Chapter X.

REFLECTIVE UNDERSTANDING

Like the acts of direct and introspective understanding, the act of reflective understanding is an insight. As they meet questions for intelligence, it meets questions for reflection. As they lead to definitions and formulations, it leads to judgments. As they grasp unity, or system, or ideal frequency, it grasps the sufficiency of the evidence for a prospective judgment.

When Archimedes shouted his Eureka, he was aware of a significant addition to his knowledge, but it is not likely that he would have been able to formulate explicitly just what a direct insight is. Similarly, we perform acts of reflective understanding, we know that we have grasped the sufficiency of the evidence for a judgment on which we have been deliberating, but without prolonged efforts at introspective analysis we could not say just what occurs in the reflective insight. What we know is that to pronounce judgment without that reflective grasp is merely to guess: again, what we know is that, once that grasp has occurred, then to refuse to judge is just silly.

Accordingly, the present section will be an effort to determine what precisely is meant by the sufficiency of the evidence for a prospective judgment. There is presupposed a question for reflection, "Is it so?". There follows a judgment, "It is so.". Between the two there is a marshalling and weighing of evidence. But what are the scales on which evidence is weighed? What does it have to weigh, if one is to pronounce a "Yes" or a "No"?

Unfortunately, the more complex judgments become, the more complex is the analysis of the grounding act of reflective understanding. The whole answer cannot be given at once and partial answers are incomplete. Hence, we shall begin from a very general statement and then illustrate its meaning from the form of deductive inference. Next, we shall turn to the concrete judgments of every day life, and consider in turn concrete judgments of fact, judgments on the correctness of insights into concrete situations, and finally the occurrence of analogies and generalizations. In the third place there will be considered the judgments of empirical science, the radical difference of such judgments from those of ordinary living, the nature of scientific generalization and verification, and what is meant by the probability of scientific opinions. Fourthly, analytic propositions and principles are distinguished and their criteria investigated. Fifthly, the nature of mathematical judgments is considered. Finally, we may add that philosophic judgments are not treated in this chapter, for they can be examined satisfactorily only after further elements in the problem have been set forth.

1. THE GENERAL FORM OF REFLECTIVE INSIGHT

To grasp evidence as sufficient for a prospective judgment is to grasp the prospective judgment as virtually unconditioned.

Distinguish then, between the formally and the virtually unconditioned. The formally unconditioned has no conditions whatever. The virtually unconditioned has conditions indeed but they are fulfilled.

Accordingly, a virtually unconditioned involves three elements, namely: 1) a conditioned, 2) a link between the conditioned and its conditions, and 3) the fulfilment of the conditions. Hence, a prospective judgment will be virtually unconditioned if 1) it is the conditioned, 2) its conditions are known, and 3) the conditions are fulfilled. By the mere fact that a question for reflection has been put, the prospective judgment is a conditioned: it stands in need of evidence sufficient for reasonable pronouncement. The function of reflective understanding is to meet the question for reflection by transforming the prospective judgment from the status of a conditioned to the status of a virtually unconditioned; and reflective understanding effects this transformation by grasping the conditions of the conditioned and their fulfilment.

Such is the general scheme and we proceed to illustrate it from the form of deductive inference. Where A and B each stand for one or more propositions, the deductive form is:

If A, then B.
But A.
Therefore B.

For instance:

If X is material and alive, X is mortal.
But men are material and alive.
Therefore, men are mortal.

Now the conclusion is a conditioned, for an argument is needed to support it. The major premise links this conditioned to its conditions, for it affirms, If A, then B. The minor premise presents the fulfilment of the conditions, for it affirms the antecedent, A. The function, then, of the form of deductive inference is to exhibit a conclusion as virtually unconditioned. Reflective insight grasps the pattern, and by rational compulsion there follows the judgment.

However, deductive inference cannot be the basic case of judgment, for it presupposes other judgments to be true. For that reason we have said that the form of deductive inference is merely a clear illustration of what is meant by grasping a prospective judgment as virtually unconditioned. Far more general than the form of deductive inference is the form of reflective insight itself. If there is to be a deduction, the link between the conditioned and its conditions must be a judgment, and the fulfilment of the conditions must be a further judgment. But judgments are the final products of cognitional process. Before the link between conditioned and conditions appears in the act of judgment, it existed in a more rudimentary state within the cognitional process itself.

Before the fulfilment of conditions appears in another act of judgment, it too was present in a more rudimentary state within cognitional process. The remarkable fact about reflective insight is that it can make use of those more rudimentary elements in cognitional process to reach the virtually unconditioned. Let us now see how this is done in various cases.

2. CONCRETE JUDGMENTS OF FACT.

Suppose a man to return from work to his tidy home and to find the windows smashed, smoke in the air, and water on the floor. Suppose him to make the extremely restrained judgment of fact: Something happened. The question is, not whether he was right, but how he reached his affirmation.

The conditioned will be the judgment that something happened.

The fulfilling conditions will be two sets of data: the remembered data of his home as he left it in the morning; the present data of his home as he finds it in the evening. Observe that the fulfilling conditions are found on the level of presentations. They are not judgments, as is the minor premise of syllogism. They involve no questions for intelligence nor insights nor concepts. They lie simply on the level of past and present experience, of the occurrence of acts of seeing and smelling.

The link between the conditioned and the fulfilling conditions is a structure immanent and operative within cognitional process. It is not a judgment. It is not a formulated set of concepts, such as a definition. It is simply a way of doing things, a procedure within the cognitional field.

The general form of all such structures and procedures has already been outlined in terms of the three levels of presentations, intelligence, and reflection. Specializations of the general form may be exemplied by the classical and statistical phases of empirical method, and by the notion of the thing, and by the differences between description and explanation. However, such accounts of the general form and its specializations pertain to introspective analysis. Prior to such an investigation and formulation, the structures and procedures exist and operate; nor, in general, do they operate any better because the analysis has been effected.

Now, in the particular instance under consideration, the weary worker ~~man~~ not only experiences present data and recalls different data but by direct insights he refers both sets of data to the same set of things which he calls his home. The direct insight, however, fulfils a double function. Not merely are two fields of individual data referred to one identical set of things but a second level of cognitional process is added to a first. The two together contain a specific structure of that process, which we may name the notion of knowing change. Just as knowing a thing consists in grasping an intelligible unity-identity-whole in individual data, so knowing change consists in grasping the same identity or identities at different times in different individual data. If the same thing exhibits different individual data at different times, it has changed. If there occurs a change, something has happened. But these are statements. If they are affirmed, they are judgments. But prior to being either statements or judgments, they exist as

unanalysed structures or procedures immanent and operative within cognitional process. It is such a structure that links the conditioned with the fulfilling conditions in the concrete judgment of fact.

The three elements have been assembled. On the level of presentations there are two sets of data. On the level of intelligence there is an insight referring both sets to the same things. When both levels are taken together, there is involved the notion of knowing change. Reflective understanding grasps all three as a virtually unconditioned to ground the judgment, Something happened.

While our illustrative instance was as simple as it could be, still it provides the model for the analysis of more complex instances of the concrete judgment of fact. The fulfilling conditions may be any combination of data from the memories of a long life, and their acquisition may have involved exceptional powers of observation. The cognitional structure may suppose the cumulative development of understanding exemplified by the men of experience, the specialist, the expert. Both complex data and a complex structure may combine to yield a virtually unconditioned that introspective analysis could hardly hope to reproduce accurately and convincingly. But the general nature of the concrete judgment of fact would remain the same as in the simple case we considered.

However, the reader probably is asking how we know whether the insights that constitute the pivot of such structures are themselves correct. To this point we have now to turn.

3. INSIGHTS INTO CONCRETE SITUATIONS

Direct and introspective insights arise in response to an inquiring attitude. There are data to be understood; inquiry seeks understanding; and the insight arises as the relevant understanding. But a mere bright idea is one thing, and a correct idea is another. How do we distinguish between the two?

The question is asked, not in its full generality but with respect to concrete situations that diverge from our expectations and by that divergence set us a problem. Thus, to retain our former illustration, the man on returning home might have said: There has been a fire. Since any fire there might have was extinguished, that judgment would suppose an insight that put two and two together. Our question is on what grounds such an insight could be pronounced correct.

First, then, observe that insights not only arise in answer to questions but also are followed by further questions. Observe, moreover, that such further questions are of two kinds. They may stick to the initial issue, or they may go on to raise distinct issues. What started the fire? Where is my wife? Observe, thirdly, that the transition to distinct issues may result from very different reasons; it may be because different interests supervene to draw attention elsewhere; but it may also be because the initial issue is exhausted, because about it there are no further questions to be asked.

Let us now distinguish between vulnerable and invulnerable insights. Insights are vulnerable when there are further questions to be asked on the same issue. For the further questions lead to

further insights that certainly complement the initial insight, that to a greater or lesser extent modify its expression and implications, that perhaps lead to an entirely new slant on the issue. But when there are no further questions, the insight is invulnerable. For it is only through further questions that there arise the further insights that complement, modify, or revise the initial approach and explanation.

Now this reveals a law immanent and operative in cognitional process. Prior to our conceptual distinction between correct and mistaken insights, there is an operational distinction between invulnerable and vulnerable insights. When an insight meets the issue squarely, when it hits the bull's eye, when it settles the matter, there are no further questions to be asked and so there are no further insights to challenge the initial position. But when the issue is not met squarely, there are further questions that would reveal the unsatisfactoriness of the insight and would evoke the further insights that put a new light on the matter.

Such, then, is the basic element in our solution. The link between conditioned and its conditions is a law immanent and operative in cognitional process. The conditioned is the prospective judgment, This or that direct or introspective insight is correct. The immanent law of cognitional process may be formulated from our analysis. Such an insight is correct, if there are no further, pertinent questions.

At once it follows that the conditions for the prospective judgment are fulfilled when there are no further, pertinent questions.

Note that it is not enough to say that the conditions are fulfilled when no further questions occur to me. The mere absence of further questions in my mind can have other causes. My intellectual curiosity may be stifled by other interests. My eagerness to satisfy other drives may refuse the further questions a chance to emerge. To pass judgment in that case is to be rash, to leap before one looks.

As there is rash judgment, so also there is mere indecision. As the mere absence of further questions in my mind is not enough, so it is too much to demand that the very possibility of further questions has to be excluded. If, in fact, there are no further questions, then, in fact, the insight is invulnerable; if, in fact, the insight is invulnerable, then, in fact, the judgment approving it will be correct.

But how is one to strike this happy balance between rashness and indecision? How is one to know when it is reached? Were there some simple formula or recipe in answer to such questions, then men of good judgment could be produced at will and indefinitely. All we can attempt is an analysis of the main factors in the problem and an outline of the general nature of their solution.

In the first place, then, one has to give the further questions a chance to arise. The seed of intellectual curiosity has to grow into a rugged tree to hold its own against the desires and fears, conations and appetites, drives and interests, that inhabit the heart of man. Moreover, every insight has its retinue of presuppositions, implications, and applications. One has to take

the steps needed for that retinue to come to light. The presuppositions and implications of a given insight have to knit coherently with the presuppositions and implications of other insights. Its possibilities of concrete application have to enter into the field of operations and undergo the test of success or failure. I do not mean, of course, that concrete living is to pursue this logical and operational expansion in the explicit, deliberate, and elaborate manner of the scientific investigator. But I do mean that something equivalent is to be sought by intellectual alertness, by taking one's time, by talking things over, by putting viewpoints to the test of action.

In the second place, the prior issue is to be noted. Behind the theory of correct insights, there is a theory of correct problems. It was to dodge this prior issue that we supposed a concrete situation that diverges from our expectations and by that divergence defines a problem. In other words, there has been postulated an inquirer that understands the background of the situation and so knows what is to be expected; there also has been postulated a problem that exists, that is accurately defined by the divergence of the situation from correct expectations, that in turn provides a definition of the pertinence of any further questions.

Now this amounts to saying that good judgment about any insight has to rest on the previous acquisition of a large number of other, connected, and correct insights. But before attempting to break this vicious circle, let us assure ourselves of the fact of its existence. Children ask endless questions; we have no

doubt about their intellectual curiosity; but so far from crediting them with good judgment, we do not suppose them to reach the age of reason before their seventh year. Young men and women have the alertness of mind that justifies their crowding into schools and universities, but the law doubts the soundness of their judgment and regards them as minors, while Aristotle denied they had enough experience to study ethics with profit. Nor is there merely the initial difficulty of acquisition but, as well, there is the subsequent necessity of keeping in touch. The man that returns to a field of commerce or industry, to a profession or a milieu, in which once he was completely at home, may try to carry on from where he left off. But unless he learns to be more wary from mistakes and minor ineptitudes, he is merely inviting blunders and disaster. Good judgment about concrete insights presupposes the prior acquisition of an organized set of complementary insights.

In the third place, then, there is the process of learning. It is the gradual acquisition and accumulation of insights bearing on a single domain. During that process one's own judgment is in abeyance. It is being developed and formed but it has not yet reached the maturity needed for its independent exercise. For the gradual acquisition and accumulation of insights are not merely a matter of advancing in direct or introspective understanding. At the same time, intellectual curiosity is asserting itself against other desires. At the same time, the logical retinues of presuppositions and implications of each insight are being expanded either to conflict and provoke further questions or else to mesh into coherence. At the same time, operational possibilities are

envisaged to be tested in thought experiments, to be contrasted with actual practice, to be executed in ventures that gradually increase in moment and scope to enlighten us by failures and to generate confidence through success.

So it is the process of learning that breaks the vicious circle. Judgment on the correctness of insights supposes the prior acquisition of a large number of correct insights. But the prior insights are not correct because we judge them to be correct. They occur within a self-correcting process in which the shortcomings of each insight provoke further questions to yield complementary insights. Moreover, this self-correcting process tends to a limit. We become familiar with concrete situations; we know what to expect; when the unexpected occurs, we can spot just what happened and why and what can be done to favor or to prevent such a recurrence; or, if the unexpected is quite novel, we know enough to recommence the process of learning and we can recognize when, once more, that self-correcting process reaches its limit in familiarity with the concrete situation and in easy mastery of it.

In the fourth place, rashness and indecision commonly have a basis in temperament. Apart from occasional outbursts, that we view as out of character, the rash man nearly always is quite sure and the indecisive man regularly is unable to make up his mind. In such cases it is not enough to point out that learning is a self-correcting process that tends to a limit or that, while the limit is not marked with a label, still its attainment is revealed by a habitual ability to know just what is up. For unless a special effort is made to cope with temperament itself, the rash man

continues to presume too quickly that he has nothing more to learn, and the indecisive man continues to suspect that deeper depths of shadowy possibilities threaten to invalidate what he knows quite well.

Finally, we note that we leave to another occasion a discussion of the philosophic opinions that no one ever can be certain. Our immediate purpose is to explain the facts. Human judgments and refusals to judge oscillate about a central mean. If the precise locus of that divide can hardly be defined, at least there are many points on which even the rash would not venture to pronounce and many others on which even the indecisive would not doubt. What, then, is the general form of such certitude of ignorance and such certitude of knowledge?

Our answer is in terms of the virtually unconditioned. There occurs a reflective insight in which at once one grasps 1) a conditioned, the prospective judgment that a given direct or introspective insight is correct, 2) a link between the conditioned and its conditions, and this on introspective analysis proves to be that an insight is correct if it is invulnerable and it is invulnerable if there are no further, pertinent questions, and 3) the fulfilment of the conditions, namely, that the given insight does put an end to further, pertinent questioning and that this occurs in a mind that is alert, familiar with the concrete situation, and intellectually master of it.

4. CONCRETE ANALOGIES AND GENERALIZATIONS

Two brief corollaries have to be drawn.

An argument from analogy assumes that some concrete situation, A, is correctly understood. It argues that some other similar situation, B, is to be understood in the same fashion.

A generalization makes the same assumption to argue that any other similar situation, X, is to be understood in the same fashion.

In both cases what is at work is the law, immanent and operative in cognitional process, that similars are similarly understood. Unless there is a significant difference in the data, there cannot be a difference in understanding the data. This point has already been made in discussing the heuristic procedure of the classical phase of empirical method. Clearly enough, it holds not merely for regularities, rules, laws, correlations but also for ideal frequencies and for things. A second look does not necessarily mean one is looking at a second thing. A second actual frequency does not necessarily mean that one will establish a second ideal frequency. For there to be a second thing or a second ideal frequency an appropriate difference in the data has to be supposed.

In the simplest possible manner then, our analysis resolves the so-called problem of induction. It makes the transition from one particular case to another or from a particular case to the general case an almost automatic procedure of intelligence. We appeal to analogies and we generalize because we cannot help

understanding similars similarly. This solution, be it noted, squares with the broad fact that there is no problem of teaching men to generalize. There is a problem of teaching them to frame their generalizations accurately: indeed, the whole point of the analogy is that it absolves one from that conceptual task and the complexities it involves. There is, above all, a problem of preventing men from generalizing on insufficient grounds, and very easily such grounds are merely putative.

For if our view makes generalization an easy matter, it also clips the generalizer's wings. There must be a correct insight with respect to the basic situation. Before similars can be similarly understood, there is needed an act of understanding; and if that act is mistaken in the first instance, it will be equally mistaken in the second. But, as we have seen, to know one's insights are correct presupposes a process of learning and the attainment of familiarity and mastery. Further, the analogous or the general situation must be similar. If there is any significant dissimilarity, then further, pertinent questions arise to complement, to modify, perhaps to revise the basic insight. Finally, and this is the real catch, what differences are significant? My familiarity and mastery of the initial situation enables me to tell whether further questions there are pertinent. Another's familiarity and mastery of the analogous situation would enable him to tell whether further questions are pertinent in that situation. But unless the two situations are similar in all respects, my familiarity with one does not enable me to tell whether or not further questions arise when my insight is transferred to the other.

To conclude, analogy and generalization are essentially valid procedures. But when their basis is an insight into a concrete situation, the conditions of their proper use can become so stringent as to render them almost useless. It is this fact that grounds the suspicion with which men greet arguments from analogy and generalizations. But, at the same time, there is a compensating factor that arises from human collaboration in the process of learning. To this we have now to turn our attention.

5. COMMON SENSE

Common sense is that vague name given to the unknown source of a large and floating population of elementary judgments which everyone makes, everyone relies on, and almost everyone regards as obvious and indisputable. Though some repetition will be involved, three points, I think, call for our attention: 1) the source of these judgments, 2) their proper object or field, and 3) their relation to empirical science.

5.1 The Source of Common-Sense Judgments.

The proximate ground and source of common-sense judgments lie in the procedures just described, of concrete judgments of fact, judgments on the correctness of insights into concrete situations, and concrete analogies and generalizations. The remote source is more complex. One has to envisage these procedures carried out, not by isolated individuals, but by members of families, of tribes, of nations, over the face of the earth for generation after generation. One has to take into account the diffusion of judgments by communication and their transmission by tradition. Finally,

one has to note that there results not merely an enlargement but also a unification and transformation of the self-correcting process of learning.

Besides, if I may repeat myself, besides the hard way of finding things out for oneself, there is the comparatively easy way of learning from others. Archimedes had to rack his brains to discover what every schoolboy can be taught. For teaching is a vast acceleration of the process of learning. It throws out the clues, the pointed hints, that lead to insights; it cajoles attention to remove the distracting images that ~~prevent~~ obstruct them; it puts the further questions that reveal the need of further insights to complement and modify and transform the acquired store; it grasps the seriation of acts of understanding to begin from the simple and work towards the more complex. But what is done explicitly and deliberately by professional teachers, also is done implicitly and unconsciously by parents with their children and by equals among themselves. Talking is a basic human art; by it each reveals what he knows and provokes from others the further questions that direct his attention to what he had overlooked. More general and more impressive than talking is doing: deeds excite our admiration and stir us to emulation; we watch to see how things are done; we experiment to see if we can do them ourselves; we watch again to discover the oversights that led to our failures. Thus it is that what anyone discovers passes into the possession of many, to be checked against their experience and to be confronted with the test of their further questions. Thus too, it is that the discoveries of different individuals enter into single, cumulative series; that the later

presupposes and improves upon the earlier; and that the starting-point of each generation is where its predecessor left off.

The remote source then of common-sense judgments is a collaboration. The self-correcting process of learning goes on in the minds of individuals, but the individual minds are in communication. The results reached by one are checked by many, and new results are allied to old to form a common fund from which each draws his variable share measured by his interests and his energy.

There is another side to the story. It is human to err, and common-sense judgments are very human. They rest upon the self-correcting process of learning as transformed by communication and collaboration. But men share not only in intellectual curiosity but also in more earthy passions and prejudices. The mixed character of human drives can generate a common deviation from the pure product of intelligence and even a common dishonesty in refusing to acknowledge the effective pertinence of further, pertinent questions. So it is that we find each tribe and nation, each group and class, prone to develop its own brand of common sense and to strengthen its convictions by pouring ridicule upon the common nonsense of others. From the contradictory varieties of common sense, men have appealed to the common consent of the human race. But one may well doubt that such a procedure goes quite to the root of the matter. If one must suspect the collaboration of groups and classes, of tribes and nations, it does not follow that one cannot suspect the collaboration of mankind. Error is not primarily a class product or a national product. It is human. The group or class, the tribe or nation, only gives a more

specific twist to the mixed motives of human effort. Undertake to select the judgments on which all men agree, and you have no guarantee either that when all men agree, they will do so from the pure and detached motives of intelligence and reason or, indeed, that you yourself in your investigation and selection have operated exclusively from that unmixed drive.

The collaboration, named common sense, not only offers enormous benefits and advantages but it also intertwines them with more than a danger of deviation and aberration. Nor do we ourselves, stand outside this collaboration as spectators. We were born into it. We had no choice but to become participants, to profit by its benefits, and to share in its errors. We have no choice about withdrawing from it, for the past development of one's own intellect can no more easily be blotted out than the past growth of one's body, and future development will have to take place under essentially the same conditions and limitations as that of the past. There is, then, a fundamental problem, and how it is to be met, we cannot discuss at once. Our immediate objective has to be confined to discerning the field or domain within which common sense might be expected to operate successfully. This brings us to our second topic.

5.2 The Objects of Common-sense Judgments.

Already a distinction has been drawn between description and explanation. Description deals with things as related to us. Explanation deals with the same things as related among themselves. The two are not totally independent, for they deal with the same things and, as we have seen, description supplies, as it were, the tweezers by which we hold things while explanations are being discovered or verified, applied or revised. But despite their

intimate connection, it remains that description and explanation envisage things in fundamentally different manners. The relations of things among themselves are, in general, a different field from the relations of things to us. There is an apparent overlapping only when we consider the relations of men among themselves; and then the different procedures of description and explanation prevent the overlapping from being more than apparent, for description is in terms of the given while explanation is in terms of the ultimates reached by analysis.

Not only are description and explanation distinct, but there are two main varieties of description. There are the ordinary descriptions that can be cast in ordinary language. There are also scientific descriptions for which ordinary language quickly proves inadequate and so is forced to yield its place to a special, technical terminology. Nor is it difficult to discern behind these linguistic differences a more fundamental difference. Both ordinary and scientific description are concerned with things as related to us, but both are not concerned with the same relations to us. The scientist selects the relations of things to us that lead more directly to knowledge of the relations between things themselves. Ordinary description is free from this ulterior preoccupation. As it begins, so also it ends with human apprehensions and interests as its center.

There exists then a determinate field or domain of ordinary description. Its defining or formal viewpoint is the thing as related to us, as it enters into the concerns of man. Its object is what is to be known by concrete judgments of fact, by judgments on the correctness of insights into concrete situations, by concrete

analogies and generalizations, and by the collaboration of common sense. It is as much an object of knowledge as any other, for it is reached by beginning from the level of presentations, by advancing through inquiry, insights, and formulation, by culminating in the critical inquiry of reflective understanding, the grasp of the unconditioned, and the rationally compelled pronouncement of judgment. To anticipate a later vocabulary, the domain of ordinary description is a section of the universe of being, of what intelligently is grasped and reasonably is affirmed. How much of that section really is reached by ordinary description, is of course, a further question. At least, it is something to know the goal at which it aims, and that has been our restricted topic.

But before going on to our third topic, it may be well to preclude possible misconceptions. First, then, the human collaboration that results in a common sense involves belief. The analysis of belief cannot as yet be undertaken. But the type of belief that is essential in this collaboration resembles that of the pupil, who believes his teacher only that later he himself may understand and be able to judge for himself. It resembles that of the scientist, who does not insist on exploring for himself all the blind allies down which his predecessors wandered but is content to test their final results either directly, by repeating experiments, or, more commonly, by operating on the principle that, if those results were erroneous, the error would be revealed indirectly in the experiments he himself does perform. Hence it is that a man pronouncing a common-sense judgment is convinced that he is uttering,

not what someone else told him, but what he himself knows.

Secondly, the human collaboration that results in a common sense is under the dominance of practical considerations and pragmatic sanctions. The further questions that arise and are considered pertinent, do not come from any theoretical realm, and the tests that are employed move within the orbit of human success and failure. Still that dominance, so far from vitiating the results, is dictated by the object to be known, by the thing as it is related to us and as it enters into the concerns of men. It was a philosophic school that invented the notion that ideas are true because they happen to work. Despite its practicality, common sense is convinced that ideas work only if they are true. Nor is this surprising, for the practical further question is a further question that leads to the modification or revision of an insight; and the pragmatic criterion of success is the absence of the failure that would reveal the necessity of thinking things out afresh.

Thirdly, the human collaboration that results in a common sense is subject to the deviations and aberrations that have their root in the mixed motives of man. But it is only in so far as I myself share in those mixed motives that my understanding and my judgment will suffer the same bias and fall in line with the same deviations and aberrations. As long as I share in them, my efforts at correction and selection will be just as suspect as the judgments I wish to eliminate. It is only when I go to the root of the matter and become efficaciously critical of myself that I can begin to become a reliable judge; and then that becoming will consist in the self-correcting process of learning <u>that</u> which has

already been described.

5.3 Common-sense Judgment and Empirical Science.

Our third main topic was the relation of common sense to science, and our fundamental assertion is that the two regard distinct and separate fields. Common sense is concerned with things as related to us. Science is concerned with things as related among themselves. In principle, they cannot conflict, for if they speak about the same things, they do so from radically different viewpoints.

When I say that in principle they cannot conflict, I mean of course, that in fact they can and do. To eliminate actual conflict, it is necessary to grasp the principle and to apply it accurately.

The basic difficulty has been to grasp the principle. The scientists of the Renaissance were quite aware that there was some difference in principle, but they expressed it by a distinction between primary and secondary qualities. Science is concerned with things and their primary qualities, that is, with things as they really are. Common sense is concerned with things, with their primary qualities, and most of all with their secondary qualities, that is, mainly with things as they merely appear. On this showing, knowledge is science, and where common sense diverges from science, partly it is the darkness of ignorance and error, partly it is the twilight soon to be replaced by a scientific dawn. Naturally enough such exclusive pretensions were met by opposite pretensions equally exclusive, and the debate raged on a mistaken issue. Today, I think, we can be not only cooler but also wiser about the whole matter. As has been argued in ~~the sections on Image and~~ earlier chapters,

~~Truth and or Description and Explanation~~, it is necessary to distinguish within knowledge between separate yet complementary domains. There is a comprehensive, universal, invariant, non-imaginable domain; its object is the thing-itself, with differences in kind defined by ~~implicit~~ explanatory conjugates, and with differences in state defined by ideal frequencies. There is also an experiential, particular, relative, imaginable domain; its object is the thing-for-us, with differences in kind defined by ~~implicit~~ experiential conjugates, and with differences in state defined by expectations of the normal. The former field of empirical science is to be reached only by abstracting from the empirical residue. ~~of the individual, the particular, the systematically aberrant, and the non-cumulative multiplicity of the continuum.~~ The latter field includes the empirical residue; it views things in their individuality, their accidental determinations, their arbitrariness, their continuity.

The significance of this distinction appears in logic as the separation of two universes of discourse. To put the matter concretely, let us take illustrative propositions and consider the three cases of 1) ignoring the distinction of the domains, 2) denying the distinction of the domains, and 3) accepting the distinction of the domains. First, if one ignores the distinction of the domains, then one has the problem of choosing between the propositions:

The planets move in approximately elliptical orbits with the sun at their focus.

The earth is at rest, and the sun rises and sets. ~~Secondly,~~

Secondly, if one denies the distinction of the domains, one is committed to the more rigorous choice between the propositions;

From every viewpoint, the planets move in elliptical orbits with the sun at their focus.

From every viewpoint, the earth is at rest and the sun rises and sets.

Thirdly, if one affirms the distinction of the domains, then one will reject all four of the preceding propositions to assert both of the following:

From the viewpoint of explanation, the planets move in approximately elliptical orbits with the sun at their focus.

From the viewpoint of ordinary description, the earth is at rest and the sun rises and sets.

On this third position there result two separate universes of discourse. All the affirmations of empirical science contain the qualifying reservation, "from the viewpoint of explanation". Similarly, all the affirmations of common sense contain the qualifying reservation,"from the viewpoint of ordinary description". Automatically, all logical conflict is eliminated, for the qualifying reservations prevent the propositions of one universe from contradicting the propositions of the other.

Underlying this logical separation, there will be more fundamental methodological differences. Both ordinary description and empirical science reach their conclusions through the self-correcting process of learning. Still they reach very different conclusions because though they use essentially the same process, they operate with different standards and criteria. What is a further, pertinent question for empirical science is not necessarily

a further, pertinent question for ordinary description. Inversely, what is a further, pertinent question for ordinary description is not necessarily a further, pertinent question for empirical science. It is this fundamental difference in the criterion of the relevance of further questions that marks the great divide between a scientific attitude and a common-sense attitude. Because he aims at ultimate explanation, the scientist has to keep asking "Why?", until ultimate explanation is reached. Because the layman aims at knowing things as related to us, as entering into the domain of human concerns, his questioning ceases as soon as further inquiry would lead to no immediate, appreciable difference in the daily life of man. Hence it is that the layman is attempting to impose his criteria on the scientist when he asks him what he is doing and follows that up with the further question: "What is the good of it?" For if the practical question can be put to engineers and technologists and medical doctors, its only effect upon pure science would be to eliminate all further progress. Inversely, the pure scientist is attempting to impose his criteria upon common sense, when he interprets a practical attitude as a lack of interest in truth: it is, indeed, a lack of interest in the truth that the scientist seeks, but that is not the sole domain in which truth is to be learned. Reflective understanding can reach the virtually unconditioned to pronounce correct judgments of concrete fact and to discern correct insights into concrete situations. Without those basic judgments, science has no starting-point and, ~~as such~~ equally, the

glorious achievements of applied science cannot be truly affirmed.

The difference of the domains appears not only in different criteria of the pertinence of further questions but also in the difference of the terms employed and in the possibilities they respectively offer for logical deduction. Because ordinary description is concerned with things-for-us, it derives its terms from everyday experience: because the elements of daily experience are constant, the terms of ordinary description are constant: visible shapes and the spectrum of colors, the volume, pitch, and tone of sounds, the hot and cold, wet and dry, hard and soft, slow and swift, now and then, here and there, do not shift in meaning with the successive revisions of scientific theories; the concrete unities that are men and animals and plants, the regularities of nature and the expectations of a normal course of events form a necessary and unchanged basis and context into which applied science introduces its improvements. # Inversely, because science seeks knowledge of the things as related among themselves, because such relations lie outside our immediate experience, because the ultimates in such relations are to be reached only when ultimate explanation is reached, each great forward step of scientific knowledge involves a more or less profound revision of its fundamental terms. Again, because science is analytic and abstractive, its terms are exact; because its correlations purport to be generally valid, they must be determined with utmost precision; because its terms are exact and its correlations general, it must be ready to bear the weight of a vast superstructure of logical deductions in which each conclusion must be equally exact and valid

new paragraph

generally. On the other hand, as we have seen, ordinary description must be perpetually on its guard against analogies and generalizations; for though similars are similarly understood, still concrete situations rarely are similar, and the synthesis of an aggregate of concrete situations is not itself a concrete situation. Because things fall away from the Pole Star in the northern hemisphere, it does not follow that they will do so in the southern. Because within the range of human vision the earth is approximately flat, it does not follow that the integration of all such views will be a flat surface. The procedure of sound common sense is not to generalize nor to argue from analogy, but to retain the insights gained in former experience and to add the complementary insights needed in fresh situations. The collaboration of common sense aims, not at establishing general truths, but at building up a core of habitual understanding that is to be adjusted by further learning in each new situation that arises.

Common sense, then, has its own specialized field or domain. It has its own criteria on the relevance of further questions. It has its own basically constant vocabulary, its proper universe of discourse, and its own methodological precepts of keeping to the concrete, of speaking in human terms, of avoiding analogies and generalizations and deductions, of acknowledging that it does not know the abstract, the universal, the ultimate. Precisely because it is so confined, common sense cannot explicitly formulate its own nature, its own domain, its own logic and methodology. These it has to learn, if it would limit properly its pronouncements, but it has to learn them in its own shrewd fashion through instances

and examples, fables and lessons, paradigms and proverbs, that will function in future judgments not as premises for deductions but as possibly relevant rules of procedure. Finally, because common sense has to be acquired, it is not possessed equally by all. It has its adept pupils that make mistakes, indeed, but also learn by them. Within their familiar field they are masters, and so well they know that their mastery ends when they step beyond its limits. Above all they know that they must master their own hearts, that the pull of desire, the push of fear, the deeper currents of passion are poor counsellors, for they rob a man of that full, untroubled, unhurried view demanded by sure and balanced judgment.

If the domains of science and common sense are distinct, so also they are complementary. If one must recognize the differences in their objects, their criteria, their universes of discourse, their methodological precepts, one must also insist that they are the functionally related parts within a single knowledge of a single world. The intelligibility that science grasps comprehensively is the intelligibility of the concrete with which common sense deals effectively. To regard them as rivals or competitors is a mistake for essentially they are partners and it is their successful cooperation that constitutes applied science and technology, that adds inventions to scientific discoveries, that supplements inventions with organizations, know-how, and specialized skills. But if common sense itself, once it is supplied with its appropriate evidence, has little difficulty in recognizing this fact, theorists of science can hardly be credited with an equal perspicacity. Misled by a confusion between heuristic and the

representative functions of imagination, they assumed that the business of science was to paint a picture of the really real. If, as we have argued, such a picture is essentially unverifiable and gratuitous, it cannot coincide with the verifiable pictures of common sense. If from this conflict the theorists of science proceeded to conclude that common sense must be some brutish survival, that it was in need of being instructed in lofty tones on the far superior virtues and techniques of the scientist, one cannot be surprised that common sense retaliated with its jokes on the ineptitude of the theorists and professors and with its quietly imperious dorsal thud, if they were to justify their existence, they had best continue to provide palpable evidence of their usefulness. But such opposition, I would contend, does justice neither to common sense nor to science; it has no better basis than a mistaken theory; and it had best be written off as an error incidental to an age of transition. During the past four centuries, empirical science has emerged and developed, to set us the twofold problem both of determining its nature and of working out the proper adjustment of the complementary functions of common sense. If such large problems cannot be solved in short order, one should not infer that they cannot be solved at all.

To conclude, common sense is one thing and common sense judgments are another. Common sense is common and specific. It is a specialized domain of knowledge with a proper universe of discourse, proper criteria on the pertinence of further questions, and proper methodological precepts. Operation within that domain is basically and fundamentally a communal collaboration in the self-

correcting process of learning. The fruit of that collaboration is a habitual core of accumulated insights into concrete situations and into the procedures needed to complement and adjust that core before one can pass judgment on further, concrete situations. Hence it is that common sense judgments are formed, not by some public authority named common sense, but only by individual judges in their own individual situations. Further, they can be known to be correct only by the individual judges in the individual situations, for no one else is in possession of the evidence as it is given and no one else is informed with the familiarity and mastery that result from the self-correcting process of l[illegible] within that situation. I can be certain that I am writing this, and you can be certain that you are reading it. But it is quite another matter for you to be certain that I am correct in affirming that I am writing, as it will be quite another matter for me to be certain that you are correct in affirming that you are reading. The common element in common sense is not some list of general truths about which all men can agree: it is not some list of particular truths about which all men can agree; but it is a collaboration in the erection of a basic structure by which, with appropriate adjustments, each individual is enabled to fill out his individual list of particular truths. Finally, each of those particular pronouncements occurs inasmuch as reflective understanding grasps the virtually unconditioned in the manner described in the sections on concrete judgments of fact and on judgments on the correctness of insights into concrete situations.

6. PROBABLE JUDGMENTS

When the virtually unconditioned is grasped by reflective understanding, we affirm or deny absolutely. When there is no preponderance of evidence in favor of either affirmation or denial, we can only acknowledge our ignorance. But between these extremes there is a series of intermediate positions, and probable judgments are their outcome.

This probability of judgment differs from the probability investigated in studying statistical method. As has been seen, the probable expectation answers a question for intelligence by assigning an ideal frequency from which actual events non-systematically diverge. But the probable judgment answers a question for reflection and, though it anticipates a divergence between the judgment and actual fact, still the ground of this anticipation lies, not in a non-systematic element in the facts, but in the incompleteness of our knowledge. Hence, judgments about things, about correlations, and about probability expectations, may be certain or may be only probable.

Probable judgments differ from guesses. In both cases knowledge is incomplete. In both cases reflective understanding fails to reach the virtually unconditioned. But the guess is a non-rational venture beyond the evidence that resembles the non-systematic aspect of events. On the other hand, the probable judgment results from rational procedures. Though it rests on incomplete knowledge, still there has to be some approximation towards completeness. Though it fails to reach the virtually

unconditioned, still it has to be closing in upon that extrinsic norm. Thus, one may say that guesses are probably true only in the statistical sense of diverging non-systematically from true judgments: but probable judgments are probably true in the non-statistical sense of converging upon true judgments, of approaching them as a limit.

It is the nature of this approximation, approach, convergence, that constitutes the problem of the probable judgment. What precisely can be meant by such metaphors? If anything is meant, then how can it be known? So one surely makes a probable judgment when he can make a certain judgment: yet how can the probable be known to approach the certain, when the certain is unknown?

Fortunately, such paradox is not as acute as it may seem. We seek the truth because we do not know it. But, though we do not know it, still we can recognize it when we reach it. In like manner we also are able to recognize when we are getting near it. As we have seen, the self-correcting process of learning consists in a sequence of questions, insights, further questions, and further insights that moves towards a limit in which no further, pertinent questions arise. When we are well beyond that limit, judgments are obviously certain. When we are well short of that limit, judgments are at best probable. When we are on the borderline, they rush are completely certain and the indecisive full of doubts. In brief, because the self-correcting process of learning is an approach to a limit of no further, pertinent questions, there are probable judgments that are probably true in the sense that they approximate to a truth that as yet is not known.

Directly, the foregoing analysis regards the probability of judgments on the correctness of insights into concrete situations. Indirectly, it can be extended to all other probable judgments. Thus, concrete judgments of fact involve some insight that links the level of presentation with the question for reflection, and so the probability of such concrete judgments may be reduced to the probability of the correctness of the insight they involve. Did something happen? Something did happen if the same set of things exhibits different data at different times. An insight is required to grasp the identity of the things, and such an identification may be certain or probable. But the data exhibited at different times either differ or do not differ. If no difference is detected, there is no ground whatever for asserting change. If any difference is detected, there are the grounds for asserting change. If you do not remember accurately the former data, then you just do not know whether or not there was a change. If you are inclined to think that the former data were different, then the issue shifts. What inclines you to think so? Any reason that can be offered will suppose some insight into the objective course of events or into the habits of your memory, and it is that insight that gives rise to probability. More complex cases call for a more complex analysis, but the general lines of the analysis will be the same.

This brings us to the probability of the empirical sciences. Two questions arise. Why are their conclusions no more than probable? In what sense are their conclusions an approximation to what is true and certain? Discussion of analytic propositions is

deferred to the next ~~sub~~section and so we have to consider the empirical sciences in their generalizations and in their particular judgments of fact.

Since similars cannot but be similarly understood, generalization itself offers no difficulty. If the particular case is understood correctly, then every similar case will be understood correctly. If the problem of induction arose because the rest of the particular cases were not inspected, then that problem would be insoluble because the rest of the particular cases never are inspected: were they, there would be no generalization. In fact, the problem of induction arises because the particular case may not be properly understood: and it is solved by seeking that correct understanding.

Still, seeking is one thing and finding another. Empirical science gets its start by hitting off significant correlations. The correlations implicitly define abstract correlatives. But precisely because they are abstract, the return to the concrete is greeted with further questions. The law of the lever is simplicity itself. But to have an independent measurement of weights, one needs the law of the spring. To test the law accurately, one needs the theorem on centers of gravity. To formulate the law, one needs the geometry of perpendiculars. ~~Automatically~~ Further, one has embarked upon a vectorial representation of forces, an assumption of Euclidean geometry, a theory of the application of forces at a point, a parallel investigation of the tension of wires, and a certain amount of dabbling with gravitation. Automatically, further questions arise. Not only do they arise from the concrete

now do they may

problems set by tension and gravitation, ~~which in turn give rise to~~ ~~[illegible]~~. That is far more significant is the presence of the highly abstract mass and procedures, ~~which give tolerance to [illegible]~~. Can every force be represented by a vector? Are all forces applied at a point? Did Euclid have the last word? The initial abstraction allows one to return to the concrete only after the exploration of successively widening circles of inquiry. Statics is mastered only to raise the problems of kinetics. Kinetics is mastered only to reveal that thermal and electro-magnetic phenomena may be the antecedents or the consequents of local movements. One begins to get the lot in line and to feel that the future of physics is a matter of determining accurately a few more decimal points when along come a Planck and an Einstein with their further questions.

The generalization of classical laws, then, is no more than probable because the application of single laws raises further questions that head towards the systematization of a whole field. In turn, such systematization is no more than probable until the limit of no further, pertinent questions is reached. But that limit is not reached, first, if there may be further, unknown facts that would raise further questions to force a revision or, secondly, if there may be further, known facts whose capacity to raise such further questions is not grasped.

Similar considerations render the generalization of statistical laws no more than probable. For statistical laws presuppose some classification of events. One is not going to advance

quantum theory by investigating baseball averages. Hence definitive statistical laws suppose definitive classifications. The future discovery of new kinds or of new subdivisions of subatomic elements will invite a revision of the statistical laws. Similarly, more accurate investigations may lead to the discernment within the statistical law of a systematic element that can be abstracted in classical form to leave a new statistical residue.

If empirical generalizations are no more than probable, what about the particular facts that ground them? Here a distinction seems necessary. In so far as such facts are expressed in the terms of ordinary description, they fall under the criteria of the concrete judgment of fact. In so far as they are relevant to be the establishment of a scientific theory, they come under the control of empirical method. What has to be observed is, not the percept with its spontaneous integration into the processes of sensitive living, but the sheer datum that is stripped of non-scientific memories, associations, and anticipations. Again, measurements must conform to the best available rules and utilize the best available instruments. Finally, the observables have to be the terms defined by the theoretical structure, and as this structure is subject to revision, so also are its definitions. Hence, one may say that empirical science is solidly grounded in fact in virtue of its concrete judgments and, at the same time, one may add that technical developments and theoretical advance can render such facts more or less obsolescent.

But if empirical science is no more than probable, still it truly is probable. If it does not attain definitive truth, still

it converges upon truth. This convergence, this increasing approximation, is what is meant by the familiar phrase, the advance of science. Questions yield insights that are expressed in hypotheses; the testing of hypotheses raises further questions that generate complementary insights and more satisfactory hypotheses. For a while the process advances in widening circles; then the coherence of system begins to close in; investigation turns from fresh ventures in new fields to the labor of consolidation, of working out implications fully, of settling issues that leave the general view unchanged. The self-correcting process of learning is palpably approaching a limit, so that [illegible] proposals for revision become again possible, the open minds of scientists are becoming closed. As Max Planck put it: " A new scientific truth does not triumph by convincing its opponents and making them see the light, but rather because its opponents eventually die, and a new generation grows up that is familiar with it." Scientific Autobiography and Other Papers, New York 1949

An ulterior question may be raised. Is scientific progress infinite? Does the self-correcting process of learning reach one limit only to discover, sooner or later, that there are further developments to be effected? If I am unable to answer this question directly, still certain observations seem relevant.

First, the advance of science through increasing accuracy would seem to head towards a limit. A measurement is not a point but an interval, not simply a number but a number plus or minus some quantity determined by a theory of errors. Hence increasing accuracy has to result from the invention of new techniques and

instruments and, while such inventions may go well beyond our present anticipations, still we have no reason to expect an infinite series of them. Once such possibilites become exhausted, the ~~principle of exclusion~~ canon of selection comes into play. Empirical method settles only the theoretical differences that imply sensible differences. If a second theory supplants a first by advancing from the second decimal place to the fourth, and a third supplants the second by advancing from the fourth decimal place to the sixth, it does not follow that there can be some nth theory established by advancing from 2n decimals to (2n 2), where n is as large a number as you please.

Secondly, as the advance of science has a lower limit in the field of presentations, so also it has an upper limit in the basic structure of the human mind. Theories can be revised if there is a reviser. But to talk about revising the revisers is to enter a field of empty speculation in which the name, revision, loses its determinate meaning. Moreover, theorists take advantage of this fact. Thus, the foundations of logic are placed in the inevitabilities of our processes of thought. Nor is logic ~~an~~ a unique example. As we have already indicated, the theory of relativity in its basic postulate rests upon a structural feature of our cognitional process. Now if the invariants governing mental process imply invariants in our theoretical constructions, there will follow an upper limit to the variation of theoretical constructions and a possibility of mapping out in advance the alternatives between which theoretical effort has to choose. To this topic we return in investigating what will be named the ~~terminal~~ elements or categories of the

range of proportionate being.

In conclusion, it may be noted that these considerations confirm the positive probability of the conclusions of empirical science. For those conclusions are probable inasmuch as the self-correcting process of learning is approaching a limit. Our argument was based upon the immanent tendency of the process itself to a limit, inasmuch as each great stage of scientific development heads for the closed coherence of system, and each successive system grips the facts with greater nuance and accuracy over wider expanses of data. Still this immanent tendency receives confirmation if there exist external limitations to the process itself. For they too, point to the possibility of some system, as yet unknown, that is increasingly determined inasmuch as it will have to meet the requirement of verification in a body of fact that is increasingly large and increasingly organized.

7. ANALYTIC PROPOSITIONS AND PRINCIPLES.

A proposition is what is proposed either for consideration or for affirmation. An analysis of propositions is reached by distinguishing what is meant from acts of meaning and from sources of meaning. Any cognitional activity is a source of meaning. Conceiving, judging, and uttering are three quite different acts of meaning. Finally, as sources lead to acts of meaning, so acts refer to terms of meaning, to what is meant.

Terms of meaning may be divided in two ways. There is the basic distinction between what is meant when one affirms or denies and, on the other hand, what is meant when one merely considers, supposes, defines. Again, in utterances there is the obvious distinction between the incomplete meaning of a word and the complete meaning of a sentence. So one is led to distinguish 1) partial terms of meaning, 2) rules of meaning, 3) formal terms of meaning, and 4) full terms of meaning.

The full term of meaning is what is affirmed or denied.

The formal term of meaning is what could be affirmed or denied but, in fact, is merely supposed or considered.

The partial term of meaning is what is meant by a word or by a phrase.

Rules of meaning govern the coalescence of words and phrases into the complete sense that may be supposed or considered, affirmed or denied.

There results at once a particular case of the virtually unconditioned. A formal term of meaning provides the conditioned.

The definitions of its partial terms provide the fulfilling conditions. And the rules of meaning provide the link between the conditions and the conditioned. Such propositions are termed analytic.

Thus, if A is defined by a relation, R, to B, and B_1 is defined by the ~~opposite~~ converse relation, R', to A, then by the rules of meaning it follows that there cannot be an A without the relation, R, to B, and that there cannot be a B without the relation, R', to A. Such conclusions ~~that rest~~ resting on definitions and rules of meaning are analytic propositions.

Fourthly, since the analytic proposition is an instance of the virtually unconditioned, reflective understanding will find in it its proper object and thereby ground a judgment. There then arises a further question: What precisely is the meaning or force or implication of such a judgment?

It would seem that its meaning is not assertoric but hypothetical. If there occur suppositions or judgments containing significant terms in the same sense as they are assigned in the analytic proposition, then such suppositions or judgments must be consistent with the analytic proposition; moreover, when that condition and other logical requirements are met, there follow valid inferences. On the other hand, the mere fact that a proposition is analytic offers no guarantee that its terms in their defined sense occur in any supposition or judgment apart from the affirmation of the analytic proposition.

It follows that analytic propositions remain in sterile isolation unless there accrues to them some form of validation. This will consist in the occurrence of the same terms in their defined

sense in some other supposition or judgment; and the precise nature of the validation will depend upon the nature of the added supposition or judgment.

There also follows the explanation of the fact that analytic propositions can be produced more or less at will and indefinitely. Partial terms of meaning are a vast multitude and further partial terms can be supplied by the art of definition. Rules of meaning provide a principle of selection of the partial terms that will coalesce into analytic propositions. And if this seems to require too much ingenuity, the task can be simplified by using symbols instead of words and by defining them by their relations in propositions. But significant increments of knowledge are not to be obtained by mere ingenuity and, in fact, the analytic proposition, by itself, is not a significant increment of knowledge; without the fulfilment of further conditions it remains in isolation and fails to enter fruitfully into the texture of knowing.

Hence, we are in substantial agreement with the contemporary view that mere analytic propositions are ~~more~~ tautologies. The use of the term, tautology, would seem to be incorrect, but the general meaning of the statement is sound. However, it may not be out of place to add that the present point was made centuries ago. Aquinas advanced that conclusions depend upon principles, and that principles depend upon their terms: but he was not ready to accept any terms whatever: he added that proper terms are selected by wisdom (I-II, 66, 5, 4m)* and by wisdom he meant an accumulation of insights that stands to the universe as common sense stands to the domain of the particular, incidental, relative, and imaginable.

*Sum. Theol. I-II, q. 66, a. 5, ad 4m.

Let us now turn from analytic propositions to analytic principles.

By an analytic principle is meant an analytic proposition of which the partial terms are existential; further, the partial terms of an analytic proposition are existential if they occur in their defined sense in judgments of fact, such as the concrete judgment of fact or the definitively established empirical generalization.

Further, since such analytic principles are hard to come by, we shall also speak of two mitigated cases.

The provisional analytic principle is an analytic proposition of which the terms are probably existential, that is, they occur in probable empirical generalizations.

The serial analytic principle is an analytic proposition of which the terms are serially existential: what is meant by the serially existential, will be clarified in our next section on mathematical judgments.

It may be remarked that the analytic principle also connotes in its terms not only an existential reference but also a basic, primitive character. I think this feature will be found to follow from the defined requirements for, as we shall proceed to argue, analytic principles lie pretty well outside the reach of common sense and empirical science.

They lie outside the reach of common sense because analytic principles are universal and common sense regards the particular. Common sense makes concrete judgments of fact and it passes judgment on the correctness of insights into concrete situations.

But in neither case does it employ terms in the sense assigned them by abstract definitions. As Socrates discovered, the average man does not define: he is suspicious of the search for definitions; and when that pursuit brings out the inference that he does not know what he is talking about, he is rather resentful.

The fact would seem to be that the structure of common sense meanings is much the same as the structure of common sense itself. There is a communal collaboration that yields a habitual core of understanding and, as well, a range of concepts and linguistic terms in ordinary use. But just as the common core of understanding has to be adjusted by complementary insights into the present, concrete situation before judgment occurs, so also common concepts and terms receive their ultimate complement of meaning from those complementary insights.

"This is a dog". "What do you mean by a 'dog' ?" The question supposes that the term "dog" has a precise meaning outside the series of statements in which it occurs. But in fact what comes first is the series of statements and that comes only later, and then only if one goes in for analysis, is the determination of the precise meaning of the single, partial term. What the average man means by a "dog" is 1) what he would with certainty pronounce to be a dog in any concrete situation with which he is familiar, 2) what he could learn to be to a "dog", and 3) what he would be willing to believe is a "dog". Hence, it is that a dictionary is constructed, not by the Socratic art of definition, but by the pedestrian, inductive process of listing sentences in which ~~the~~ each word occurs in good usage.

It may be objected that one cannot make a brick house without first making bricks. But one is only arguing from a false analogy if one claims that the mind develops in the same fashion as the wall of a house is built. Prior to concepts there are insights. A single insight is expressed only by uttering several concepts. They are uttered in conjunction, and reflection pronounces whether the insight and so the conjunction is correct. The isolation and definition of concepts is a subsequent procedure and common sense does not undertake it.

Because we have denied that common sense reaches analytic principles, it is not to be inferred that the average man has no principles. Analytic principles suppose analysis; analysis supposes accurate conceptualization. But prior to analysis, to concepts, to judgments, there are the native endowments of intelligence and reasonableness and the inherent structures of cognitional process. These are the real principles on which the rest depends. Moreover, all understanding has its universal aspect, for similars are similarly understood. But it is one thing to exploit this universal aspect in a professional manner; it is another to exploit the intelligibility, which is by itself universal, by adding further intelligibilities until one comes to grips with concrete situations. The latter line of development we have named common sense so that, by definition, common sense deals with the particular. Again, the latter line of development is conspicuous in the average man. But what else the average man knows and how he knows it, are further questions. As has been remarked already, one cannot treat all issues at the same time.

Next, analytic principles lie outside the reach of empirical science. It is true of course, that every insight yields several concepts linked together through the insight: it also is true that the empirical scientist formulates definitions, postulates, and inferences: but the trouble is that the empirical scientist knows his insights not as certainly correct but only as probable. Hence his defined terms, in the sense they are defined, are as much subject to revision as the probable judgments of fact that contain them and validate them.

Thus, consider the assertions: 1) water probably is H_2O; 2) what I mean by water is H_2O; 3) this water contains impurities; 4) there are two kinds of water, heavy and ordinary.

The first is an empirical conclusion. The second is a definition. The third is a concrete judgment of fact; its meaning is that this sample is water in the sense of the empirical conclusion but it is not solely water in the sense of the definition. The fourth introduces a new basis of definition that has its ground in fresh experimental work. Now both the initial definition and the later definitions yield analytic propositions, namely, that what does not satisfy certain specifications is not pure water, or it is not pure water of molecular weight eighteen, or it is not pure heavy water. Moreover, none of these are merely analytic propositions; they are not the sort of thing that can be produced at will and indefinitely. On the other hand, they are not strictly analytic principles, for though their terms possess validating judgments of fact, still those judgments are subject to revision, and, indeed, the discovery of heavy water has already forced such a revision.

Generally one may say that the advance of empirical science is an instance of the advance of the self-correcting process of learning. But in this instance the previous insights yield correlations, definitions, and inferences. It is in terms of such formulations that are framed the further questions that will complement and modify the previous insights by later insights. In like manner the later insights receive their formulation which is presupposed by the further questions that lead to a still fuller understanding. Now in this process the successive formulations have three distinct aspects. First, they are the expression of insights that grasp the intelligible form of data; thus, they are probable empirical conclusions. Secondly, they are the presupposition of the further questions that lead to further insights; from this viewpoint they are provisional analytic principles. Thirdly, they are revised in the light of the further insights and so cease to be probable empirical conclusions and provisional analytic principles to pass into the limbo of the analytic propositions whose terms have no existential reference.

The reader interested in further illustrations of this process will find numerous examples in Arthur Pap's "The A Priori in Physical Theory", New York, 1946.

8. MATHEMATICAL JUDGMENTS

In mathematical thought one may readily discern the difference between operations on the level of intelligence and operations on the level of reflection.

The level of intelligence is the level of discovery and invention, of catching on and learning, of grasping problems and coming to grasp their solutions, of seeing the point made in each of a series of mathematical statements and then seeing how the successive points hang together.

The level of reflection is the complementary process of checking. One understands and now one wishes to know whether what is understood is also correct. One has grasped the point and one asks whether it is right. One has seen how the successive steps hang together and one is out to make sure that what hangs together is really cogent.

Now the process of checking can be developed into an elaborate technique. What is checked becomes a whole department of mathematics. Definitions are worked out. Postulates are added. From the definitions and postulates it is shown that all the conclusions of the department can be reached by the rigorous procedure of deductive inference.

But what is the goal of checking? Clearly, it is to marshal the evidence in the shape in which reflective understanding can grasp the virtually unconditioned and so ground rational judgment. In so far as the checking reduces conclusions to premises, there is the virtually unconditioned of the form of deductive inference.

In so far as the definitions and postulates coalesce into a self-justifying meaning, there is the virtually unconditioned of analytic propositions. Both of these types of the virtually unconditioned have already been considered and so, for us, the problem of mathematical judgment consists in determining what else is required for such judgment.

First of all, something else is required. For if the premises of mathematical thought are analytic propositions, still not all analytic propositions are mathematical premises. Analytic propositions can be produced at will and indefinitely. But the premises of mathematical thought are to be reached only through the discoveries of genius and the labor of learning what genius has grasped. Further, it does happen that abstruse regions of mathematics are occasionally pulled out of their cold and airy regions to become the tools of empirical hypotheses and theories and to share with such formulations the probable existential reference that they possess. But prior to a probable existential reference there is a possible existential reference before a department of mathematics can be applied, it must possess an inherent possibility of being applied. What, then, is that inherent possibility? And what is its criterion?

Secondly, then, we have to undertake an examination of mathematics to determine what this further element is and what its criterion is. Let us say, then, that there is a mathematical series, that each term in the series is a department of mathematics, that each department consists 1) of rules governing and so defining operations and 2) of operations proceeding from some terms to

others and so relating and defining them. Further, we may presuppose each department of mathematics to be formalized, that is, to be stated in a set of definitions, postulates, and deductions. Finally, we shall presuppose that there are other formalizations, equally rigorous, equally elegant, but in fact not members of the mathematical series. Our problem thus becomes the question, In the light of our general analysis of knowledge, how is one to recognize some formalizations as mathematical and others as not mathematical?

Our answer contains three elements, and it will be convenient to refer to them respectively as the material element, the formal element, and the actual element.

The material element is what we have named the empirical residue. There are aspects of data from which understanding always abstracts. Such have been seen to be the individual, the continuum, particular places and times, and the non-systematic divergence of actual frequency from probable expectations.

The formal element may be designated by abstraction as enriching. It has been seen that insight goes beyond images and data by adding intelligible unities and correlations and frequencies, which, indeed, contain a reference to images or data but, none the less, add a component to knowledge that does not exist actually on the level of sense or imagination.

Finally, the actual element lies in the conjunction of the material and the formal elements. ~~From the viewpoint of the mathematician, this~~ By the mathematician the formal element commonly is viewed as dynamic. There is a

laborious process named "learning mathematics". It consists in gradually acquiring the insights that are necessary to understand mathematical problems, to follow mathematical arguments, to work out mathematical solutions. This acquisition occurs in a succession of higher viewpoints. One department of mathematics follows upon another. Logically, they are discontinuous, for each has its own definitions, postulates, and inferences. But intellectually they are continuous, inasmuch as the symbolic representation of operations in the lower field provides the images in which intelligence grasps the idea of the new rules that govern operations in the higher field.

However, this expansion of intelligence does not seem to be completely free. Not only is there the link between higher viewpoints and preceding lower viewpoints, but also there is a bias from the particular to the general, from the part to the totality, from the approximate to the ideal. If there exist concrete instances of one, two, three, the mathematician explores the totality of positive integers, of real numbers, of complex numbers, of ordered sets. If there exist edges and surfaces, the mathematician works out not merely one geometry but the total series of possible geometries. If there are various fields in which it seems mathematics may be applied, the mathematician sets out to explore the whole of each regions in which the fields occur.

Again, besides its preference for the general, the complete, the ideal, the development of mathematical thought also seems restricted by its material element. By

this I do not mean that the mathematician is confined to individuals that exist, to continua that exist, to places and times that exist, to non-systematic divergences that occur, or to any other actual elements in the empirical residue that may be discovered through the introduction of new techniques of abstraction. For it is quite clear that mathematical thought in its pursuit of the general and complete and ideal reveals a profound unconcern for the existent. Still it does seem to be true that the empirical residue does supply mathematics with samples of the type of stuff on which mathematical ideas confer intelligibility and order. For unless the mathematician is investigating the pure intelligibilities that Aquinas identified with angels, there must be some mathematical matter; and since there are other sciences that deal with data as of determinate kinds, there remains for the mathematician the empirical residue of all data.

If we have succeeded in characterizing the material and formal elements of mathematics, there remains the question of the significance of their conjunction. Briefly, this may be indicated by recalling that we found the heuristic structures of empirical method to operate in a scissors-like fashion. Not only is there a lower blade that rises from data through measurements and curve-fitting to formulae, but also there is an upper blade that moves downward from differential and operator equations and from postulates of invariance and equivalence. Moreover, it is no secret that the upper blade owes its effectiveness to the labors of mathematicians. But what is the possibility

of that upper blade?

To grasp the answer to that question, two complementary tendencies have to be envisaged at once. On the one hand, there is the movement of empirical science from description to explanation, from proper domains of data to systems of laws that implicitly define the terms they relate; and at the end of this movement there is the ideal goal that is to be attained when all aspects of data, except the empirical residue, will have their intelligible counterpart in systems of explanatory conjugates and ideal frequencies. On the other hand, there is the movement of mathematical thought that begins from the empirical residue and endeavors to explore the totality of manners in which enriching abstraction can confer intelligibility upon any materials that resemble the empirical residue. Clearly, these two movements are complementary. For the mathematician begins from the empirical residue with which the empirical scientist would end; and if the mathematical exploration of intelligible systems is thorough, then it is bound to include the systems of explanatory conjugates that the empirical sciences will verify in their respective domains.

Let us now revert to our distinction between outright analytic principles, provisional analytic principles, and serially analytic principles. All are analytic propositions, i.e., instances of the virtually unconditioned in which the conditioned is linked to its conditions by syntactical rules and the conditions are fulfilled by defining terms. None are mere analytic propositions that are obtained by devising any definitions or

syntactical rules that one pleases. For the terms and relations of outright analytic principles occur, in their defined sense, in certain judgments of fact. The terms and relations of provisional analytic principles occur, in their defined sense, in probable judgments of fact. Finally, the terms and relations of serially analytic principles ground the deductive expansions that explore completely, generally, and ideally, the total range of fields to which outright and provisional analytic principles give access in a particular, fragmentary, or approximate manner.

Next, it seems possible to identify the basic propositions of mathematics with serially analytic principles. For there is a material element in mathematical thought, and it bears some similarity to the empirical residue in the data of the empirical sciences. Again, there is a formal element in mathematical thought, and it tends towards a general, complete, and ideal account of the manners in which enriching abstraction can add intelligibility and order to the material element. But the empirical sciences are in search of the intelligibility and order that, when combined with the empirical residue in the data of their several domains, will provide a complete and definitive explanation of those data. It follows that the mathematician is concerned to establish generally, completely, and ideally, the range of possible systems that include verifiable scientific systems as particular, fragmentary, or approximate cases.

Thirdly, if the basic propositions of mathematics are serially analytic principles, then we have the answer

to our principal question that asked the difference between free formalizations and mathematical formalizations.

Fourthly, there readily follows an account of the possibility of isomorphism between mathematical relations and the relations of the empirical sciences. Both sets of relations are products of enriching abstraction, and both possess a relevance to the empirical residue in data .

Finally, it seems appropriate to add a note on the difference between the foregoing account of the field of mathematics and current views. Commonly, it would be agreed that mathematics is based on mere analytic propositions, and it would be explained that, ~~apart from~~ if one disregards merely arbitrary definitions and syntactical rules, one can distinguish logic which deals with such relations as "and", "or", "if..., then...", mathematics which deals with relations of equivalence or congruence in individuals and sets, and a more general subject, call it mathesis, which deals with rules common to logic and to mathematics.

The principal difference in our approach is that it goes behind concepts and affirmations to the grounding acts of direct and reflective understanding. From this feature there follows its dynamic character, for it contains an invitation to mathematicians to explore the possibility of setting up the series of deductive expansions that would do as much for other empirical sciences as has been done for physics. On the other hand, while we have emphasized a relation between mathematics

and empirical science, it must be insisted that we have not done so by restricting materially the field of mathematics. The mathematician remains free to take as his materials anything that resembles the empirical residue. He is free to discover further additions to the residue that, at present, is known. He is free to explore with full generality, completeness, and ideality, the enrichments that the exercise of human intelligence can add. Yet his creations will remain serially existential, for they will exhibit the series of systems to some of which the empirical scientist will be able to say "Yes".

9. SUMMARY

Prospective judgments are propositions 1) that are the content of an act of conceiving, thinking, defining, considering, or supposing, 2) that are subjected to the question for reflection, to the critical attitude of intelligence, and 3) that thereby are constituted as the conditioned.

There is sufficient evidence for a prospective judgment when it may be grasped by reflective understanding as virtually unconditioned. Hence sufficient evidence involves 1) a link of the conditioned to its conditions, and 2) the fulfilment of the conditions. These two elements are supplied in different manners in different cases.

In formal inference the link is provided by the hypothetical premiss: If the antecedent, then the consequent. The fulfilment is the minor premiss.

In judgment on the correctness of insights, the link is that the insight is correct if there are no further, pertinent questions, and the fulfilment lies in the self-correcting process of learning reaching its limit in familiarity and mastery.

In judgments of fact the link is the correct insight or set of insights and the fulfilment lies in present and/or remembered data.

In generalizations the link is the conditional law that similars are similarly understood and the fulfilment lies in such similarity that further, pertinent questions no more arise in the general case than in the correctly understood particular case.

In probable judgments the link is that insights are correct when there are no further pertinent questions and the fulfilment is some approximation of the self-correcting process of learning to its limit of familiarity and mastery.

In analytic propositions the link lies in rules of meaning that generate propositions out of partial terms of meaning and the fulfilment is supplied by the meanings or definitions of the terms.

Analytic propositions become analytic principles when their terms are existential; and terms are existential when they occur in definitive, factual judgments.

Provisional analytic principles are analytic propositions whose terms are probably existential.

~~Serial analytic principles are analytic propositions whose terms pertain to series, some of which, in some fashion, formally or virtually, approximately or approximately, normally or abnormally, somewhere or somewhen exist.~~

Serially analytic principles are the ~~propositions~~ analytic propositions from which follow the ranges of systems some of which in some fashion exist.