

Page Ch. 3: Commodity Market Equilibrium and the Multiplier

51 The theory is very simple. The division of disposable income between consumption (C) and saving (S) follows a regular pattern. C and S are passive (dependent variables). The active components are private investment and government spending (I and G). Both are autonomous (independent variables). Following a \$1.00 change in I or G, the change in GNP is generally greater than \$1.00, because C, part of GNP, is dragged up or down by changes in I and/or G. The ratio of changes in GNP to the initiating changes in I or G is called the multiplier.

52  $Q_D$  Household disposable income after taxes  
 $a$  autonomous consumption (minimum sustinence)  
 $c$  propensiy to consume (see below P. 55)  
 $C = a + cQ_D$   
 $s$  propensity to save (cf. 55)  $s = 1 - c$   
 $S = Q_D - C = Q_D - a - cQ_D = (1 - c)Q_D - a = sQ_D - a$

55 Average  $c$  is ratio of total consumption to total product  
 Marginal  $c$  is increment in C for \$1.00 increment in  $Q_D = \Delta C / \Delta Q_D$   
 Average  $s$  is the ratio,  $S/Q_D$   
 Marginal  $s$  is the increment in S induced by \$1.00 increment in  $Q_D$ ;  
 it is  $\Delta S / \Delta Q_D$

59  $I_p$  Planned investment spending  
 $I_u$  Unintended inventory accumulation, whence pressure for change  
 $A_p$  Autonomous planned spending ( $= a + I_p$ )  
 $E_p$  Planned expenditure ( $= C + I_p$ )  $= a + cQ + I_p$

62 Income (Q) equals expenditures (E)  
 (E) equals desired expenditures ( $E_p = C + I_p$ ) plus  
 uninded inventory accumulation ( $I_u$ )  
 Autonomous planned spending ( $A_p$ ) equals induced saving ( $a + I_p$ ) for  
 $Q = E_p$  (condition of equilibrium) hence  
 $Q - cQ = E_p - cQ = A_p = a + I_p$

67 Multiplier (k) is  $\Delta Q / \Delta A_p$  which equals  $1/s$ .

since  $\Delta A_p = \Delta(a + I_p) = \Delta I_p = \Delta S$

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Natural real output is defined (ch 1-5) as the highest level of real output attainable without causing an accelerating inflation.

A multiplier expansion or contraction of output is favorable if it moves the economy nearer to natural output and unfavorable if it pushes the economy away from it.

Let natural real output be \$1600 billion, where  $A_p$  is \$400 billion and  $\Delta C$  is  $0.75 \Delta Q$ .

Let actual real output be \$1200 billion with  $A_p$  at 300 billion. How raise actual to natural rate?  $\Delta I = G = \$100$  billion.

Government spending ( $G$ ) changes  $E_p$ , from  $C + I_p$  to  $C + I_p + G$ .

Taxes reduce  $Q_D$  to  $Q_D - T$  so that the consumption function changes from  $(a + cQ_D)$  to  $(a + c(Q - T))$  so that

$$E_p = a + cQ - cT + I_p + G$$

$$\text{but } A_p + cQ = E_p$$

$$\text{hence } A_p + cQ = a + cQ - c\bar{T} + I_p + G$$

where the  $cQ$ 's cancel and the bar over the  $T$  means taxes that do not vary with income.

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$A_p$  accordingly varies with  $a$ , with  $c\bar{T}$ , with  $I_p$ , & with  $G$ .

Since  $c$  is  $0.75$ , the taxes reduce autonomous spending only 75% of  $\bar{T}$ ; the remaining 25% comes out of saving.

From this value of  $A_p$  the multiplier equation gives the change in  $Q$ :

$$\Delta Q = \Delta A_p / \frac{1}{4} = 4 \Delta A_p$$

The expenditure  $G$  raised  $Q$  from 1200 billion to 1500 billion and the taxes reduce 1600 billion by 300 billion to 1300 billion.

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One might suppose that the taxes mentioned above were to recoup the government deficit; this does not seem to be Gordon's intention; on page 74 he explicitly denies a change in taxes or in investment. Government purchases cost \$100 billion; tax revenues remain at zero; and so the deficit is \$100 billion.

From (3.22) there is repeated the formula:  $S - I = G - T$ ; changes on one side must balance changes on the other side; so

$$\Delta S - \Delta I = \Delta G - \Delta T$$

but  $I$  and  $T$  do not change so that

$$\Delta S = \Delta G = \$100 \text{ billion}$$

so that the savings will be available to buy the new Gov't bonds.

Diagram  
 36  
 page 72

Tax increases and the balanced budget multiplier

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The gov't may prefer increasing taxes to deficit spending. If equilibrium was at point D, with  $G = \$100$  billion, what happens if autonomous taxes are raised from zero to \$100 billion ( $\Delta \bar{T} = 100$ ) and everything else remains the same? First the change in  $A_p$  in equation 3.20 is:

$$\begin{aligned} \Delta A_p &= \Delta a - c\Delta T + \Delta I_p + \Delta G \\ &= 0 - c\Delta T + 0 + 0 \\ &= -0.75(100) = -75 \end{aligned}$$

\$325

So autonomous spending drops from \$400 to \$325 billion. Further equilibrium income drops from \$1600 to \$1300 billion.

$$\Delta Q = \Delta A_p / s = -c\Delta \bar{T} / s$$

$$\Delta Q = \frac{-0.75(100)}{0.25} = -300$$

The multiplier for an increase in taxes is the income change in the preceding equation divided by  $\Delta \bar{T}$ :

$$\frac{\Delta Q}{\Delta \bar{T}} = \frac{\Delta A_p}{s\Delta \bar{T}} = \frac{-c\Delta \bar{T}}{s\Delta \bar{T}} = \frac{-c}{s} \quad \text{i. s.} \quad \frac{-0.75}{0.25} = -3.0$$

The balanced budget multiplier is the gov't spending multiplier ( $1/s$ ) plus the autonomous tax multiplier ( $-c/s$ ):

$$\frac{\Delta Q}{\Delta G} = \frac{\Delta Q}{\Delta \bar{T}} = 1/s - c/s = (1 - c)/s = 1$$

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Cautions

The gov't may collect part of its taxes from income tax and then the balanced budget multiplier is less than unity.

Presupposed has been caeteris paribus; but ~~x~~ changes in  $s$  (change  $I_p$ ) can cause major changes in equilibrium income; the larger  $s$  is, the smaller is the spending multiplier.  $\therefore K = \frac{1}{s}$

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Ch 3-8 Case Study: Spending, leakages, and the multiplier in the Great Depression (1929-33)

Numerical values of  $Q$ ,  $A_p$ ,  $I_p$ ,  $G$ ,  $-c\bar{T}$ ;  $G - T = S - I$  in 1929 and in 1933 and the differences with graphical representation of the movement.

Fig. 3-7 p. 78

Robert Gordon, Macro

Appendix to ch 3 pp 81 -85

Income taxes, Foreign trade, and the Multiplier

$$T = \bar{T} + \bar{t}Q$$

Taxes equal the sum of taxes that are independent of income and that vary with variations in income.

$$Q_D = Q - T = Q - \bar{T} - \bar{t}Q = (1 - \bar{t})Q - \bar{T}$$

When income tax rate is 0.2, the fractions going to

1. Induced consumption	$c(1 - \bar{t})$	
2. Induced saving	$s(1 - \bar{t})$	
3. Income tax revenue	$\bar{t}$	$= \frac{\bar{t}}{1 + \bar{t} - \bar{t}} = \frac{\bar{t}}{1}$
Total	$(c + s)(1 - \bar{t}) + \bar{t}$	$= \frac{\bar{t}}{1 + \bar{t} - \bar{t}} = \frac{\bar{t}}{1}$

For equilibrium:  $Q = E_p$

Hence  $Q - \text{induced consumption} = E_p - \text{induced consumption}$   
and from the table

the LHS is  $Q(s(1 - \bar{t}) + \bar{t})$  i. e.  $Q$  times the marginal leakage rate  
and the RHS is  $A_p$

$Q$  and  $A_p$  are related by  $\frac{A_p}{s(1 - \bar{t}) + \bar{t}} = Q$

Hence when  $A_p$  is 400 and  $\bar{t}$  is 0.2, then  $Q$  is 1000.

But if  $\bar{t}$  were zero, then  $Q$  would be 1600

Why? Because  $\bar{t}$  changes the multiplier.

$$\text{Multiplier} = \frac{\Delta Q}{\Delta A_p} = \frac{1}{s(1 - \bar{t}) + \bar{t}} = \frac{1}{0.4} = 2.5$$

Hence  $A_p$  must be increased by 240 billion to give a  $Q$  of 1600 billion since 600 over 240 equals 2.5 (the new multiplier.)

Exports add a fifth factor to  $A_p = a - cT + I_p + G + X$  (X is real value of exports)  
where imports are  $H = hQ$ , imports add factor "h" to marginal leakage rate increasing effect of income tax.

$$Q = \frac{A_p}{\text{Marginal leakage rate}} = \frac{a - cT + I_p + G + X}{s(1 - \bar{t}) + \bar{t} - h}$$

$$\frac{\Delta Q}{\Delta A_p} = \frac{1}{s(1 - \bar{t}) + \bar{t}} = \frac{1}{\frac{1}{4} \cdot 0.8 + 0.2} = \frac{1}{\frac{1}{4} \cdot \frac{4}{5} + \frac{1}{5}} = \frac{1}{\frac{2}{5}} = 2.5$$