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On the suppositions of the Special Theory of Relativity, some revision is necessary. We shall consider how it affects 1) lengths of standard units, 2) lengths of measurable objects, 3) measurements, and 4) sizes.

First, a length results from fitting a size into a geometrical construction. On the Special Theory of Relativity, the relevant geometry is that of Minkowski space. The following characteristics of the lengths of standard units follow from the properties of this space or, what comes to the same thing, from the Lorentz-Einstein transformation.

1. In all ~~inertail~~ inertial frames of reference a standard rod determines a four-dimensional interval of magnitude, unity. Similarly, in all ~~inertail~~ inertial frames of reference a standard clock determines a four-dimensional interval of magnitude, <u>ic</u>, where <u>i</u> is the square root of minus one, and <u>c</u> is the velocity of light <u>in vacuo</u>.

2. A reference frame will be said to be normal to a standard rod, when the rod in the frame determines an interval with spatial component of magnitude, unity, and with temporal component of magnitude, zero.

Similarly, a reference frame will be said to be normal to a standard clock, when the clock in the frame determines an interval with a spatial component of magnitude, zero, and a temporal component of magnitude, unity.

3. Reference frames that are not normal to standard rods or standard clocks are in relative motion to normal reference frames.

Inversely, in reference frames in relative motion to normal frames, standard rods determine the same invariant interval but now possess spatial components, H, and temporal ~~component~~ components, $- Hu/c^2$ or Hu/c^2, according to the direction of the relative motion.

Similarly, in reference frames in relative motion to normal frames, standard clocks determine the same invariant interval, which, however, now possesses a spatial component, - Hu or Hu, and a temporal component, H.

Secondly, there are to be determined the characteristics of the lengths of other measurable objects. Clearly, these lengths will have the same properties as the lengths of standard units. For both sets of lengths are subject to the same transformation equations.

Accordingly, for every measurable spatial object there is a group of normal reference frames, relatively at rest, and in them the object determines an interval with spatial component, A, and with temporal component, zero. In other reference frames in relative motion, the same object will determine an interval of the same magnitude but with spatial component, AH, and with temporal component, $- AHu/c^2$ or AHu/c^2, according to the direction of the ~~x~~ relative motion.

Similarly, for every measurable temporal object, there is a group of normal reference frames, relatively at rest, and in them the object determines an interval with spatial component, zero, and with temporal component, B. In other inertial frames in relative motion, the same object will determine

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the same invariant interval, namely icB, but with a spatial component, - BHu or BHu, and with a temporal component, BH.

In the third place, measurements are to be considered, and they offer two distinct aspects.

For, in so far as measurements are numbers to be substituted into equations or to be derived ~~from~~ by solving equations, they are identical with lengths. This follows from the nature of the coordinate system which, in the present case, deals only with measured lengths. Accordingly, all that has been said about lengths may now be repeated about measurements. A spatial magnitude will determine an invariant interval, A, with components, [AH, - AHu/c^2], and a temporal magnitude will determine an invariant interval, icB, with components, [BH, - BHu]. In normal reference frames, H becomes unity, and u becomes zero, so that the components are [A, 0] and [0, B] respectively. Finally, in transformations to the left, the sign of u changes.

However, there is a further aspect to measurements. The numbers substituted into equations have to be derived from data, and the numbers derived ~~in~~ from equations have to be verified in data. Thus, there arises the question whether Special Relativity modifies the concrete operation of ~~measurement.~~ measuring.

The general answer would seem to be that it does not. A measurement remains the number that stands to unity as the measurable object stands to a standard unit. However, within the frame-work of that general answer it will be well to advert to particular cases.

~~Ordinarily, spatial measurements occur in normal reference frames. In other words, neither the measurable object nor the standard unit is moving relatively to the frame. Moreover, since A : 1 :: AH : H, it would make no difference if both measurable object and standard unit had the same relative velocity with respect to the frame~~

Ordinarily, simultaneity is determined in the same manner in selecting the point-instants at the ends of the standard unit and in selecting those at the ends of the measurable object. It will follow that spatial measurements ordinarily occur with the standard unit and the measurable object in the same reference frame and, since A : 1 :: AH : H, the result of measuring will be the number, A.

Still, this is not inevitable. Further, it may be ~~far~~ fairly common to use a clock, stationary in a reference frame, to time a process that begins at one place in the frame and ends at another. Hence, besides the measurements that result when the object and the standard are taken in the same frame, namely, A/1, AH/H, B/1, BH/H, there are the measurements that result when they are in different frames. If one of these frames is normal, the results will be AH/1, A/H, BH/1, B/H; if neither frame is normal, one must distinguish two values of H, say H' and H", so that the results may be AH'/H", AH"/H', BH'/H", BH"/H'. In other words, the actual process of measuring can involve the same ~~fallacies~~ ambiguities as are contained in the elementary paradox and, indeed, even more elaborate ~~fallacies~~ ambiguities.

Accordingly, we are brought to the conclusion that, while Special Relativity demands an operation of measuring that fundamentally is similar to measuring under Newtonian assumptions, still it adds new rules that either eliminate or correct some results which, on Newtonian assumptions, would be valid.

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Foot-note

In the fourth place, there are the sizes of spatial and of temporal magnitudes. Do rods contract or expand? Do clocks run slow or fast? Our answer will be negative, and our reasons run as follows.

First, it is difficult to suppose that rods and clocks should undergo such variation without a proportionate variation occurring in the objects that they measure; and if the proportionate variation occurs, then no explanation is provided for the relativity of lengths to reference frames.

~~secondly, variations in size are unequal to~~

Secondly, even if rods and clocks varied while other sizes do not vary, the required explanation would not be forthcoming. For rods and clocks and other sizes determine intervals that are invariant for all inertial reference frames. Moreover, these intervals exhibit temporal components for rods and other spatial magnitudes; and they exhibit ~~in~~ spatial components for clocks and other temporal magnitudes. How does a contracting rod generate a temporal component? How does a decelerated clock generate a spatial component?

Thirdly, the evidence for contracting rods and decelerated clocks lies in the elementary paradox. Now we have no doubt that, on the suppositions of Special Relativity, it would be possible to reach such measurements as A/H, AH/1, B/H, BH/1, which are the lengthened and shortened rods and the faster and slower clocks. But the obvious explanation lies, not in any variation of the sizes of rods or clocks, but in the relativity of lengths and in the use of a standard unit in one reference frame to measure an object in another, significantly different frame.

Fourthly, there is no aspect of the Special Theory of Relativity that is not accounted for by distinguishing between size and length, where length is constructed in accord with the geometry of Minkowski space. There follow immediately both the invariant intervals and the relativity of spatial and temporal components to reference frames. Moreover, this construction of length ~~presupp~~ presupposes, not a variation in size, but a relativity of simultaneity. It was ~~from~~ from a relative solution to the problem of synchronization that Special Relativity was evolved; and whenever such a solution is adopted, Special Relativity will follow even though no variation in size is admitted.

This point is worth illustrating. Suppose two planes flying in the same direction with the same constant velocity, so that the distance between them is constant. Let that distance be regarded as the standard unit, and suppose two observers, K and K', that determine simultaneity differently. Now consider the instant at which the first plane is at a point, P. ~~T~~ Let us say that for K the second plane at the same instant is at some point, R. Then for K', since he determines simultaneity differently, the second plane must be at some nearer or further point, S, at the instant when the first plane is at P.

~~relativity of simultaneity generates a relativity of distance.~~

~~Such is the problem. If this relativity of simultaneity is regarded as ultimate, then the solution will be to appeal to the invariance of principles and laws and to select the geometry that grants principles and laws their invariance. Accordingly, one makes Einstein's postulates and draws his conclusions. But one does so, not by supposing that the second plane altered its speed and so expanded or contracted a size, but, because statements of simultaneity are not regarded as invariant.~~

Chapter V: Space & Time §4 Rods & Clocks

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The question should be clarified. Size has been ~~[illegible]~~ defined as an experimental conjugate that varies both from inner change in the object and from change of position of the observer. In the text I do not mean to deny perspectival variation of size. Similarly I do not mean either to affirm or deny what I regard as meaningless, namely that there is or is not an inner change of the object as referred to some absolute space. The question is whether an acceptance of special relativity logically entails any change in rods or clocks, and my answer is that no such change can be deduced. "Lengths" vary because reference frames vary, and reference frames vary because modes of determining simultaneity vary.

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Accordingly, though there is only one size, though this size is constant, though both observers agree that there is only one size and that it is constant, none the less in virtue of different determinations of simultaneity there are two lengths, PR and PS, and they are unequal with an inequality in some proportion to the relative velocities of the planes and the divergence between the two determinations of simultaneity.

While this illustration is, I believe, to the point, still it is only an illustration. One cannot take a relativity of simultaneity as postulate and from it deduce the Special Theory of Relativity. On the contrary, a relativity of simultaneity merely sets a problem; confronted with that problem, one adverts to the invariance of principles and laws; and it is by postulating the invariance of principles and laws under inertial transformations that one reaches the basic premise from which Special Relativity follows.

4.4 Summary.

The aim has been to work out a general theory of measurement and there by clarify the notions of measurable object, standard unit, measuring, and measurement peculiar to Special Relativity.

Measurement was seen to be the technique by which the scientist moves from the description of things as related to our senses to the explanation of things as related to one another.

Standard units were conceived as measurable objects that intrinsically stand on the same footing as other measurable objects but conventionally are given a unique status to simplify and systematize the formulation of the relations of things to one another.

The definitions of measurable objects of various kinds, the standardization of their respective units, the rules of measuring, and the nature of measurement were seen to depend on abstract presumptions and laws and, therefore, to be subject to revision along with revisions of the presumptions and the laws.

This generic notion of measurement was then applied to measurements of spatial and temporal magnitudes.

A basic distinction was drawn between the experiential conjugate, size, and the pure conjugate, length. The former is correlative to our experience. The latter is implicit in a geometrical structure of definitions, postulates, and inferences.

The transition from Newtonian to Einsteinian physics is a transition from length, as implicit in Euclidean geometry, to length, as implicit in Minkowski space. It drops invariant spatial and temporal lengths. It introduces invariant four-dimensional intervals with variable spatial and temporal components. While it grants no special significance to reference frames at rest, still it does imply a position of privilege for normal reference frames, in which spatial magnitudes have a zero temporal component and temporal magnitudes have a zero spatial component. Thus, an interval, A, which is a real number, has the components $[AH, - AHu/c^2]$ which become $[A, 0]$ in a normal reference frame; and an interval, icB, which is an imaginary number, has the components $[- BHu, BH]$, which become $[0, B]$ in a normal reference frame. It is to be noted that the distinction between the spatial and the temporal is as sharp as

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of/

the distinction between real and imaginary numbers, that the lengths of standard units are but particular cases of the lengths of other measurable objects, that the transformation properties of unit and/other lengths are the same, that in a Minkowski manifold lengths are already measured so that measurements are coincident with lengths, that in the operation of measuring there arise ~~an~~ in Special Relativity ambiguities that do not exist and so do not have to be solved on Newtonian suppositions.

However, while Special Relativity involves a revision of the notions of lengths and of measurements and while it introduces a new caution in the operation of measuring, it does not imply the expansion or contraction of rods or the acceleration or deceleration of clocks. In other words, the unit divisions of the axes in the coordinate systems are constituted, not by the size, but by the length of standard distances and standard durations. Such lengths are relative to reference frames, but this relativity of length arises, not from change of size, but from the ~~dependence~~ inter-dependence of determinations of ~~length~~ length and of simultaneity. What corresponds to change of size is, not a mere transformation of reference frames, but a variation in the intervals, A or icB. A variation in some of these intervals corresponds to a variation in some measurable objects; a proportionate variation in all of these intervals suggests that the standardization of units needs to be corrected and revised.

Might I suggest that, on this showing, there vanishes the arbitrary division of the world of physics into rods and clocks and, on the other hand, all other objects? Such arbitrariness is noted and regretted by Prof. Einstein in his Autobiography [Albert Einstein, Philosopher-Scientist, ed. P.A.Schlipp, The Library of Living Philosophers, New York 1949 and 1951, p. 59]. It would seem to vanish inasmuch as physics is set the problem of accounting for experienced extensions and durations by invariantly expressed abstract relations in the same manner as it accounts for experienced colors and sounds and experienced phenomena of heat and electricity, ~~and~~ inasmuch as this problem is met by formulating hypotheses and by verifying them in the data of experience, inasmuch as notions of measurement and rules of measuring form part of the hypothesis~~es~~ to be verified

I left this out ?

It would seem to vanish 1) inasmuch as physics is set the task of assigning invariantly expressed abstract relations to account not only for experienced colors and sounds but equally for experienced extensions and durations, 2) inasmuch as these relations are reached by formulating and verifying hypotheses, 3) inasmuch as notions of length and measurement and the standardization of units form internal parts of the hypothesis to be verified, ~~and~~ 4) inasmuch as the hypothesis assigns the same properties to lengths of standard units as to lengths of other measurable objects, and 5) inasmuch as frames of reference have their units constituted, not by the sizes of rods and clocks, but by their theoretically defined lengths.

? p.283 – Dennis ?

Finally, it would seem that the foregoing account of rods and clocks in Special Relativity might easily be adapted to the requirements of General Relativity. In General Relativity there remains the invariant four-dimensional interval; there remain its spatial and its temporal components; there remains the covariance of these components in different reference frames. The basic differences are that the components now are curvilinear and that specifications of coordinates are not virtual measurements of distance or duration.

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Chapter V: Space and Time.
§4. Rods and Clocks

Foot-note to p. 239

The meaning of the question should be clarified further. I have defined "size" as an experiential conjugate that varies both perspectivally and from an inner change of the object. There is no doubt about the perspectival relativity of size and, as has been concluded, there is no meaning to the statement that an object, as compared with absolute space, is larger or smaller.

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Chapter V: Space and Time. §4.3 Rods and Clocks.

Foot-note to p. ~~259.~~ Towards end of #4.3. Questions are: "Do rods contract and expand? Do clocks run fast and slow?"

The questions should be clarified. "Size" has been defined as an experiential conjugate that varies both from inner change in the object and from change of position of the observer. In the text I do not mean to deny perspectival variation of size. Similarly, I do not mean either to affirm ~~as~~ or to deny what I regard as meaningless, namely, that there is or is not an inner change of the object as referred to some absolute space. The question is whether an acceptance of special relativity logically entails any change in rods or clocks, and my answer is that no such change can be deduced. "Lengths" vary because reference frames vary; and reference frames vary because modes of determining simultaneity vary.

On the other hand, it is not to be claimed that our account of measuring deleted is completely general. Rather that distinction ~~should~~ seems to pertain to Quantum Theory viewed as a theory of measurements. For if it is true that all ~~measurement is~~ measuring is abstractive both in the sense that it replaces sets of data by series of approximate numbers and in the sense that it relates the numbers not to our senses but to one another, still the relations may be systematic or non-systematic; and non-systematic relations, no matter what their origin, can be manipulated theoretically only in a context that ~~ex~~ envisages statistical laws.

End of 4.4.