The conceptual answer is the conceptualization of the psychological answer. We recall that analytic principles are analytic propositions whose terms are existential; and that existential terms are terms that occur in factual judgments. Further, serial analytic principles are analytic propositions whose terms are serially existential; and terms are serially existential in a series of manners.

What is that series of manners? As the reader will suspect, it is the series that corresponds to the series of mathematical departments; again, it is the series that corresponds to the psychological series of learning mathematics. But our present task is to attempt to assign rules by which that series can be recognized, and our procedure will be, first, to offer illustrations of the serially existential and, secondly, to assign rules that characterize the series of manners in which terms may be serially exitential.

A first illustration of the serially existential is offered by the positive integers.

Obviously, not all positive integers are existential, for not all occur in factual judgments. Indeed, no matter how many millions of them do so, there is an infinity of other positive integers that do not.

On the other hand, the whole series of positive integers is grounded in the existential. For in the first place many positive integers do occur in factual judgments; and, in the second place, the generative principle of the series, the operation of adding "one more," also occurs in factual judgments.

The first case of the serially existential is, then, a series of terms that satisfies two conditions. Some of the terms are existential. The generative principle of the series is in some instances existential.

A second illustration of the serially existential is offered by the totality of "real" numbers.

First, some of the real numbers, namely, the positive integers, are serially existential. Secondly, from the positive integers by appropriate operations one can reach any of the real numbers. Thirdly, these appropriate operations are in some instances equivalent to existential operations.

The third point may need elucidation. "Adding" in the sense in which it certainly occurs in factual judgments can base definitions of substracting, multiplying, dividing, powers, and roots. However, these operations, so defined, will not yield the totality of the real numbers. What is needed is the algebraic generalization of these operations, and one may doubt that such generalized operations occur in factual judgments. But what is beyond doubt is that in certain instances both the initial and the generalized operations yield the same results. The square root of 16 is 4, whether you define square root by a theory of indices, or take it as the inverse of "multiplying a number by itself" and "multiplying a number by itself" as "taking that number the same number of times and adding the resultant aggregate."

A third illustration of the serially existential is offered by geometry. Let us say that that Euclid's geometry consists of elements and constructions. Now such elements as points, finite straight lines, finite planes, in their defined sense, do not occur in factual judgments. Still what approximates to them does occur in factual judgments. Again, the exact execution of Euclidean constructions, if it occurs, cannot be ascertained as in factexact. Still what approximates to such exact execution does occur in factual judgments. In this case the serially ~~existential will consist of terms that can be generated approximately from terms that approximate to~~ existential will be a ~~series~~ of terms, some of which approximate to the existential, and the rest/can be generated by constructions that approximate to the existential. From that basis one can proceed to add further manners of the serially existential. ~~Thus, lines produced to infinity, planes infinitely extended~~ Thus, lines or planes extended to infinity will be serially existential because their starting-point and their generative principle are serially existential. Again, geometries that include Euclidean as a particular case would be serially existential because of that case. On the other hand, specifically non-Euclidean geometries could be serially existential on the ground that, like the Euclidean, they approximate to the existential. Finally, one might conceive conceive a grand series of geometries and hold all to be serially existential because some are serially existential and/the ~~xxxxxx~~ from generative principle of the lot is serially existential inasmuch as it relates the few that are serially existential.

totality// of which /

because/

Three illustrations of the serially existential have been offered. It is time to try to grasp the notion itself.

First, then, the serially existential takes the existential as its starting-point.

Secondly, the existential is merely a starting-point. It represents the knowledge of present factual judgments. But present factual judgments are limited by present ignorance. The vast field of the unknown that remains to be known is ~~woth~~ worth exploring. If, so to speak, the mathematician has one foot solidly placed on the earth, the other is raised and groping for further footholds.

Thirdly, this exploration is conducted systematically. Its rule is to be perfectly general. Some positive integers occur in factual judgments. Others might occur. Which ones might, we do not know. Explore the lot. In like manner, the totality of real numbers is envisaged, the totality of geometries, and so forth.

Fourthly, the sets of operations ~~woth~~ with their terms, that are explored with perfect generality, are referred to the known existential, as the complete series to the incomplete, as the general to the particular, as the ideal to the approximate; moreover, the reference may be direct or mediated, and the mediation may involve any number of steps.

Fifthly, the series of different manners of the serially existential is not closed but open. Future developments of mathematics are still possible. Their precise nature cannot ~~be determined in advance. Their precise reference to the known existential cannot be fixed beforehand.~~

be foreseen, and so their precise reference to the existential is to be left to future discovery or invention.

Sixthly, if ever a department of mathematics is correctly referred to the existential, then it will ~~remain~~ always remain a department of mathematics. Still, this does not preclude revision of the manner in which that reference to the existential is conceived; for subsequent developments may involve more satisfactory manners of ~~conception~~ conceiving earlier developments.

Seventhly, since sound judgment presupposes an accumulation of insights, sound judgment on the validity of new departments of mathematics is to be expected, not at their inception when the self-correcting process of learning has hardly begun, but only when the new field has been adequately explored and its full implications have been grasped.

Eightly, the possibility of applying even abstruse mathematics ~~to empirically by~~ within empirical theories rests on the reference of even the abstruse to the existential. The fact of such application, however, does not establish the validity of mathematics but only serves to reveal its usefulness.

A few further questions may be considered.

First, is the object of mathematics the possible?

If by the possible is meant what can exist, the answer is negative. For mathematics does not consider the totality of conditions of real possibility. What can really exist, must meet physical or chemical or biological or some similar set of conditions; and the mathematician does not bother about any of them.

If by the possible is meant the mathematically possible, then it remains to be explained what precisely that is.

Perhaps behind this question lies the desire to give mathematics an objective, ontological foundation, for instance, in the ideas of the divine mind. Such a desire seems out of place. An essential element in mathematics as it happens to exist is ignorance. The mathematical field is worth exploring because the existential, as we have defined it, is not privileged. It is not whatever exists but whatever we know exists, whatever supplies a term in our factual judgments. In so far as the mathematician has an ulterior purpose, he does not aim at knowing how God would conceive possible universes but at determining how men might grasp more comprehensively the existing universe. Finally, correct mathematics can have their exemplar in the divine mind not only as correct conceptions of possible things but also as correct human operations to envisage with complete generality the series of terms some of which in some fashion present an aspect that is existential.

Second, what about axiomatic coherence?

The real ground of coherence of a set of axioms is the cluster of insights in an intelligence. Inasmuch as a department of mathematics emerges through the development of mathematical understanding, its coherence seems to be guaranteed automatically. On the other hand, a merely formalizing approach to mathematical foundations hasthe problem of coherence on its hands.

Thirdly, what about the principle of excluded middle?

Our requirement of reference to the existential is opposed in principle to setting up objects that are defined