INSIGHT

CHAPTER IV

THE COMPLEMENTARITY OF CLASSICAL AND STATISTICAL INVESTIGATIONS

A review of the main points that have been made will prove, perhaps, the most expeditious introduction to the problem of the present chapter.

Our study of human intelligence began from an account of the psychological aspects of insight. It turned to geometrical definitions as products of insight and thence to the re-definitions that result from higher view-points. The argument then twisted to the queer type of insight that grasps that the understanding of given data or of the answer to a given question consists in understanding that there is nothing to be understood. Finally, ~~from an examination of definitions and of limits,~~ there was effected a generalization that acknowledged in all data an empirical residue from which intelligence always abstracts.

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The second chapter switched to insights in the field of empirical science. After a brief contrast between mathematical and scientific developments of understanding, attention centered on the origin of the clues that form the first moment of insight. It was seen that, by inquiring, intelligence anticipates the act of understanding for which it strives. The content of that anticipated act can be designated heuristically. The properties of the anticipated and designated content constitute the clues intelligence employs to guide itself towards discovery. Finally, since there are not only direct insights that understand what is to be understood but also the queer type of insights that understand that there is nothing to be understood, heuristic structures fall into two groups, namely, the classical and the statistical. A classical heuristic structure is intelligent anticipation of the systematic-and-abstract. A statistical heuristic structure is intelligent anticipation of the systematic-and-abstract setting a boundary or norm from which the concrete cannot systematically diverge.

Of themselves, heuristic structures are empty. They anticipate a form that is to be filled. Now just as the form can be anticipated in its general properties, so also can the process of filling be anticipated in its general properties. There exist, then, canons of empirical method. If insight is to be into data, there is a canon of selection. If insights into data accumulate in a circuit of presentations, insights, formulations, experiments, new presentations, there is a canon of operations. If applied science involves insights into materials, purposes, agents, and tools, then pure science, as prior to applied, will be concerned solely with the immanent intelligibility of data and so will be subject to a canon of relevance. If pure science goes beyond the data inasmuch as it grasps their immanent intelligibility, still it adds to the data no more than that intelligible content; there results a canon of parsimony, which excludes any affirmation that goes beyond what can be verified in the data. If some data are to be understood, then all are to be understood; the scientific goal is the understanding of all phenomena, and so scientific method is subject to a canon of complete explanation; it follows that no exception is to be made for experienced extensions or for experienced durations; and this conclusion implies a shift from a Galilean to an Einsteinian viewpoint. Finally, though all data are to be explained, it remains that certain aspects of all data are explained in the queer fashion already noticed. There exist statistical residues, for the totality of the systematic is abstract, the abstract is applied to the concrete only by the addition of further determinations and, from the nature of the case, the further determinations cannot be systematically related to one another.

Now this bare enumeration of the points that have been made in our first three chapters confronts us with a problem. Both the heuristic structures of science and the canons of empirical method involve a duality. Besides grasping the intelligibility immanent in data in a positive fashion, human intelligence also grasps a domination of the concrete by the abstract-and-systematic. However, though one admits this duality as a fact, one still may ask whether it is ultimate, whether classical and statistical inquiries are isolated or related procedures, whether they lead to isolated or related results. An answer to these questions is sought in the present chapter, and it falls into three parts.

First, it will be advanced that classical and statistical investigations are complementary as types of knowing. In their heuristic anticipations, in their procedures, in their formulations, in their differences of abstractness, in their verification, and in their domains of data, each will be shown to complement and to be complemented by the other.

Secondly, besides the complementarity in knowing, there is a complementarity in the to-be-known. Whether one likes it or not, heuristic structures and ~~empirical~~ canons of method constitute an <u>a priori</u>. They settle in advance the general determinations, not merely of the activities of knowing, but also of the content to be known. Just as Aristotle's notions on science and method resulted in his cosmic hierarchy, just as the Galilean reduction of secondary to primary qualities necessitated a mechanist determinism, so too our simultaneous affirmation of both classical and statistical investigations involves a world view. What is that view?

Thirdly, there is a clarification that results from contrast. Accordingly, after endeavoring to determine the world view, to which one commits oneself by accepting the heuristic structures and the canons of empirical method, there are set forth its differences from the world views of Aristotle, Galileo, Darwin, and contemporary indeterminists.

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1. Complementarity in the Knowing.

Complementary Heuristic Structures.

1.1/ First, the heuristic anticipations of classical and of statistical procedures are complementary. For the systematic and the non-systematic are the contradictory alternatives of a dichotomy. Inquiry of the classical type is an anticipation of the systematic. Inquiry of the statistical type is an anticipation of the non-systematic. Now the relations between data must be either systematic or non-systematic. It follows that in any given case either the classical or the statistical anticipation must be correct.

Two corollaries follow.

The first is the openness of empirical method. The mere fact of inquiry is itself a presupposition, for it implies that there is something to be known by understanding the data. Still this presupposition is inevitable, for it marks the difference between the scientific and the non-scientific attitudes to experience. Moreover, this presupposition is minimal. For it does not determine *a priori* whether any selected range of data is to be reduced to system in the classical fashion or, on the other hand, is to be accounted for by showing how the concrete diverges non-systematically from systematic expectations.

The second corollary is the relevance of empirical method. For empirical method is a matter of trial and error, and the only way to settle whether a given aggregate of observations are or are not reducible to system is to formulate both hypotheses, work out their implications, and test the implications against observed results.

Complementary Procedures.

1.2/ Next, classical and statistical investigations are complementary procedures. For they separate systematically and non-systematically related data, and the isolation of either type is a step towards the determination of the other.

With such separation everyone is familiar when it is effected physically by experimentation. As has been seen, the aim of the experimenter is to isolate a definable conjunction of elements and to exhibit their operations as they occur when uninfluenced by extraneous factors.

Again, physical separation is not always possible, and then one attempts to do by thought what one cannot achieve by deed. In this fashion, as soon as a science has made some progress, it invokes its known laws in seeking the determination of the unknown. Thus, once Boyle's law is known, one assumes it in determining Charles' law; once both are known, one assumes both in determining Gay-Lussac's law. Similarly, in all departments, known laws are employed to guide experiment, to eliminate the consideration of what already has been explained, and to provide premises for the interpretation of observed results.

Moreover, such separation, whether physical or mental, is not confined to classical laws. All laws belong to a single complementary field. For this reason it has been possible to invoke the laws of probable errors and thereby to eliminate a non-systematic component in observations and measurements. In like manner, Mendel's statistical laws of macroscopic, genetic characters led to the postulation of microscopic entities named genes; to each gene was assigned, on the classical model, a single, determinate effect and manifestation; genes with incompatible effects were classified as dominant and recessive; and so statistical combinations of classically conceived genes became the explanation of non-systematic, macroscopic phenomena.

The reader may be surprised that we lump together the laws of probable errors and the Mendelian laws of heredity. But from our viewpoint they belong together. In both cases a component in the data is brought under law. In both cases the discovery of the law grounds a mental separation of the component, subject to known law, from other components still to be determined. In both cases this mental separation opens the way to the determination of further laws. In both cases, finally, it is the discovery of a statistical law that grounds the mental separation and that can lead to the discovery no less of classical than of statistical laws.

This complementarity of classical and statistical procedures has an important corollary. For the experimental, physical exclusion of extraneous factors is not always possible. When it is not, there exists the alternative of discovering the law of the extraneous factor and then allowing for its influence in interpreting one's result. Now the corollary, to which we would draw attention, is that statistical laws can be employed in this fashion to the determination of classical laws. For knowledge of statistical laws enables one to separate mentally the non-systematic component in the data and so it leaves one free to investigate the remaining systematic component.

It will be asked, then, whether the statistical investigations of Quantum Mechanics, may be expected to prepare the way for a later resurgence of classical thought in the field of subatomic physics.

This question is, I think, ambiguous. One may mean a return to the former type of classical thought with its imaginable models, its belief in the universal possibility of imaginative synthesis, its affirmation of a mechanist determinism, and its concept of explanation as the reduction of secondary to primary qualities. On the other hand, it is possible to speak of "classical" thought in a transposed and analogous sense. In that case, one would grant to imagination a notable heuristic value, for images supply the materials for insights; but, at the same time, one would deny to unverified and unverifiable images any representative value: classical laws would be conceived as abstract, the abstraction would be conceived as enriching, and so full knowledge of classical laws would not preclude the existence of statistical residues.

Once this distinction is drawn, our answer to the foregoing question becomes obvious. In the light of the canons of complete explanation, of parsimony, and of statistical residues, we cannot expect any return to the older type of classical thought. Again, in the same light, we must expect Quantum Mechanics, if interpreted statistically, to open the way to a new development of "classical" thought in a transposed and analogous sense. Indeed, Pauli's exclusion principle provides a premise for the determination of the states of electrons in atoms; and while changes of these states seem to occur statistically, still the series of states is as regular and systematic as the periodic table of chemical elements.(See Lindsay and Margenau, pp. 488f).

In like manner, one might note classical tendencies in the discovery of new subatomic entities over and above the more familiar electrons, protons and neutrons.

1.3 Complementary Formulations. Thirdly, classical and statistical formulations are complementary. For classical formulations regard conjugates, which are verified only in events. And statistical formulations regard events, which are defined only by conjugates.

The dependence of classical upon statistical formulation comes to light, when one probes into the meaning of the classical proviso, other things being equal. What are the other things? In what does their equality consist? These questions cannot be given an answer that is both detailed and systematic. For the proviso, which limits classical laws, is effectively any relevant pattern of a diverging series of conditions. Such series vary with circumstances, and the aggregate of patterns of such series is both enormous and non-systematic. In other words, classical laws tell what would happen if conditions were fulfilled; statistical laws tell how often conditions are fulfilled; and so the phrase, other things being equal, amounts to a vague reference to the statistical residues, which are the province of the complementary statistical laws.

The inverse dependence of statistical upon classical formulations comes to light, when one asks which statistical investigations possess scientific significance. Thus, anyone would acknowledge a difference in such significance between determining the frequency of red hair in trombone players and, on the other hand, measuring the intensity of line spectra. In either case one arrives at a number that may be regarded as an actual frequency, but it is not apparent that in both cases one has an equal chance in contributing to the advance of science. For the advance of science is secured by operating in the light of present knowledge and towards the solution of well-formulated problems. As soon as any department of science has passed beyond its initial stages, it begins to desert the expressions of ordinary language and to invent technical terms of its own. Such technical terms have their origin in the correlations that have been found significant; they are or, in some fashion, they depend upon what we have named pure conjugates. Accordingly, inasmuch as the statistical investigator proceeds in the light of acquired knowledge and towards the solution of well-formulated problems, he will be led to define events by appealing, directly or indirectly, to the pure conjugates that are implicit in classical laws.

However, the reader may ask whether this view can be regarded as definitive. It is true enough that the scientific classifications and definitions of the present are dependent on the discovery and formulation of classical laws. But may one not expect that a fuller development of statistical inquiry will result in the implicit definition of technical terms by statistical and not classical laws?

While there seem to be those that would answer this question affirmatively, I cannot see my way to agreeing with them. My reason runs as follows. The answer, "Yes", to a question for reflection obtains a determinate meaning only by reverting from the "Yes" to the question and to its origin in the descriptive or explanatory answer to a question for intelligence. Now the event, the happening, the occurring, corresponds to the bare "Yes". To say what happens, what occurs, one must raise a question that cannot be answered by a "Yes" or a "No". One must appeal either to the experiential conjugates of description or to the pure conjugates of explanation. On this showing, then, one cannot expect events to generate their own definitions any more than one can expect "Yes" or "No" to settle what is affirmed or denied. Finally, if events cannot generate their own definitions, then, frequencies of events cannot do so; for there seems no reason to expect that different types of events must have different numerical frequencies or, indeed, that the numerical frequencies could serve to specify the kinds of events to which one wishes to refer.

There is, then, a complementarity of classical and statistical formulations. For if statistical formulations are to be significant contributions to the advance of science, they will appeal to the experiential and pure conjugates of classical classifications and definitions. Inversely, the conjugates of classical formulations are verifiable only in statistically occurring events and their immanence in statistical residues is revealed by the proviso, "other things being equal".

It may not be out of place to conclude this subsection by clarifying a slight puzzle. It is true enough that statistical laws also are immanent in statistical residues, and so hold under the general proviso, "other things being equal". If "P follows Q" has the probability, p/q, still there are conditions for the occurrence of the occasion, Q, and it is only when those conditions are fulfilled that the probability, p/q, is verifiable. The frequency of such fulfilment might be indicated by saying that "Q follows R" has the probability, q/r, so that one statistical law would be dependent on another. Still this inter-dependence of statistical laws, while true enough, is beside our present point. It in no way invalidates the significant contention that the dependence of classical upon statistical formulations is revealed by the proviso, "Other things being equal".

Complementary Modes of Abstraction.

1.4 Fourthly, there is a complementarity in modes of abstraction.

Classical heuristic procedure rests on the assumption that to some extent the relations between data are systematic, and it devotes its efforts to determine just what those systematic relations are.

Statistical heuristic procedure rests on an assumption of non-systematic relations and it aims at determining an ideal frequency from which actual frequencies may diverge but only non-systematically.

In both cases the result obtained is abstract. For the classical law represents the systematic and prescinds from the non-systematic. On the other hand, the statistical law represents, not the actual frequency of actual events, but the ideal frequency from which actual frequencies diverge.

Now while both types of law are abstract, still their modes of abstraction differ. The classical law is concerned simply with the systematic; it disregards the non-systematic. The statistical law, on the contrary, assumes the non-systematic as a premise. By itself, of course, such a premise could yield no conclusions such as the abstract, ideal, universal frequencies named probabilities. What concerns the statistical inquirer is, then, neither the purely systematic, nor the purely non-systematic, but the systematic as setting ideal limits from which the non-systematic cannot diverge systematically.

Clearly, these two modes of abstraction are complementary. In its first movement, inquiry aims at determining the systematic component in data; in its second movement, inquiry turns to the more concrete task of determining the manner in which the systematic component in data moderates the non-systematic. The complete view results only from the combination of the two movements, and so the two are complementary.

There is another aspect to this complementarity. The systematic relations, with which classical inquiry is concerned, mainly are the relations of things, not to our senses, but to one another. In so far as the relations of things to one another are considered in the abstract and so as independent of their relations to our senses, there arises a principle of equivalence for all senses since all equally are abstracted from. On the other hand, statistical laws deal, not simply with occasions and events, but with observable occasions and observable events. They are not, in principle, independent of the relations of things to our senses, and they cannot be subjected to a full principle of equivalence. There follows the already mentioned formal opposition between Quantum Mechanics, interpreted statistically, and General Relativity; the two theories may deal with the same things, but they deal with them from radically different view-points; they are complementary in so far as General Relativity is concerned with things as independent of their relations to our senses while Quantum Mechanics views things in a manner that includes those relations. (Cf. Chapter III, # 6.64).

1.5 Complementarity in Verification. Fifthly, classical and statistical laws are complementary in their verification. This may be stated roughly by saying that classical laws determine what would happen if conditions were fulfilled, while statistical laws determine how often one may expect conditions to be fulfilled. However, a fuller account of this complementarity may be given by showing how the determination of either classical or statistical laws leaves room for the determination of the other.

Thus, if one were to suppose exact and complete knowledge of all ~~cl~~ classical laws, one would not preclude the possibility of the verification of statistical laws. For a set of classical laws, say P, would be exact and complete, if there were no possibility of replacing them by some different set, say Q. Now, there would be no possibility of replacing P by Q, if there were no systematic divergence between the data and the set of laws, P; for the sets, P and Q, differ as laws and so differ systematically: and so the verification of the set, Q, in place of the set, P, supposes a systematic divergence between the set, P, and the data. Finally, though there is no systematic divergence between the set, P, and the data, there can be a non-systematic divergence that would provide the field for the investigation and verification of statistical laws.

Again, as has been seen (Cf. Chapter III, 6), exact and complete knowledge of classical laws not merely can leave room for possible statistical investigation but also must do so. For such exact and complete knowledge would embrace all the systematic relations between determinate data; none the less, such knowledge would be abstract and so in need of further determinations to be applied to concrete instances; it follows that the further determinations cannot be systematically related to one another; and so ~~that~~ there must be a field for statistical laws.

Finally, statistical investigations in their turn have no genuine tendency to totalitarian aspirations. For besides statistical predictions, there exist the fully accurate predictions that are exemplified by astronomy and that rest on the existence of schemes of recurrence. Moreover, the intelligent manner of making these predictions is to analyze the schemes into their component classical laws. Copernicus corrected Ptolemy's imaginative scheme; Kepler corrected the circles of Copernicus; but it was Newton that worked out the underlying laws and Laplace that revealed the periodicity of the planetary system. From that discovery of laws the great movement of thought, named modern science, received its most powerful confirmation. It did so because it ended, at least for two centuries, the more common human tendency to speak, not of precise laws, but of the common run of events or the ordinary course of Nature. At the present moment, the profound significance of statistical laws is coming to light. But if this new movement is not to degenerate into the old talk about what commonly happens, it must retain its contact with the empirically established precision of classical formulations. For statistical laws are of no greater scientific significance than the definitions of the events whose frequencies they determine; unless these definitions are determined scientifically, statistical thought lapses into pre-scientific insignificance.

1.6 Complementarity in Data Explained. Sixthly, classical and statistical laws are complementary in their domains of data. By this is meant, not that some data are explained by classical laws and other data by statistical laws, but rather that certain aspects of all data receive the classical type of explanation while other aspects of the same data are explained along statistical lines.

As has been seen (Cf. Chapter II, # 2.3), the classical heuristic assumption is that similars are similarly understood. Consequently, preliminary classifications are based on similarity to sense. However, the scientist is interested in the relations of things, not to our senses, but to one another. Accordingly, the preliminary classifications are superseded by the emergence and development of technical terms that are derived, not from sensible similarity, but from similarities of constant ~~and~~ OR regularly varying proportion; and in the limit there are reached what we have named pure conjugates, that is, terms implicitly defined by the empirically established correlations in which they occur.

Still to account for data as similar is not to account for data in all their aspects. Each datum is just this instance of the given. It emerges within a continuous manifold. It is in a particular place and at a particular time. It occurs rarely or frequently. Now these aspects of all data are disregarded in explanations of the classical type. The law of the lever tells us nothing about the frequency of levers, about the places where they are to be found, about the times at which they function. Hence, explanations of the classical type~~s~~ have to complemented by explanations of a further, different type.

Nor is it difficult to see, at least in some general fashion, that statistical laws can provide the complementary explanation. For the general form of the statistical law is that on p occurrences of the occasion, P, there tend to be q occurrences of the event, Q. Now the occasion, P, is itself an event or a combination of events. In either case it will possess its probability. In like manner, the occasions on which P is probable, will have their probability, and so there arises an indefinite regress of probabilities from events of the type, Q. More generally, for events of any type, X, there are corresponding indefinite regresses of probabilities.

Now, it is not immediately apparent that such regresses can be combined into a single view. But it suffices for present purposes to remark that, were such a combination possible, one would be on the way to attaining a statistical explanation of data in their numbers and in their spatio-temporal distribution. To invoke only the simplest considerations, low probabilities are offset by large numbers of occasions, so that what is probable only once on a million occasions, is to be expected a million times on a million million occasions. In like manner, the rarity of occasions is offset by long intervals of time, so that if occasions arise only once in a million years, still they arise a thousand times in a thousand million years. At once there emerges the explanatory significance of statistical laws. Why are there in the world of our experience such vast numbers and such enormous intervals of time? Because probabilities are low, numbers have to be large; because occasions are rare, time intervals have to be long.

By itself, this is a very modest conclusion. Still, though the achievement is quite negligible, the potentialities are extremely significant. Statistical laws possess a capacity to generate explanation. Their heuristic assumption is simply that the non-systematic cannot diverge systematically from the systematic. But this incapacity for systematic divergence, when combined with large numbers and long intervals of time, is equivalent to a positive tendency, to an intelligible order, to an effective thrust, that is no less ~~explained~~ explanatory than the rigorous conclusions based on classical laws. In other words, probability is one thing, and chance is another. Probability is an ideal norm that, for all its ideality, is concretely successful in the long run. Chance is merely the non-systematic divergence of actual frequencies from the ideal frequencies named probabilities. Chance explains nothing. It pertains irretrievably to the merely empirical residue, to the aspects of data from which intelligence always abstracts. But probability is an intelligibility: it is, as it were, rescued from the merely empirical residue by the roundabout device in which inquiring intelligence sets up the heuristic anticipations of the statistical type of investigation.

1.7 Summary. We have been considering the complementarity of classical and statistical investigations as forms of knowing. We have found such complementarity to exist at each of the stages or components of the process of inquiry. There is the classical heuristic anticipation of the systematic; there is the complementary statistical heuristic anticipation of the non-systematic. Next, to determine either a classical or a statistical law is to prepare the way for the determination of further laws of either type; for both classical and statistical laws pertain to a single complementary field, and to know either is to effect a mental separation between types of data that have been accounted for and types that still remain to be explained. Thirdly, there is a complementarity of formulations; the experiential and pure conjugates of classical laws can be verified only in events; the events occur only if other things are equal; and the ~~equality of~~ failure to specify the other things amounts to an unconscious acknowledgement of the non-systematic aggregate of patterns of diverging series of conditions. Inversely, as conjugates are verified only in events, so events are defined only by conjugates, and statistical laws of events can possess scientific significance only in the measure that they employ definitions generated by classical procedures. Fourthly, there is a complementarity in modes of abstraction; classical laws regard the systematic in abstraction from the non-systematic, the relations of things to one another in abstraction from their relations to our senses; but statistical laws consider the systematic as setting bounds to the non-systematic and they are confined to the observable events that include a relation to our senses. Fifthly, the two types of law are complementary in their verification: exact and complete knowledge of classical laws cannot successfully invade the field of statistical laws; and statistical investigations are confronted with regular recurrences that admit explanations of the classical type. Finally, there is complementarity in the aspects of data explained by the different types of laws; data as similar are explained on classical lines; but their numbers and their distributions become intelligible only by some synthesis of statistical considerations.

The Complementarity of Classical and Statistical Investigations

Complementarity in the Known.

2.0 Just as the first part of this chapter was devoted to exhibiting the complementarity of classical and of statistical investigations from the view-point of knowing, so now the second part is to be directed to the determinations of the corresponding complementarity from the view-point of what is to be known. For knowing and known, if they are not an identity, at least stand in some correspondence and, as the known is reached only through knowing, structural features of the one are bound to be reflected in the other. Aristotle's world-view stemmed from his distinction between the necessary laws of the heavenly bodies and the contingent laws of things on this earth. Mechanist determinism had its scientific basis in the Galilean concept of explanation as the reduction of secondary to primary qualities. In similar fashion some parallel implication cannot be avoided by any fully conscious methodology and so, if we are not to play the ostrich, we must face the question, what world-view is involved by our affirmation of both classical and statistical laws.

General Characteristics of the View.

2.1 Certain general characteristics of our position may be indicated immediately.

In the first place, it will be concerned with the intelligibility immanent in the universe of our experience. For it will be a conclusion from the structure of empirical method and, by the canon of relevance, empirical method is confined to determining such immanent intelligibility. Hence, we shall have nothing to say in this chapter about the end or purpose of this universe, about the materials from which it was fashioned, about the principal or instrumental agents responsible for it. Our efforts will be limited to determining the immanent design or order characteristic of a universe in which both classical and statistical laws obtain.

In the second place, our account of this design or order will be generic. A specific account would have to draw upon the content of the empirical sciences. It would have to appeal, not to classical and statistical laws in general, but to the precise laws that can be empirically established. Our account, on the other hand, will rest not on the results of scientific investigations but simply and solely upon the dynamic structure of inquiring intelligence. Accordingly, if in the course of the exposition any particular scientific conclusions are invoked, their function will be not determinative but merely illustrative. Just as mechanist determinism has been a world-view that is independent of the precise content of classical laws, so too, our objective is a similarly generic structure that is compatible not only with present classical and statistical laws but also with their future revisions.

In the third place, our account of the design or order of this universe will be relatively invariant. The content of the natural sciences is a variable. There has been the science of the Renaissance. There has been the science of the Enlightenment. There is the science of today. There will be the successive stages of scientific development in the future. But knitting together these diverse manifestations of scientific thought, generating each in turn only to bring forth the revision and transformation of each, there is the underlying invariant that loosely may be named scientific method and more precisely, I think, would be designated as the dynamic structure of inquiring intelligence. For, as has been seen, it is the desire to understand that results both in the heuristic structure of classical procedure and in the complementary structure of statistical investigation; and it is the nature of insight that accounts for the six canons of selection, operations, relevance, parsimony, complete explanation, and statistical residues, in accord with which the heuristic structures generate the series of scientific theories and systems. Now our premise is to be, not the variable contents of the sciences, but the invariant forms governing scientific investigation. It follows that the design of the universe, to which we shall conclude, will enjoy the invariance of the premise which we shall invoke.

Still, I have said that our account will be only relatively invariant, and the reason for this restriction is plain enough. For our appeal will be, not to the structure of the human mind itself, but only to our account of that structure. Just as the natural sciences are subject to revision, so too, one may expect our account of inquiring intelligence to be subjected to rearrangements, modifications, and improvements. In the measure that such changes will affect the premises of the present argument, in the same measure they will also affect the conclusions. Accordingly, the world-view to be presented will be invariant, inasmuch as it will be independent of ~~the contents~~ changes in the content of the empirical sciences; but it will be only relatively invariant, for it cannot be independent of revisions of our analysis of empirical method

In the fourth place, our account of a world-view within the limits of empirical science will not be complete in this chapter. In treating the canon of parsimony, we postponed the question of the validity of the notion of the thing. In a later chapter, that question will have to be met, and then a further complement to the present account will be added.

In the fifth place, our account will not claim to be deductive. Perhaps one might argue in strictly deductive fashion from the complementary structure of the knowing to the corresponding complementarity of the known. But, if that procedure is possible, it also requires an elaboration that for present purposes would be excessive. Accordingly, our appeal will be to insight. We shall begin from the problem of showing how both classical and statistical laws can coalesce into a single, unified intelligibility commensurate with the universe of our experience. Against this problem we shall set our clue, namely, the scheme of recurrence. On the one hand, the world of our experience is full of continuities, oscillations, rhythms, routines, alternations, circulations, regularities. On the other hand, the scheme of recurrence not only squares with this broad fact but also is related intimately both to classical and to statistical laws. For the notion of the scheme emerges in the very formulation of the canons of empirical method. Abstractly, the scheme itself is a combination of classical laws. Concretely, schemes begin continue, and cease to function in accord with statistical probabilities. Such is our clue, our incipient insight. To develop it we shall consider 1) the notion of a conditioned series of schemes of recurrence, 2) the probability of a single scheme, 3) the emergent probability of a series of schemes, and 4) the consequent characteristics of a world order.

2.2 Schemes of Recurrence The notion of the scheme of recurrence arose when it was noted that the diverging series of positive conditions for an event might coil around in a circle. In that case, a series of events, A, B, C,..... would be so related that the fulfilment of the conditions for each would be the occurrence of the others. Schematically, then, the scheme might be represented by the series of conditionals, If A occurs, B will occur; if B occurs, C will occur; if C occurs, ... A will recur. Such a circular arrangement may involve any number of terms, the possibility of alternative routes, and in general, any degree of complexity.

Two instances of greater complexity may be noted. On the one hand, a scheme might consist of a set of almost complete circular arrangements, of which none could function alone yet all would function if conjoined in an interdependent combination. On the other hand, schemes might be complemented by defensive circles, so that if some event, F, tended to upset the scheme, there would be some such sequence of conditions as, If F occurs, then G occurs; if G occurs, then H occurs; if H occurs, then F is eliminated.

In illustration of schemes of recurrence the reader may think of the planetary system, of the circulation of water over the surface of the earth, or of the nitrogen cycle familiar to biologists, of the routines of animal life, of the repetitive, economic rhythms of production and exchange. In illustration of schemes with defensive circles, one may advert to generalized equilibria. Just as a chain reaction is a cumulative series of changes terminating in an explosive difference, so a generalized equilibrium is such a combination of defensive circles that any change, within a limited range, is offset by opposite changes that tend to restore the initial situation. Thus, health in a plant or animal is a generalized equilibrium: again, the balance of various forms of plant and animal life within an environment is a generalized equilibrium: again, economic process was conceived by the older economists as a generalized equilibrium.

However, we are concerned, not with single schemes, but with a conditioned series of schemes. Let us say that the schemes, P, Q, R,.... form a conditioned series, if all prior members of the series must be functioning actually for any later member to become a concrete possibility. Then, the scheme, P, can function though neither Q nor R exist; the scheme, Q, can function, though R does not yet exist; but Q cannot function unless P is already functioning; and R cannot function unless Q is already functioning.

Thus, by way of a simple illustration, one may advert to the dietary schemes of animals. All carnivorous animals cannot live off other carnivorous animals. Hence, a carnivorous, dietary scheme supposes another herbivorous, dietary scheme but, inversely, there could be herbivorous animals without any carnivorous animals. Again, plants cannot in general live off animals; the scheme of their nourishment involves chemical processes; and that scheme can function apart from the existence of any animals. Finally, chemical cycles are not independent of physical laws yet, inversely, the laws of physics can be combined into schemes of recurrence that are independent of chemical processes.

Such in briefest outline is the notion of the conditioned series of schemes of recurrence. Let us seek a slight increase in precision by drawing a threefold distinction between 1) the possible seriation, 2) the probable seriation and 3) the actual seriation.

The actual seriation is unique. It consists of the schemes that actually were, are, or will be functioning in our universe along with precise specifications of their places, their durations, and their relations to one another.

The probable seriation differs from the actual. For the actual diverges non-systematically from probability expectations. The actual is the factual, but the probable is ideal. Hence, while the actual seriation has the uniqueness of the matter of fact, the probable seriation has to exhibit the cumulative ramifications of probable alternatives. Accordingly, the probable seriation is not a single series but a manifold of series. At each stage of world process there is ~~are~~ a set of probable next stages, of which some are more probable than others. The actual seriation includes only the stages that occur. The probable seriation includes all that would occur without systematic divergence from the probabilities.

The possible seriation is still more remote from actuality. It includes all the schemes of recurrence that could be devised from the classical laws of our universe. It orders them in a conditioned series that ramifies not only along the lines of probable alternatives but also along lines of mere possibility or negligible probability. It is equally relevant to our universe and to any other universe subject to the same classical laws, no matter what its initial numbers, diversities, and distribution of elements.

Of the three seriations, then, the possible exhibits the greatest complexity and variety. It depends solely on a consideration of classical laws. It suffers from the indeterminacy of the abstract, and so exhibits the process of any universe with laws similar to ours. The probable seriation depends on statistical as well as classical laws, and, indeed, on the statistical laws that arise from the initial or basic situation of our world. Still, if it is not as abstract as the possible seriation, none the less, it is ideal. For each moment of world history, it assigns a most probable future course. But it also assigns a series of less probable courses, and it has to acknowledge that any of these may prove to be the fact. Finally, the actual seriation is unique, but it purchases its uniqueness by going beyond the field of all laws, classical and statistical, and entering the field of observation, in which alone non-systematic divergences from probability are determinate.

2.3 The Probability of Schemes. Our outline of the notion of a conditioned series of schemes of recurrence supposes that one can attribute a probability to the emergence and to the survival of a scheme of recurrence. However, our account of probability has been in terms of the frequency, not of schemes, but of events. Have schemes any probability? If they have, is there a distinct probability for their emergence and another for their survival? Such questions must be met.

Consider a set of events of the types, A, B, C,.... and a world situation in which they possess respectively the probabilities, p,q,r,... . Then by a general rule of probability theory, the probability of the occurrence of all the events in the set will be the product, pqr...., of their respective probabilities.

Now let us add a further assumption. Let us suppose that the set of events, A,B,C,.... satisfy a conditioned scheme of recurrence, say K, in a world situation in which the scheme, K, is not functioning but, in virtue of the fulfilment of prior conditions, could begin to function. Then, if A were to occur, B would occur. If B were to occur, C would occur. If C were to occur,..... A would occur. In brief, if any of the events in the set were to occur, then, other things being equal, the rest of the events in the set would follow.

In this case, we may suppose that the probabilities of the single events are respectively the same as before, but we cannot suppose that the probability of the combination of all events in the set is the same as before. As is easily to be seen, the concrete possibility of a scheme beginning to function shifts the probability of the combination from the product, pqr...., to the sum, p + q + r + ... For, in virtue of the scheme, it now is true that A and B and C and ..... will occur, if either A or B or C or ..... occur; and by a general rule of probability theory, the probability of a set of alternatives is equal to the sum of the probabilities of the alternatives.

Now a sum of a set of proper fractions, p,q,r,.... is always greater than the product of the same fractions. But a probability is a proper fraction. It follows that, when the prior conditions for the functioning of a scheme of recurrence are satisfied, then the probability of the combination of events, constitutive of the scheme, leap from a product of fractions to a sum of fractions.

There exists, then, a probability of emergence for a scheme of recurrence. That probability consists in the sum of the respective probabilities of all the events included in the scheme, and it arises as soon as the prior conditions for the functioning of the scheme are satisfied.

There also exists a probability for the survival of schemes that have begun to function. For, of itself, a scheme tends to assure its own perpetuity. The positive conditions for the occurrence of its component events reside in the occurrence of those events. Even negative conditions, within limited ranges, can be provided for by the development of defensive circles. None the less, the perpetuity of a scheme is not necessary. Just as classical laws are subject to the proviso, other things being equal, so also are the schemes constituted by combinations of classical laws: and whether or not other things will continue to be equal, is a question that admits an answer only in terms of statistical laws. Accordingly, the probability of the survival of a scheme of recurrence is the probability of the non-occurrence of any of the events that would disrupt the scheme.

2.4 Emergent Probability. There have been formulated the notion of a conditioned series of schemes of recurrence and, as well, the general sense in which one can speak of the probability of the emergence and the survival of single schemes. From these considerations there now comes to light the notion of an emergent probability. For the actual functioning of earlier schemes in the series fulfills the conditions for the possibility of the functioning of later schemes. As such conditions are fulfilled, the probability of the combination of the component events in a scheme jumps from a product of a set of proper fractions to the sum of those proper fractions. But, what is probable, sooner or later occurs. When it occurs, a probability of emergence is replaced by a probability of survival; and as long as the scheme survives, it is in its turn fulfilling conditions for the possibility of still later schemes in the series.

Such is the general notion of emergent probability. It results from the combination of the conditioned series of schemes with their respective probabilities of emergence and survival. While by itself it is extremely jejune, it possesses rather remarkable potentialities of explanation. These must now be indicated in outline, and so we attempt brief considerations of the significance for emergent probability of spatial distribution, absolute numbers, long intervals of time, selection, stability, and development.

The notion of a conditioned series of schemes involves spatial concentrations. For each later set of schemes becomes possible in the places where earlier schemes are already functioning. Accordingly, the most elementary schemes, which are earliest in the series, can occur anywhere in the initial distribution of materials. But the second batch can occur only where the first have in fact occurred, the third can occur only where the second have in fact occurred, and so on. Moreover, since the realization of the schemes is in accord with the probabilities, which may be low, one cannot expect all possibilities to be actuated. Hence, elementary schemes will not be as frequent as they could be, to narrow the possible basis for schemes at the second remove. These will not be as frequent as they could be, to narrow again the possible basis for schemes at the third remove, and so forth. It follows that, however widespread the realization of elementary schemes, there will be a succession of constrictions of the volumes of space in which later schemes can be found. Similarly, it follows that the points, so to speak, of greatest and least constriction occur where the probabilities of emergence of the next set of schemes are respectively the lowest and the highest. Finally, it follows that, since the latest schemes in the series have the greatest number of conditions to be fulfilled, their occurrence will be limited to a relatively small number of places.

Secondly, there is the significance of absolute numbers. For large numbers offset low probabilities. What occurs once on a million occasions, is to be expected a million times on a million million occasions. Now the minimum probability pertains to the latest schemes in the series, for their emergence supposes the emergence of all earlier schemes. It follows that the lower the probability of the last schemes of the conditioned series, the greater must be the initial absolute numbers in which elementary schemes can be realized. In brief, the size of a universe is inversely proportionate to the probability of its ultimate schemes of recurrence.

Thirdly, there is the significance of long intervals of time. No matter how great the universe and how widespread the functioning of elementary schemes, there is an increasing concentration of the spatial volumes in which later schemes can be realized. Sooner or later, the initial benefit of large numbers is lost by the successive narrowing of the basis for further developments. But at this point long intervals of time become significant. Just as a million million simultaneous possibilities yield a million probable realizations, whose probability is one in a million, so also a million million successive possibilities yield a million probable realizations under the same expectation.

Fourthly, there is a selective significance attached to the distinction between probabilities of emergence and probabilities of survival. If both are low, the occurrence of the scheme will be both rare and fleeting. If both are high, the occurrences will be both common and enduring. If the probability of emergence is low and that of survival is high, the scheme is to be expected to be rare but enduring. Finally, in the opposite case, the expectation is that the scheme will be common but fleeting.

Fifthly, this selectivity has its significance for stability. The functioning of later schemes depends upon the functioning of earlier schemes, so that if the earlier collapse, then the later will collapse as well. It follows that the line of maximum stability would be of common and enduring schemes while the line of minimum stability would be of rare and fleeting schemes.

Sixthly, no less than stability, the possibility of development must be considered. Unfortunately, these two can conflict. Schemes with high probabilities of survival tend to imprison materials in their own routines. They provide a highly stable basis for later schemes, but they also tend to prevent later schemes from emerging. A solution to this problem would be for the earlier conditioning schemes to have a high probability of emergence but a low probability of survival. They would form a floating population, on which later schemes could successively depend. Because their probability of survival is low, they would readily surrender materials to give later schemes the opportunity to emerge. Because their probability of emergence was high, they would readily be available to fulfill the conditions for the functioning of later schemes.

Needless to say, the foregoing considerations are extremely rudimentary. They are limited to the emergent probability of any conditioned series of schemes of recurrence. They make no effort towards developing that notion in the direction of its application to the conditions of the emergence and survival of modes of living. However, while absolutely such a fuller exposition would be desirable, still it has no place in a merely generic account of world order. For the premise of a generic account is, not the content of the natural sciences, but the possibility and validity of their assumptions and method.

The point we are endeavoring to make, within the limits of our narrow premise, is that the notion of emergent probability is explanatory. Intelligent inquiry aims at insight. But classical laws alone offer no insight into numbers, distributions, concentrations, time intervals, selectivity, uncertain stability, or development. On the contrary, they abstract from the instance, the place, the time, and the concrete conditions of actual functioning. Again, statistical laws, as a mere aggregate, affirm in various cases the ideal frequency of the occurrence of events. They make no pretence to explaining why there are so many kinds of events or why each kind has the frequency attributed to it. To reach explanation on this level, it is necessary to effect the concrete synthesis of classical laws into a conditioned series of schemes of recurrence, to establish that such schemes, as combinations of events, acquire first a probability of emergence and then a probability of survival through the realization of the conditioned series, and finally to grasp that, if such a series of schemes is being realized in accord with probabilities, then there is available a general principle that promises answers to questions about the reason for numbers and distributions, concentrations and time intervals, selectivity and uncertain stability, development and break-downs. To work out the answers pertains to the natural sciences. To grasp that emergent probability is an explanatory idea, is to know what is meant when our objective was characterized as a generic, relatively invariant, and incomplete account of the immanent intelligibility, the order, the design of the universe of our experience.

Consequences of Emergent Probability.

2.5/ There remains the task of working out the generic properties of a world process in which the order or design is constituted by emergent probability. This we shall attempt in two main steps. First, we shall summarize the essentials of the notion of emergent probability. Secondly, we shall enumerate the consequences of that notion to be verified in world process.

The essentials of the notion of emergent probability may be indicated in the following series of assertions.

1. An event is what is to be known by answering "Yes" to such questions as: Did it happen? Is it occurring? Will it occur?
2. World process is a spatio-temporal manifold of events. In other words, there are many events

and each has its place and time.

3. Events are of kinds. Not every event is a new species, else there could be neither classical nor statistical laws.
4. Events are recurrent. There are many events of each kind, and all are not at the same time.
5. There are regularly recurrent events. This regularity is understood, inasmuch as combinations of classical laws yield schemes of recurrence. Schemes are circular relationships between events of kinds, such that if the events occur once in virtue of the circular relationships then, other things being equal, they keep on recurring indefinitely.
6. Schemes can be arranged in a conditioned series, such that the earlier can function without the emergence of the later, but the later cannot emerge or function unless the earlier already are functioning.
7. Combinations of events possess a probability, and that probability jumps, first when a scheme becomes concretely possible in virtue of the fulfilment of its prior conditions, and secondly when the scheme begins actually to function.
8. The actual frequencies of events of each kind in each place and at each time do not diverge systematically from their probabilities. However, actual frequencies may diverge non-systematically from probabilities, and that non-systematic divergence is chance. Accordingly,

probability and chance are distinct and are not to be confused.

9. Emergent probability is the successive realization in accord with successive schedules of probability of a conditioned series of schemes of recurrence.

The consequent properties of a world process, in which the design is emergent probability, run as follows:

1. There is a succession of world situations. Each is characterized 1) by the schemes of recurrence actually functioning, 2) by the further schemes that now have become concretely possible, and 3) by the current schedule of probabilities of survival for existing schemes and of probabilities of emergence for concretely possible schemes.
2. World process is open. It is a succession of probable realizations of possibilities. Hence, it does not run along the iron rails laid down by determinists nor, on the other hand, is it a non-intelligible morass of merely chance events.
3. World process is increasingly systematic. For it is the successive realization of a conditioned series of schemes of recurrence, and the further the

series of schemes is realized, the greater the systematization to which events are subjected.

4. The increasingly systematic character of world process ~~is~~ can be assured. No matter how slight the probability of the realization of the most developed and most conditioned schemes, the emergence of those schemes can be assured by sufficiently increasing absolute numbers and sufficiently prolonging intervals of time. For actual frequencies do not diverge systematically from probabilities; but the greater the numbers and the longer the time intervals, the clearer the need for a systematic intervention to prevent the probable from occurring.

5. The significance of the initial or basic world situation is limited to the possibilities it contains and to the probabilities it assigns its possibilities. By the initial world situation is meant the situation that is first in time; by the basic world situation is meant the partial prolongation through time of initial conditions, such as arises, for instance, in certain contemporary hypotheses of continuous creation.

In either case, what is significant resides in possibilities and their probabilities, for in all its stages, world process is the probable realization of possibilities. While the determinist would desire full information, exact to the nth decimal place, on his initial or basic situation, the advocate of emergent probability is quite satisfied with any initial situation in which the most elementary schemes can emerge and probably will emerge in sufficient numbers to sustain the subsequent structure.

6. World process admits enormous differentiation. It envisages the totality of possibilities defined by classical laws. It realizes these possibilities in accord with its successive schedules of probabilities. And, given sufficient numbers and sufficient time, even slight probabilities become assured.

7. World process admits break-downs. For no scheme has more than a probability of survival, so that there is for every scheme some probability of a break-down; and since earlier schemes condition later schemes, a break-down of the former entails ~~obtains~~ the break-down of the latter.

8. World process includes blind alleys. For schemes with a high probability of survival have some probability of emergence. In so far as they emerge, they tend to bind within their routines the materials for the possibility of later schemes and so to block the way to full development.

9. The later a scheme is in the conditioned series, the narrower is its distribution. For actual realization is less frequent than its concrete possibility; and each later set of schemes is concretely possible only where earlier, conditioning schemes are functioning.

10. The narrower the basis for the emergence of each later set of schemes, the greater the need to invoke long intervals of time. For in this case, the alternative of large numbers is excluded.

11. The greater the probabilities of blind alleys and break-

downs, the greater must be the initial absolute numbers, if the realization of the whole series of schemes is to be assured. For in this case the device of long time intervals might not be efficacious. Blind alleys with their inert routines could last for extremely long periods and, when they suffered break-downs, they might result in another blind alley. Again, a situation which led to some development only to suffer break-down might merely repeat this process more frequently in a longer interval of time. On the other hand, the effect of large initial numbers is to assure at least one situation in which the whole series of schemes will win through.

12. The foregoing properties of world process are generic. They assume that there are laws of the classical type, but they do not assume the determinate content of any particular classical law. They assume that classical laws can be combined into the circular relationships of schemes, but they do not venture to analyze the structure of any scheme whatever. They assume that there are statistical laws, but there is no assumption of the determinate content of any statistical law.

Moreover, these properties are relatively invariant. They rest on the scientist's necessary presupposition that there are classical and statistical laws to be determined. But they in no way prejudge the determination of those laws nor the manner in which they are to be combined to yield schemes of recurrence and their successive probabilities. It follows that the foregoing properties of world process cannot be upset by any amount of scientific work in the determination of classical or statistical laws.

Again, these properties are explanatory of world process. They reveal an order, a design, an intelligibility. For they account in generic fashion, for numbers and time intervals, for distributions and concentrations, for blind alleys and break-downs, for enormous differentiation, for increasing systematization, for stability without necessity, for assurance without determinism, for development without chance.

Finally, the intelligibility, offered by the explanation, is immanent in world process. It exhibits the inner design of world process as an emergent probability, and from that design it concludes to the outstanding, generic features of the same process. Accordingly, since empirical method aims at such an immanent intelligibility, emergent probability is a view of world order within the limits of empirical method. As we began by inviting the reader to grasp the intelligibility immanent in the image of a cart-wheel, so now we are inviting him to perform again the same kind of act. The only difference is that, for the image of the cart-wheel, he now must substitute the main features of the universe of our experience.

Clarification by Contrast.

3.0/ There is a clarification of ideas through contrast with their opposites. As we have argued that an acceptance of both classical and statistical laws leads to some such world view as emergent probability, so now we have to see how different methodological positions result in different world views.

The Aristotelian World View.

3.1/ Aristotle recognized both natural laws and statistical residues. But his natural laws lumped together in primitive confusion not only classical laws and schemes of recurrence but also an element or aspect of statistical laws. His distinction was between the necessary and the contingent. The necessary was what always happens, as in the movements of the stars. The contingent was what usually happens; thus, usually, heavy bodies fall to the earth but, sometimes, they are propped up and so do not fall.

Not only did Aristotle fail to grasp the abstract laws of nature of the classical type, but explicitly he repudiated the possibility of a theory of probability. For him all terrestrial events were contingent. No doubt, effect follows from cause: but it does so, only if some other cause does not intervene; and such intervention is a mere coincidence. ~~that~~ It is true that any coincidence can be traced back to earlier coincidences and that from ~~From~~ the earlier coincidences one can regress to still earlier coincidences; but one can never get out of the category of the merely coincidental, and within that category there is nothing to be grasped by any science. Hence, while Aristotle recognized statistical residues and concrete patterns of diverging series of conditions, he had no theory of probability to bring them to heel within the field of scientific knowledge.

Still, Aristotle had no intention of allowing terrestrial process to bog down in a mere morass of coincidental interferences. To exorcize such entropy, he argued from seasonal variations to the influence of celestial bodies upon terrestrial activities. Because the sun and moon, the planets and stars, operated necessarily; because they operated from successively different positions: they supplied him with a sufficient ground and cause for the periodicity and perpetuity of terrestrial change. In this fashion there arose his notion of an eternal heaven, an eternal earth, and an eternal cyclic recurrence.

Emergent probability differs from the Aristotelian world view, because it rests on a different notion of science and of law. Classical laws are abstract. The alleged necessary movements of the heavens are merely schemes of recurrence that arose through the unfolding of probabilities and will survive in accord with probabilities. The regularities of terrestrial process are essentially similar, though here the schemes are more complex and the probabilities lower. Finally, eternal cyclic recurrence vanishes and in its place there comes the successive realization, in accord with successive schedules of probabilities, of a conditioned series of ever more complex schemes of recurrence. It is not celestial necessity that assures the success of terrestrial process, but emergent probability that provides the design of all process; and that design is not an eternal, cyclic recurrence, but the realization through probability of a conditioned series of ever more developed schemes.

The Galilean World View.

3.2/ Galileo discovered our law of falling bodies, but he failed to recognize its abstractness. Correctly, he grasped that explanation lies beyond description, that the relations of things to our senses must be transcended, that the relations of things to one another must be grasped, and that a geometrization of nature is the key tool in performing this task. Still Galileo did not cast his methodological discoveries in the foregoing terms. Instead of speaking of the relations of things to our senses, he spoke of the merely apparent, secondary qualities of things. Instead of speaking of the relations of things to one another, he spoke of their real and objective primary qualities, and these he conceived as the mathematical dimensions of matter in motion.

Thus Galilean methodology is penetrated with philosophic assumptions about reality and objectivity and, unfortunately, those assumptions are not too happy. Their influence is evident in Descartes. Their ambiguities appear in Hobbes and Locke, Berkley and Hume. Their final inadequacy becomes clear in Kant, where the real and objective bodies of Galilean thought prove to constitute no more than a phenomenal world.

Hitherto, on the other hand, our procedure has been to prescind severely from philosophic questions about reality and objectivity. In due course we shall have to meet them. But our present concern is the fact that Galilean laws of nature are not conceived in abstraction from sensible or, at least, imaginable elements and, consequently, that the Galilean law stands in the field, not of our abstract classical laws, but rather of our schemes of recurrence in which abstract laws and imaginable elements can combine.

From this concreteness of the conception of natural laws there follows a twofold consequence. On the one hand, there arises the hostility of incomprehension against statistical laws. On the other hand, there results a mechanistic view of the universe. For, in the abstract, classical laws possess universality and necessity. The Galilean acknowledges this universality and necessity but cannot recognize its abstractness. For him, it is attached immediately to imaginable particles or an imaginable ether or both. For him, it is already concrete, and so it is not in need of further determinations to reach concreteness. ~~Further,~~ For him, the further determinations, which would be non-systematically related to one another, simply do not exist. Accordingly, since he has no doubt of the existence of classical laws, he cannot but regard statistical laws as mere formulations of our ignorance. There is some vast aggregate of discrete or continuous but imaginable elements; they are subject to universal and necessary laws; and the business of the scientist is the hard task of determining those laws and so predicting what cannot but occur.

Moreover, within this context, the negation of statistical laws involves mechanism. A machine is a set of imaginable parts, each of which stands in determinate systematic relations to all the others. In like manner, the universe, implicit in Galilean methodology, is an aggregate of imaginable parts and each is related systematically to all the others. The sole difference is that, apart from the machine, there are other imaginable elements that can interfere with its operation, but apart from the universe of imaginable elements, what imaginable interventions can there arise? Mechanism accordingly becomes a determinism.

Until recently, this Galilean view has been dominant in scientific circles. It easily survived the rather veiled implications of Darwinism. But it seems to have suffered a cripling wound from the overt claims of Quantum Mechanics. Our argument, however, moves on a different terrain. It appeals to Darwinism and to Quantum Mechanics only as illustrations of scientific intelligence. Its proper premises lie in the dynamic structure of empirical inquiry and in the canons that govern its unfolding. In that field it has noticed that abstraction is not impoverishing but enriching, that in the sense of enriching abstraction classical laws are abstract, that a systematic unification of classical laws does not imply the possibility of imaginative synthesis, that the concentration of systematic relationships in the abstract field leaves the further determinations, needed for concrete applications, non-systematically related to one another. It follows that classical and statistical laws, so far from being opposed, are complementary. It follows that the regularities of our universe result, not from classical laws alone, but from the combination of such laws with suitable constellations of concrete circumstances. Finally, it follows that these schemes of recurrence - just as the machines that men make - emerge and function, survive and vanish, in accord with the successive schedules of probabilities for

crippling

the realization of a conditioned series of schemes.

3.3 There are those that date the dawn of human intelligence from the publication of Darwin's Origin of Species in 1859. In fact, though the work does not contain any systematic statement of methodological foundations, it does present the outstanding instance of the employment of probability as a principle of explanation. For, in the first place, Darwinism proposes to explain. It offers to tell why species differ, why they are found in their observable spatio-temporal distributions, why the numbers in each species increase, or remain constant, or diminish even to the point of extinction. In the second place, the explanation presents an intelligibility immanent in the data, grounded in similarities and differences, in numbers and their rates of change, in distributions over the surface of the earth and through the epochs of geology. In the third place, this immanent intelligibility differs radically from the immanent intelligibility offered, for instance, by Newton's theory of universal gravitation or Laplace's affirmation of a single mathematical formula by which a suitably endowed intelligence might deduce any world situation from complete information on a single situation. For the follower of Laplace cannot reach any determinate conclusions, unless he is provided with fully accurate information on the basic situation. But the follower of Darwin is indifferent to the details of his basic situation, and he obtains his conclusions by appealing to the natural selection of chance variations that arise in any of a large variety of terrestrial processes from any of a large variety of initial situations.

It is not difficult to discern in Darwin's natural selection of chance variations a particular case of a more general formula. For it is not the single, isolated variation but rather a combination of variations that is significant for the evolutionary process. Again, while such combinations of variations may be attributed to chance, in the sense that the biologist is concerned, not with efficient causality, but with an immanent intelligibility, still, what is significant for evolution is the probability of emergence of such combinations of variations and not the non-systematic divergence from their probability, which is our meaning of the name, chance. Finally, as chance variation is an instance of probability of emergence, so natural selection is an instance of probability of survival. Artificial selection is the work of the breeder, who mates the plants or animals possessing the characteristics he wishes to encourage. Natural selection is the work of nature, which gives a shorter life expectancy and so less frequent litters to the types that are less well equipped to fend for themselves. Still, nature effects this selection, not with the exact predictability of the changing phases of the moon, but only by a general tendency that admits exceptions and that increases in efficacy with the increase of numbers and the prolongation of time intervals. In a word, natural selection means survival in accord with the probabilities.

Moreover, these combinations of variations, which possess probabilities of emergence and of survival, are relevant to schemes of recurrence. For the concrete living of any plant or animal may be regarded as a set of sequences of operations. Such operations are of kinds; there are many of the same kind; and those of the same kind occur at different times. There are, then, in each set of sequences recurrent operations, and the regularity of the recurrence reveals the existence and functioning of schemes.

Within such schemes the plant or animal is only a component. The whole schematic circle of events does not occur within the living thing, but goes beyond it into the environment, from which sustenance is won, and into which offspring are born. No doubt, the higher the type, the greater the complexity and the greater the proportion of significant events that occur within the animal. But this greater complexity only means that the larger circle connects a series of lesser and incomplete circles. The vascular circulation occurs within the animal, but it depends upon the digestive system, which depends upon the animal's capacity to deal with its environment and, in turn, that capacity depends on the growth and nourishment secured by the vascular system.

Again, the plant or animal is a component for a range of schemes. Unlike the planets which stick to their courses in the solar system, and like the electrons which may be imagined to hop from one orbit to another, the plant or animal enters into any of a range of sets of alternative schemes. This range is limited by immanent structure and capacity. Still, though it is limited, it remains open to alternatives. For without change of structure or of basic capacity, the plant or animal continues to survive within some variations of temperature and pressure, of circumambient water or air, of sunlight and soil, of the floating population of other plants or animals on which it lives.

At this point, however, the differences between Darwinism and emergent probability begin to come to light. Emergent probability affirms a conditioned series of schemes of recurrence that are realized in accord with successive schedules of probabilities. Darwinism, on the other hand, affirms a conditioned series of species of things to be realized in accord with successive schedules of probability. The two views are parallel in their formal structures. They are related, inasmuch as species of living things emerge and function within ranges of alternative sets of schemes of recurrence. None the less, there is a profound difference. For Darwinism probabilities of emergence and survival regard, not schemes of recurrence, but underlying potential components for any schemes within a limited range, and the Darwinian series of species is a sequence of higher potentialities that exhibit their development by their capacity to function in ever greater ranges of alternative sets of schemes.

This difference prompts us to recall that the present account of emergent probability did not aim at completeness. We had not raised the question, What are things? We had not determined whether there is an answer to that question that satisfies the scientific canon of parsimony. Accordingly, we presented emergent probability in the present chapter with the qualification that later, when the notion of thing had been investigated, there might be needed a further development of the analysis.

Darwinism would indicate the necessity of such a further development. Accordingly, if a satisfactory notion of the thing can be reached, there will arise the following questions. Are things potential components for ranges of schemes of recurrence? Are they variable in these potentialities? Are such variations of potentiality capable of transmission? Is there a series of combinations of transmissible variations of potentiality? Are there the appropriate, successive schedules of probabilities for the emergence and the survival of the series of combinations of transmissible variations of potentiality? Finally, if these questions can be answered affirmatively, can these affirmations rest on general, methodological grounds?

3.4 Indeterminism.

By indeterminism is meant a contemporary tendency that owes its ~~origen~~ origin to the verified equations of Quantum Mechanics but goes beyond its source inasmuch as it pronounces on the nature of scientific knowledge and even on philosophic issues. While it is opposed radically to mechanical determinism, its positive features do not admit summary description and, perhaps, our purpose will best be served by discussing successively a series of issues.

First, as Galileo distinguished between merely apparent secondary qualities and, on the other hand, the real and objective dimensions of matter in motion, so too there are ~~detert~~ indeterminists that offer a somewhat parallel disclosure of the nature of reality. The old distinction between the real and the apparent is retained, but now the real is microscopic and random, while the merely apparent is the macroscopic in which classical laws seem to be verified. However, we mention this issue only to decline an immediate discussion. Later in a philosophic context we shall attempt an explanatory account of the almost endless variety of views on reality and objectivity. For the present we shall have to be content with the canon of parsimony. The scientist ~~can say~~ may affirm what he can verify and he may not affirm what he cannot verify.

Secondly, indeterminists tend to reject the old imaginable particles and waves and to favor some type of conceptual symbolism. Here again the issue is ~~thr~~ the precise nature of reality but now, by appealing to the canon of parsimony, we can reach two conclusions. On the one hand, it would seem that the only possible verification of the imagined as imagined lies in a corresponding sensation; accordingly, if the particles are too small and the waves too subtle to be sensed as particles and waves, then the particles as imagined and the waves as imagined cannot be verified; and if they cannot be verified, they may not be affirmed by the scientist. On the other hand, it is possible to verify conceptual formulations if they possess sensible implications; for in the measure that an increasing number and ~~va~~ variety of such implications are found to correspond to sensible experience, the verification of the conceptual formulation is approached. Thus, Special Relativity is said to be probable, not because many scientists feel that they have had a fairly good look at a four-dimensional space-time manifold, but because many scientists working on different problems have found procedures and predictions based on Special Relativity to be highly successful.

Thirdly, there occurs an argument from the haziness of data to the ultimate unverifiability of classical laws. While I do not believe it to be cogent, it is well worth attention. For it appeals to the criterion of verifiability; it rests on the solid fact of the haziness of data; and it does exclude misconceptions of the nature of classical laws.

To begin, the haziness of data is not to be denied. What of itself is determinate never is a datum and always is a concept. Of themselves, data may be said to be determinate materially or potentially; but they become determinate formally only in the measure that they are subsumed under concepts; and this process of subsumption can be prolonged indefinitely. Thus, a greater formal determinateness of data is possible as long as scientific concepts can be revised to yield more precise objects for measurement and as long as scientific techniques can be improved to make measurements more accurate. But as long as a greater formal determinateness is possible, the determinateness that actually is attained is conjoined with an unspecified remainder of merely potential determinateness. That unspecified remainder is the haziness of data, and it will be with us as long as new concepts and more accurate measurements are possible.

However, the haziness of data alone cannot prove the unverifiability of classical laws. Logically, it is impossible ~~to~~ for a valid conclusion to contain a term that does not appear in the premises. More concretely, it could be true that, ~~whent~~ whenever data became more determinate formally, new classical laws were discovered; and in that case the haziness of data would prove, not that classical laws were unverifiable, but that existing classical laws were always due to be revised in favor of other [illegible] classical laws.

One comes closer to the issue when one argues that classical laws are conceptual formulations, that they possess all the precision and formal determinateness of concepts, that they cannot be stripped of that precision and determinateness without ceasing to be classical laws. In contrast, data are irreducibly hazy. Because measurements never can be accurate to n decimal places, where n is as large as one pleases, classical laws never can be more than approximative. Their essential determinacy is in radical conflict with the haziness of data; and so classical laws essentially are unverifiable.

Now this argument is valid if classical laws are interpreted concretely. For on concrete interpretation classical laws are supposed to state relations between data or between elements in strict correspondence with data. But there cannot be completely determinate relations between essentially hazy terms; and so, but on concrete interpretation, classical laws must be regarded as no more than approximative.

Still, there is no need to interpret classical laws concretely. They can be statements of elements in abstract system where 1) the abstract system is constituted by implicitly defined relations and terms, 2) the abstract system is connected with data not directly but through the mediation of a complementary set of descriptive concepts, and 3) the laws of the abstract system are said to be verified inasmuch as they assign limits on which, other things being equal, vast varieties of data converge. On this showing, the completely determinate relations of classical laws are between the completely determinate terms they implicitly define. This closed structure is referred to data through a set of descriptive and so approximative concepts. Finally, the closed structure is proved relevant to data, not by exact coincidence, but by assigning the limits on which data converge.

Fourthly, the affirmation of convergence is also an admission of divergence. Is not that admission equivalent to the statement that ultimately classical laws are not verifiable?

Again, the issue is the precise nature of verification It hardly would be claimed that any single law was not verified because it did not account for the whole of our experience. But what can hold for single laws, also can hold for the totality of classical laws. The existence of the divergence proves that classical laws are not the whole of our explanatory knowledge. But though they are not the whole, they can be a part; and the classical laws that in fact are such a part are the ones that are verified in the sense that they assign the limits on which data do converge.

Fifthly, it is claimed that Quantum Mechanics is the more general theory and that it includes, say, Newtonian mechanics as a particular case.

Here I would suggest the relevance of a distinction between logical inclusion and concrete application. I see no reason for disputing the contention that Schrödinger's time equation can plausibly be simplified into Newton's second law of motion. But it need not follow that the simplification has no analogue in the world of events. On the contrary, it would seem that such an analogue would exist if schemes of recurrence were realized perfectly; and in that case it would seem difficult to maintain that the accuracy of basic observations was not the sole limit to the accuracy of predictions.

Sixthly, it may be argued that determinism must be true or false and that we are dog dodging the issue. But every-sole limit to the accuracy of predictions. More realistically, in so far as schemes are not realized perfectly or perfect realization cannot be ascertained, at least the reason for objective divagations or subjective ignorance would be assigned.

Sixthly, it may be argued that determinism must be true or false and that we seem to be dodging the issue. But if the disjunction is admitted, one finds oneself forced into philosophic questions. At least in the present context, our contention would be that the old determinism with its philosophic implications has to give way to a new, purely methodological view that consists in a developing anticipation of a determinate object.

Such a view would remain within the limits of empirical science. It would distinguish between an antecedent component of methodological assumptions and a consequent component of probably verified laws and frequencies. Both components would be regarded as variable. The antecedent component develops; initially it consists in such vague generalities as the assertion that there is a reason for everything; subsequently, as science advances, it takes on the increasing precision of ever more accurately differentiated heuristic structures. Again, the consequent component is subject to variation, for what is regarded as verified at any time may be called into question and subjected to revision. The concrete conjunction of the two components in the minds of scientists constitutes at any time their anticipations of a determinate object; and when the components are undergoing profound change, there naturally will be some uncertainty in their anticipations.

On this view the old determinism was mistaken not only because it was involved in philosophic issues but also because it failed to envisage the possibility of development in in heuristic structures. It supposed the universal validity of a type of explanation that is possible only when schematic situations are realized perfectly. It overlooked the possibility of a type of explanation in which the probabilities of the non-schematic account for the emergence of the schematic.

Indeterminism is true as a negation of the old determinism. But it cannot escape the necessity of methodological assumptions and precepts; it cannot ~~escap~~ prevent their conjunction in thought with laws and frequencies that are regarded as verified; and so it cannot succeed even in delaying the day when, from a new viewpoint, scientific anticipations once more will envisage a determinate object to be known.

However, at the present time, there is some difficulty in specifying in a universally acceptable fashion just what is the determinate object that science is to anticipate. A student of human knowledge can make suggestions that regard the antecedent component, and so I have offered a unified view that anticipates both the systematic and the non-systematic without excluding in particular cases insight into concrete ~~situations~~ non-schematic situations. The possibility of ~~sened~~ concrete insight into the non-schematic situations of the subatomic order probably will be called into question on both practical and theoretical grounds. However, I do not propose to discuss this aspect of the issue, principally because it regards the consequent component of methodological anticipations, but also because I believe all discussions of concrete possibility to suffer from a radical ambiguity. For on any concrete issue further insight is always possible and, when it occurs, what previously seemed impossible, turns out to be quite feasible after all.

4. Let us bring this long chapter to an end. It began from the problem of apparent duality that arose from the existence of two types of insight, two heuristic structures, and two distinct methods of empirical investigation. There was no question of eliminating the duality, for the direct and the ~~inverted~~ inverse types of insight both occur. There remained, then, the task of relating diverse procedures and results into a single whole. In a first section it was argued that classical and statistical investigations are complementary as cognitional activities. In a second section it was revealed how their results, whatever their precise content, can be combined into a single world view. In a third section this world view was contrasted with the Aristotelian, with that of mechanist determinism, with the Darwinian view, and with contemporary tendencies to affirm an indeterminism. In the course of the argument the problem of the thing and, with it, the problem of objectivity became increasingly apparent. But before tackling such large issues, it will be well to broaden the basis of our operations and so we turn to the notions of space and time.