K has been transformed into a distance between positions that are not simultaneous in K'. But even Cinderella's foot would seem large if one measured the distance between the tip of her toe at one instant and the back of her heel at another; and such is the view in K' of the standard unit of length in K. Similarly, equations (9) and (10) are taken together to reveal that, what for K is a time interval on the same stationary clock, for K' is a difference in time between clocks in different positions. It follows that the difference in time given by equation (10) results not only from the difference in time given by equation (8) but also from the fact, underlying the transformation equations, that in every frame of reference clocks in different positions are synchronized by assuming the velocity of light to be the same constant in all frames. Indeed, while one may find this method of synchronization to be strange, while one may even find it strange that there is any problem of synchronization, still, granted that initial oddity, there is no further oddity brought to light by equations (3) to (10) or by the similar equations obtained when one transforms from *K'* to *K*.

A third interpretation is in terms of Minkowski space. It asserts that, within the context of Special Relativity, it is a blunder to suppose that a difference of position is a merely spatial entity or that a difference of time is a merely temporal entity. Hence, a standard rod is spatio-temporal: it is not merely a distance between two positions; it is a distance between ~~s~~ a position, x_1, at a time, t_1, and a position, x_2, at a time, t_2. Similarly, a standard clock is spatio-temporal: it does not assign merely temporal differences; it assigns a difference between a time, t_1, at a position, x_1, and a time, t_2, at a position, x_2. ~~2~~ Moreover, a unit on any standard rod determines ~~and~~ one and the same invariant spatio-temporal interval for all frames of reference, namely, unity; and a unit on ~~a~~ any standard clock determines one and the same invariant spatio-temporal interval for all frames of reference, namely, *ic*. This invariant interval, *s*, may be obtained from the ~~m~~ equations

$$s^2 = (x_2 - x_1)^2 - c^2(t_2 - t_1)^2 = (x'_2 - x'_1)^2 - c^2(t'_2 - t'_1)^2$$

and one? will find that substitutions from equations (3) and (4) will yield the same result, unity, as ~~m~~ substitutions from equations (5) and (6); similarly, substitutions from equations (7) and (8) will yield the same result, *ic*, as substitutions from equations (9) and (10). However, while standard rods and clocks determine the same spatio-temporal intervals for all frames of reference, still these invariant ~~are~~ intervals divide differently into spatial and temporal components in different frames of reference. Hence one may distinguish between normal and abnormal frames by introducing the definitions:

> A reference frame is normal to measurements if differences of position have a temporal component that is zero and differences of time have a spatial component that is zero.
>
> A reference frame is abnormal to measurements if differences of position have a temporal component that is not zero and differences of time have a spatial component that is not zero.

Operationally this means that reference frames, rods, clocks, and measurable objects should be relatively at rest if one's measuring is not to be complicated by the ambiguities of the elementary paradox.

Finally, it may be noted that, while the first interpretation differs from the other two, the second and third are compatible and complementary. For the second explains the differences that arise on transforming units of distance and time by remarking that, when the relative velocity is not zero, the transformation equations cover over a peculiar technique in synchronization, while the third interpretation systematizes the whole matter by adverting to spatio-temporal invariants and by noting that these invariants divide differently into spatial and temporal components in different reference frames. It remains, however, that something be said on the general notion of measurement presupposed by the second and third interpretations.

next comes § 42.

4.2/

4.2 The Generic Notion of Measurement.

Empirical inquiry has been conceived as a process from description to explanation. We begin from things as related to our senses. We end with things as related to one another. Initial classifications are based upon sensible similarities. But as correlations, laws, theories, systems are developed, initial classifications undergo a revision. Sensible similarity has ceased to be significant, and definitions consist of technical terms that have been invented as a consequence of scientific advance. In this fashion biological classifications have felt the imprint of the theory of evolution. Chemical compounds are defined by appealing to chemical elements. Chemical elements are defined by their relations to one another in a periodic table that has room for elements that, as yet, have not been discovered or synthesized. The basic notions of physics are a mass, that is distinct from weight, a temperature, that differs from the intensity of the feeling of heat, and the electro-magnetic vector fields.

~~Now the principal technique in effecting the transition from description to explanation is measurement. If we get beyond things as related to our senses, it is by some praeternatural gift of new senses, but by a pedestrian use of the senses we already have by only by introducing a new technique in the use of the senses we happen to have. not by some praeternatural gift of new senses, but only by introducing a new technique in the use of the senses we happen to have. We move away from colors and shapes as seen, from sounds as heard, from heat and pressure as felt. We seek relations between data. We evade the necessity of merely sensible relations between merely sensible terms by measuring and relating the numbers named measurements.~~

Now the principal technique in effecting the transition from description to explanation is measurement. We move away from colors as seen, from sounds as heard, from heat and pressure as felt. In their place, we determine the numbers named measurements. In virtue of this substitution, we are able to turn from the relations of sensible terms, which are correlative to our senses, to the relations of numbers, which are correlative to one another. Such is the fundamental significance and function of measurement.

Further, in constructing these numerical relations of things to one another, there is introduced an almost necessary simplification of arrangement. If it would be theoretically possible, it would not be practicable to relate things to one another by stating separately the relations of each to all the others. The procedure that is both simpler and more systematic is to select one type of thing or magnitude, to relate all others directly to it, and to leave to deductive inference the relations of the others among themselves. Thus, instead of noting that Tom is 1/10 taller than Dick, Dick 1/20 shorter than Harry, and Harry 1/20 of Dick shorter than ~~Harry~~ Tom, one selects some arbitrary magnitude as standard unit and measures Tom, Dick, and Harry, not in terms of one another, but in terms of feet or centimeters.

A standard unit, then, is a physical magnitude among other similar physical magnitudes. Its position of privilege is due to the systematic simplicity of implying the relations of each of these magnitudes to all the others by stating only the relations of all to some one.

In selecting and determining standard units, there is a conventional, arbitrary element and, as well, there is a far larger theoretical element. It is a matter of convention that the standard ~~foor~~ foot is the length between notches on a bar at a certain temperature in a given place. It is arbitrary that the foot happens to have the length it has, neither more nor less. On the other hand, the remaining aspects of the standard unit have their basis in presumed or acquired theoretical knowledge. What is length? Does length vary with temperature? Does length vary with change of place or of time? Does length vary with changes of frames of reference? ~~Thes~~ These are relevant questions. If their answers rest on the results of empirical science, they are subject to revision when those results are revised. If their answers can be obtained only by appealing to the field of basic presuppositions and presumptions, they will be methodological and subject to the revisions of methodology.

The fundamental point to be grasped here is a point that already has been made. The absolute resides not on the level of sensible presentations but in the field of abstract propositions and invariant expressions. The constancy in time of the length of a standard metal bar cannot be ascertained by comparing its length yesterday with its length today; the field of observables is limited to the present place and time; today's length of the bar can be observed, if today you are in the right place; but yesterday's length has passed out of the field of observables and tomorrow's has not yet been ushered in. It remains that the constancy in time of the length of the bar is a conclusion based on general knowledge. One ascertains, as best one can, all the manners in which metal bars can change in length; one takes precautions to prevent the occurrence of any such changes in the standard; and, one concludes that, as far as one knows, no such change has taken place. In other words, the constancy of the standard is a conclusion based upon the invariance of laws, and a revision of the laws will lead to a new determination of standard requirements.

This possible revision of standards sets a logical puzzle. How, one may ask, can one reach new laws except through measurements based on old standards? How can the new laws be correct if the old standards are wrong? How can incorrect laws lead to the correction of old standards? Behind such questions there lies a mistaken presupposition. Science does not advance by deducing new conclusions from old premises. Deduction is an operation that occurs only in the field of concepts and propositions. But the advance of science, as we have seen, is a circuit, from data to inquiry, from inquiry to insight, from insight to the formulation of premises and the deduction of their implications, from such formulation to material operations, which yield fresh data and, in the limit, generate the new set of insights named a higher viewpoint. A basic revision, then, is a leap. At a stroke, it is a grasp of the insufficiency both of the old laws and of the old standards. At a stroke, it generates both the new laws and the new standards. Finally, by the same verification, it establishes ~~both~~ that both the new laws and the new standards satisfy the data.

What holds for standards, also holds for their use. It is necessary to define as accurately as possible the precise type of magnitude that is to be measured. It is necessary to define the precise procedure that leads from the measurable magnitude and the standard unit to the determination of the number named a measurement. At each stage in the development of a science, these definitions will be formed in the light of acquired or presumed knowledge. But at every subsequent stage, there is the possibility of further ~~acquisition~~ acquisitions and of new presumptions and so of a revision of the definitions. Such a revision involves, not the deduction of new conclusions from old premises, but a leap to fresh premises.

Such, then, is the generic notion of measurement. Clearly, it contains within itself the possibility of successive differentiations that result from revisions that occur in the abstract field of definitions, principles, and laws. We have now to turn our attention to the revision involved in the notions of spatial and temporal measurements by the Special Theory of Relativity.

4.3 Differentiations of the Generic Notion of Measurement.

Let us begin by distinguishing 1) size, 2) length, and 3) measurement.

By size will be meant magnitude apart from any geometrical conceptions. It is an elementary, experiential conjugate, and it is to be characterized in terms of simple experiences.

Thus, spatial size may be indicated sufficiently by saying that it varies in two manners. It varies in an external fashion, inasmuch as the nearer it is, the bigger it looks. Also it varies in an internal fashion, inasmuch as it expands or contracts.

Temporal size similarly varies in two manners. There is the external variation, named psychological time, which rushes by when we are interested and lags when we are bored. ~~There is also an internal variation that is independent of our subjective states; andxisxmeasured notxinxtermsxof thexlengthxofxdays a process may be prolonged or quickly end by a dilatoriliness or a expeditiousness of its own; and so the size of a year is much greater than the size of a day.~~ There are also internal ~~difference~~ differences between the sizes of durations; twenty years is a long time, even if one is not in jail; and a second is a short time, even if one is.

By length will be meant size as fitted into a geometrical construction.

Spatial length, at a first approximation, seems simply to be ~~distance~~ size in a single direction or dimension. Still, one does have to use some such expression as direction or dimension. This fact recalls, not only the analysis of size into length, breadth, and depth, but also the requirement that length has to be taken along a straight line or geodetic. Further, the ends of a straight line or geodetic are points, but the ends of a size are hardly just points; it follows that the size of the material object must have been submitted to some detailed geometrical analysis, so that boundaries of the size stand in some unique correspondence with points on a straight line. Finally, material objects may be varying
may/ internally in size, and they/be moving locally; an expanding or contracting object has a ~~length~~ series of lengths at a series of instants; a moving object successively lies between two series of bounding positions; ~~and~~ its length is not the distance between ~~a~~ present and past bounding positions; and so it follows that
an/ the length of/object~~s~~ depends, not only on a geometry of space, but also upon determinations of the instant and of simultaneity.

The length of a duration can be determined only by adding mechanical to geometrical analysis. There has to be discovered some constant velocity or some regular periodicity. The spatial size traversed by the velocity has to be conceived in terms of length and divided into equal parts. Finally, while the length of a single duration may be determined by counting traversed parts or recurring periods, still there are many durations; they have to be related to one another in some fashion; and so there must be worked out some general determination of simultaneity or synchronization.

It has ~~x~~ been noted that sizes differ in two manners; internally, in virtue of expansions and contractions, prolongations and curtailments; externally, in virtue of the relative position of our senses and the quality of our subjective states. The ~~x~~ obvious advantage of the notion of length is that it eliminates merely external differences of size. Still, one must not jump to the conclusion that, therefore, length will prove invariant. As has been seen, determinations of length depend upon determinations of simultaneity, and it may be that simultaneity is not invariant. Again, determinations of length depend upon the supposition of some specific geometry, and it may happen that the specific geometry, verified in Space and Time, does not regard length as invariant.

~~There remains measurement. It is a number that stands to unity, as the length of a measurable magnitude stands to the length of a standard unit~~

There remains measurement. On Newtonian suppositions, a measurement is a number that stands to unity as the length of the measured magnitude stands to the length of a standard unit. Thus, to say that a room is twenty feet long is to say that the length of the room stands to the length of a foot-rule as the number, twenty, stands to unity. Again, to say that a process lasts five seconds is to say that the length of the process stands to the length of a standard second as the number, five, stands to unity. Finally, lengths are invariant under permissible transformations, and so measurements valid in one reference frame are valid in all permissible frames.

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are invariant under permissible transformations, and so measurements valid in one reference frame are valid in all permissible frames.

Now the transition to the suppositions of Special Relativity may be effected very simply by noting an oversight in the foregoing account of measurement. Two rods, AP and BQ, are equal in length if and only if A coincides with B at the same time as P coincides with Q. In particular, if A coincides with B at one moment and P coincides with Q at another moment, relative motion could occur during the interval and so equality could not be asserted. Similarly, two clocks, R and S, are synchronous if and only if readings taken at the same time agree. In particular, synchronization cannot be asserted on the ground ~~time~~ that the ~~[illegible]~~ readings from R at one series of moments agree with the readings from S at another series of moments.

Moreover, not only is an exact determination of the meaning of simultaneity an essential condition in measuring spatial and temporal differences but also, as had been seen, it cannot be presumed that that meaning ~~simultaneity~~ is identical for all spatio-temporal standpoints. Indeed, since simultaneity is a relation between particular events occurring at particular times in particular places, it may be expected to be ~~that simultaneity is~~ analogous to such notions as "now" and "then."

Further, to escape the relativity of simultaneity, appeal must be made to some absolute. But the absolute in measurement as the absolute in space and time resides in the realm of principles and laws. For principles and laws, because they abstract from particular places and particular times, cannot vary with variations in place and time.

Hence, the basic supposition of measurement in

Special Relativity will coincide with its basic postulate that the mathematical expression of physical principles and laws is invariant under inertial transformations. It follows that the appropriate geometry into which sizes must be fitted to yield lengths will be Minkowski space. Further, it follows that the correct notion of simultaneity will be the notion implicit 1) theoretically in the Lorentz-Einstein transformation and 2) operationally in the fact that in all reference frames clocks are synchronized by light signals and the velocity of light is always the same constant.

Hence, in Special Relativity the measurement of any spatial or temporal difference determines a spatio-temporal ~~tum~~ interval 1) that is invariant for all reference frames but 2) that resolves into different spatial and temporal components in different relatively moving frames.

Further, a distinction may be drawn between normal and abnormal reference frames. For if a measured magnitude is purely spatial, in a normal frame it will have a temporal component that is zero, but in an abnormal frame it will have a temporal component that is not zero. Similarly, if a measurued magnitude is purely temporal, in a normal frame it will have a spatial ~~compoen~~ component that is zero, but in an abnormal frame it will have a spatial component that is not zero. It follows that in actual measuring only normal frames should be used if one is to avoid the complexity of discovering the temporal component in a spatial difference and the spatial component in a temporal difference.

It may be remarked that on the present analysis there seems to vanish the apparently arbitrary division of the

~~universe~~

universe into rods and clocks on the ~~sam~~ one hand and, on the other, everything else (*). For the fundamental point is the

(*) See the autobiography in Albert Einstein, Philosopher-Scientist, edited by P. A. Schilpp, The Library of Living Philosophers. New York. 1949 and 1951. P. 59.

relativity of simultaneity, and that relativity enters into the very notion of a determinate measurement. Hence, while measurements are relations between rods and clocks on the one hand and, on the other, all other spatial and temporal magnitudes, still there is no peculiarity in rods that is lacking in other spatial magnitudes and there is no peculiarity in clocks that is lacking in other temporal magnitudes.

Finally, it is perhaps unnecessary to note that our account of measurement makes no attempt to treat either the notion of measurement implicit in General Relativity or the problems that arise when the activity of measuring introduces a coincidental or non-systematic element into the objects under investigation. No doubt, these issues could not be omitted in a general treatment of the subject, but our purpose has been to reinforce the point that absolutes do not lie in the field of sensible particulars and to disassociate our account of the abstract intelligibility of Space and Time from the paradoxes that too readily have been supposed to be inherent in the Special Theory of Relativity.

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[Continue at page 284: §5. The Concrete Intelligibility ....]

5. The Concrete Intelligibility of Space and Time.

Space and Time have been defined as ordered totalities of concrete extensions and of concrete durations.

They are distinct from imaginary space and imaginary time, which are totalities of merely imagined extensions and of merely imagined durations. Moreover, the existence of this ~~extension~~ distinction reveals that notions of Space and Time begin from experienced extensions and experienced durations and employ reference frames to reach out and embrace the totality of other concrete extensions and concrete durations.

Since reference frames are an endless multiplicity, their intelligible order cannot be more than descriptive. If one would understand, not men's notions of Space and Time, but the intelligibility immanent in Space and in Time, then one must advance from reference frames to the geometrical principles and laws whose expression is invariant under transformations. Moreover, the geometry to be reached will coincide with the geometry determined by physicists in securing invariant expression for physical principles and laws.

However, such a geometry is abstract. It is abstract, not indeed in the sense that it is not verified (for what is wanted is a geometry verified by physicists), but in the sense that it consists in a set of abstract propositions and invariant expressions and that, while applicable to concrete extensions and durations, still is applied differently from different spatio-temporal view-points. Thus, as long as men remain on the level of invariant expressions, they are not considering any concrete extension and duration; inversely, as soon as men consider concrete extensions and durations, each views them differently. The endless multiplicity of different spatio-temporal stand-points and of different frames of reference, so far from being transcended, re-appears with every return from the abstract to the concrete.

There is a parallel point to be made. The abstract intelligibility of Space and Time is coincident with the solution of a problem in physics. It is the intelligibility, not so much of Space and of Time, as of physical objects in their spatio-temporal relations. May one not expect an intelligibility proper to Space and proper to Time?

Such, then, is the ~~ex~~ question envisaged by this section on the concrete intelligibility of Space and Time. What is wanted is an intelligibility grasped in the totality of concrete extensions and durations and, indeed, identical for all spatio-temporal view-points.

The answer is easily reached. One has only to shift from the classical type of inquiry, which has been under consideration, to the complementary statistical type. It has been argued that a theory of emergent probability exhibits generically the intelligibility immanent in world ~~per~~ process. Emergent probability is the successive realization of the possibilities of concrete situations in accord with their probabilities. The concrete intelligibility of Space is that it grounds the possibility of those simultaneous multiplicities named situations. The concrete intelligibility of Time is that it grounds the possibility of successive realizations in accord with probabilities. In other words, concrete extensions and concrete durations are the field or matter or potency in which emergent probability is the immanent form or intelligibility.