Now this general argument can be set forth in more concrete fashion inasmuch as the reader can be offered the materials for two insights. The first insight will be a grasp of the non-systematic in a familiar case. The second will be a grasp of the same lack of system in the aggregate of concrete patterns of diverging series of conditions.

The familiar case may be defined by the question, How many ways are there to cast a "five" with a single die? One might attempt to answer this question empirically. One would get a high-speed camera, suitable lighting, a transparent box, and proceed to take pictures. Next, one would study the pictures of all cases in which a "five" was thrown and calculate the linear and angular momenta in each movement of the die. The more diligent one was, the greater the number of distinct manners in which a "five" can be thrown would be known. But no matter how great one's industry, one could hardly arrive at the point where one could say one knew all of the ways in which a "five" could be thrown with this die from this box on this surface. Accordingly, one would shift to an a priori method. One would work out a formula that gave the maximum and minimum momenta for the last stage of a throw, and the formula would contain constants that received different numerical values for different surfaces and different dice. From the formula one could list all the possible combinations of specifications for the last stage of throwing a "five." By introducing a convenient supposition to prevent the list from containing a non-countable infinite multitude of cases, one could proceed to the second last stage of the process; it would end in any of the manners in which the last could begin; and a further formula would enable one to assign a multitude of ways in which the second last could begin for each way in which the last could begin. With this multitude of multitudes on one's hands, one could turn to the third last stage, and so forth.

Now we happen to know that throwing a "five" is a non-systematic process. While each movement in the process is determinate, while the relations between successive movements are determinate, still these relations cannot be subsumed under any rule or law. The purpose of the preceding paragraph was, not to show that throwing a "five" is non-systematic, but to grasp in that instance of the non-systematic some of its distinctive characters or symptoms. Our first discovery, then, was that an empirical method of observation and analysis could reveal a great number of ways in which the result might occur, but it offered no promise of providing a complete list of all the ways. Our second discovery was that an a priori method yielded an unmanageable variety of different combinations of distinct alternatives. Even though distinct stages of the process were summed up in formulae, still every possible combination of numerical values satisfying the formulae offered a different alternative, and combinations of these alternatives defined the different ways.

However, the reader is already acquainted with this general argument, and he now desires a more detailed account of its meaning and implications. Accordingly, we shall first illustrate set forth an illustration of the non-systematic and then provide an opportunity

Let us turn now to the second insight. Consider any event, X, and let it be defined as a determinate numerical value of some variable in some classical law. ~~at a specified place and time~~ Next, consider all the laws in which this variable occurs, and list all the alternative combinations of numerical values for the other variables in these laws when the event, X, is occurring.

Thirdly, consider the different manners in which each of the alternative combinations may be approached. Thus, if there are n variables involved and they may have the numerical values, a, b, c,... when the event, X, is occurring, then the a, b, c,... specify one of the alternative combinations. Now there are different combinations of rates of change in these variables, such that the rates of change are compatible and, as well, they bring the variables to the values, a, b, c,... A complete list of such combinations of rates of change, first, when the rates are regular, and secondly when they are not, would serve to define the different approaches to one of the alternative combinations.

Fourthly, repeat the foregoing performance for all kinds of events. Then, one will have worked out all the manners in which one may approach at all possible combinations of rates of change all the alternative combinations of numerical values for the other relevant variables when each variable in each law assumes every possible numerical value.

Fifthly, by comparing different processes, one can draw up a list of incompatible events.

Sixthly, by combining compatible processes in all possible manners, one can ~~construct~~ list diverging series of positive conditions for all kinds of events to as many removes as one pleases.

Perhaps this is enough. One is working out a plan for setting up an unmanageable variety of different combinations of distinct alternatives. The intelligent procedure in dealing with such combinations of alternatives is to acknowledge their non-systematic character and turn to the calculation of probabilities. For an a priori method of working out diverging series of conditions yields the concrete patterns that occur, not only in this visible universe, but also in every possible universe subject to the same laws. On the other hand, an a posteriori method would be both impracticable and inconclusive.

6.54 Summary. Such is the argument in the general case. Classical laws hold in the concrete only if conditions are fulfilled. To invoke the same or different laws to show that conditions will be fulfilled, merely sets up a diverging series of conditions. The further one goes back along the series, the more numerous become the conditions and the more they are dispersed not only in space but also in time. Even if one knew the patterns of the diverging series, and the fulfilment of all conditions at some nth remove, the only possible deduction would be in virtue of the inverse, converging series. Finally, such patterns form a non-systematic aggregate; they are an enormous series of different combinations of distinct alternatives; their intelligibility is reached, not by working them out in detail, but by acknowledging their non-systematic character and turning to probabilities.

The Possibility of Accurate Prediction.

6.55 However, besides the foregoing general case, there is also a range of particular cases. In the last analysis, they reduce to the general case. But the last analysis is not reached at once and, in the meantime, there is the possibility of the accurate deduction and prediction of fully determinate events. Accordingly, we have to define the particular cases, show how it escapes the logic of the diverging series of conditions, and finally argue that this escape is never complete.

The particular case will be named a scheme. Its abstract or theoretical component is some classical law or combination of laws, such that there arises a mutual fulfilment of conditions. Its concrete or factual component is such a conjunction of things or events that, in virtue of the law or laws, the conjunction leads to another, the other leads to a third, the third leads to a fourth, until eventually the initial conjunction recurs. Such schemes may be extremely simple or extremely complex. They may involve any number of intermediaries or, in the case of straight-forward continuity, none at all. Moreover, schemes may be combined, so that all will function if any one or two or n function. Finally, schemes may emerge in a conditioned series, such that the later become possible when the earlier are functioning.

The conspicuous example of the scheme of recurrence is of course the planetary system. But the whole of nature seems full of oscillations, rhythms, alternations, recurrences, from the elementary processes of physics to the technological, economic, and political inventions of man. Finally, when such patterns of recurrent and routine activity are submitted to analysis, they are found to involve the two elements of a scheme, the theoretical component of inter-related laws and the factual component of a conjunction that through the laws brings forth its own recurrence.

Clearly, such schemes do not suppress the principle that no event is unconditioned. Nor do they prevent each event from having many conditions. None the less, though the diverging series of conditions remains, it has been brought to heel. For the scheme itself takes care of its positive conditions, all of which are included within classes of events, and every event within the classes keeps recurring because the others do in a perpetual vicious circle.

There is, then, an escape from the unpredictability implicit in the diverging series of conditions. Were astronomers merely in possession of full and exact knowledge of all natural laws, they still would be stuck with their 3-body problem, that is, with the task of finding a general solution to the problem of determining the trajectories of three bodies when their initial positions and momenta were given. In fact, astronomers operate in the light of an imaginative synthesis; Ptolemy's mistaken imaginative synthesis yielded fair results; Copernicus' simpler imaginative synthesis combined with a more accurate knowledge of laws enables men to predict with remarkable accuracy the movements, not merely of three bodies, but of the sun, the planets, their satellites, the comets, and even asteroids.

Still, this escape is not complete. The periodicity of our planetary system offers no guarantee against internal disruption of its members or against the intrusion of some external body like a bull into our china shop. The planetary system secures its own perpetuity only if certain negative conditions are fulfilled, and over those negative

conditions it exercises no control. Moreover, just as the planetary system is not a proof of its own survival, so it is not the ground of its own emergence. A scheme is a matter, not merely of a combination of laws, but also of a happy conjunction of things or events. That conjunction has to take place before the scheme can begin to function, and so the scheme has its origin in a combination which it did not generate.

Now one might like to suppose that, just as there are schemes, so too there is an over-all scheme, an ultimate imaginative synthesis, on which there could be based accurate predictions of the emergence and survival of lesser ~~schemes~~ schemes. Such would be the affirmation of mechanist determinism. But, as we have seen, complete and exact knowledge of all laws would include a systematic unification of laws but without involving an imaginative synthesis either of the concrete unfolding of this universe or any other subject to the same laws. Moreover, an over-all scheme would have not only a theoretical component, constituted by laws in combination, but also a factual component, constituted by an initial conjunction that the over-all scheme itself could not bring about. Finally, the issue before us is to be settled, not by what one might like to think, but by the evidence; and the evidence is that the concrete, historical unfolding of this world process involves a conspicuous use of the statistical techniques of large numbers and long intervals of time. It seems to follow that the over-all intelligibility of our world process is, not in accord with the assumptions of mechanist determinism, but some different view that assigns a due place to statistical laws. After all, machines are constructed and function within schemes, and schemes emerge, survive, and are superseded without systematic divergence from the probabilities.

6.56 It is to be noted that the foregoing analysis cuts a middle path between traditional determinism and recent proposals of indeterminism.

It agrees with the indeterminist in acknowledging statistical laws, not merely from the viewpoint of our limited and imperfect knowledge, but also as grounded objectively. But it places this objective ground, not in some indeterminateness in things, but in a lack of systematic relationships.

No less, it agrees with the determinists in acknowledging that whatever is, is determinate, and that whatever follows from determinate antecedents or conditions, follows determinately. Every event rests on the occurrence and non-occurrence of a diverging series of conditions that may be examined as far back as anyone cares; and once the conditions are fulfilled, the event is in some sense inevitable. ~~Still, this inevitability is not a matter of law alone.~~ For events are linked to their concrete conditions by laws, but the conditions to which they are linked are settled in each case by concrete circumstance. The inevitability of events rests, then, on laws in conjunction with a merely empirical structure of circumstance, and such structures are the non-systematic aggregate of concrete patterns of diverging series. ~~Finally, the inevitability of events not only is some infra-intelligible necessity~~

The Indeterminacy of the Abstract.

6.56 We have been endeavoring to indicate in precise terms both the indeterminacy of the abstract and the consequent statistical residues.

In brief, the indeterminacy of the abstract is the indeterminacy of the blanket proviso, "other things being equal." Classical laws are said to hold in the concrete, provided other things are equal, but no one specifies what the other things are or in what their equality consists.

There is good reason for this ~~emitt~~ omission. For a fully determinate event in the general case depends upon the fulfilment of a diverging series of positive and negative conditions. The conditions at each remove in the series not only become more numerous but also scatter in space and in time. Finally, the patterns of such diverging series form an enormous, non-systematic aggregate.

It is true that there are schemes of recurrence. Granted any of a long series of suitable initial conjunctions, the operation of classical laws will tend to repeat the initial conjunction indefinitely. Still, there is only a tendency and not an absolute necessity, for here too there rules the blanket proviso, other things being equal, or marginal and negligible etc. Nor is there any evidence to support the affirmation of some over-all scheme to regularize the emergence and the survival of lesser schemes.

The general case, then, is the universal case. In the last analysis, events depend upon a non-systematic aggregate of patterns of diverging series of conditions. ~~Because that aggregate is non-systematic, it provides statistical investigation with its premise that the non-systematic cannot be systematized. Because the non-systematic component in concrete conditions is what insight abstracts from in arriving at classical laws, this component pertains to the merely empirical residue. It remains that the canon of statistical residues has its meaning clarified~~

Because that aggregate is non-systematic, it is a residue abstracted from by the totality of classical laws. Because the non-systematic is the premise of statistical inquiry, this residue may be named statistical. Hence, the canon of statistical residues may be said to affirm the non-systematic character of the aggregate of patterns of diverging series of conditions that govern concrete events.

A Mathematical Analogy.

6.57 ~~A mathematical analogy may exist, inasmuch as classical laws would be represented by differential equations~~

6.57 A mathematical analogy may exist. For complex combinations of differential equations are likely to be soluble only through the introduction of special suppositions and, even then, only by a method of approximations. Hence, if one said that classical laws corresponded to differential equations, that concrete problems demanded combinations of such equations, and that the totality of special suppositions and approximate solutions was non-systematic, one would have in the field of mathematics an analogy to the canon of statistical residues.