EARLY NOTES ON ECONOMICS

- Bernard J.F. Lonergan, S.J.

1942

A 338

(= Files 58, 59, 60, Batch I)
Rec. 11-2-95

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Sept. 21/83

Mike:

Re BL's papers on economics. I'll take them separately.

The 1944 (presumed date?) work could go on the shelves as "public" material

- on the grounds that it is the first "edition" of the works from 1978 on, which we have made public (following Bernie's own "public" use of them),
- and that it was a planned and completed essay, as far as it went.

I will make out a card for it, therefore, with the title and date in brackets, to indicate that we assign them on a presumption.

But I think you should make up another title page, and maybe a little introduction on its history, so far as we know it.

Certainly, it should have (C) Bernard Lonergan 1983 on the title page.

And the usual note on getting permission for publication of quotations from BL's non-published (in the strict sense of "published") writings.

But the 1942 papers are another matter, and I would not agree to their being made public. They are not a planned and complete work, but various drafts, with pages started and not finished, with later pages covering the same ground as earlier (and, obviously, a different attack on the same subject), etc. To put them on the shelves, while Bernie is still alive, would be like digging Fulton Sheen's trial drafts of a sermon out of his waste papers and publishing them while he was alive.

On the other hand, they should be available to really earnest students (not just curious tourists) in the way the other "Archives" papers are, so you might want to add them to the Catalogue, and I would locate them with the latest "Batch"--whatever that is, as being put together from various earlier files.

Again, an introduction would be helpful. And, since this is a bound copy, better to have the copyright sign on it too. (Notice that there are marginal notes that don't seem to be Bernie's --at least, there is one on p. 13. Notice also the phrase, p. 101, "the present war"--internal evidence of the date!)

N.B. The 1944 volume, p. 56, has cut off the bottom line, written in by Bernie. It's hardly legible in my copy, but seems to be:

'Typist's haplographia, insert at

"and in the second interval objects in the same class were sold in the quantity, q_i"'

[The double quot. marks (") are Bernie's, the single (') are mine. He may have had commas, cut off in my xerox.]

Fred C.

LONERGAN'S EARLY NOTES ON ECONOMICS - 1942

* Pagination supplied in Aug. 83 (by N.Graham) on a copy made from the files. The originals had no page numbers.

SECTION PAGE *

BATCH II, folder 60

"Essay Towards a Pure Theory of Social Economics"

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C O N T E N T S

File 58, 7, 60

An Outline of Circulation Analysis.

1. View-point. It is the view-point of the present inquiry that, besides the pricing system, there exists another economic mechanism, that relative to this system man is not an internal factor but an external agent, and that present economic problems are peculiarly baffling because man as external agent has not the systematic guidance he needs to operate successfully the machine he controls.

On classical analysis economic mechanism is the pricing system. It coordinates spontaneously a vast and ever shifting manifold of otherwise independent choices of demand and decisions of supply. But man does not stand outside this machine; he is part of it; his choices and decisions are themselves the variables in the system. It follows that there is no possibility of setting down ~~systematically~~ methodically, on the one hand, the exigences of the machine and, on the other, the consequent performance of man. A study of the mechanics of motor-cars yields premises for a criticism of drivers, precisely because the motor-cars, as distinct from the drivers, have laws of their own which drivers must respect. But if the mechanics of motors included, in a single piece, the anthropology of drivers, criticism could be no more than haphazard.

There is at present an abundance of economic criticism. It is haphazard criticism. It does not proceed systematically from solid premises. It is the intuition of socialists who find a radical incoherence in individual choices and decisions and leap

with a gay profusion of rhetoric to the simpliste solution of subordinating preferences and expectations to the benevolence of a tyranny. In contrast the criticism of traditional economists is the soul of sobriety. It is acute, informed, exact, subtle. But perhaps one may doubt that it is inspired, that it suffers from the imperious pressure of really significant ideas. Too often does one learn that problems are very complex indeed, that this or that element in the complexity may be singled out as especially troublesome, that such and such a makeshift perhaps meets the issue more satisfactorily than others which have been advocated. For as makeshift follows makeshift, it becomes increasingly difficult to distinguish between a democratic and a totalitarian economy.

But economists can be champions of democracy as well as advisers to dictators or planning boards. The proof of the possibility is an historical fact: the old political economists were champions of democracy; and if the content of their thought has been found inadequate, its democratic form is as valid to-day as ever. That form consisted in the discovery of an economic mechanism and in the deduction of rules to guide men in the use of the economic machine, a rule of laisser faire for governments and a rule of thrift and enterprise for individuals. It is now fully apparent that these rules serve their purpose only in particular cases, but it is still insufficiently grasped that new and more satisfactory rules have to be devised. Without them human liberty will perish. For either men learn rules to guide them individually in inthe use of the economic machine, or else they surrender their liberty to be ruled along with the machine by a central planning board.

The reality of that dilemma measures the significance of an effort, however tenuous and incomplete, to formulate the laws of an economic mechanism more remote and, in a sense, more fundamental than the pricing system. Now there is little dispute that the dilemma is real, for the liberal dream of an automatic economy has, like all dreams, at long last broken. The necessity of rational control has ceased to be a question, and the one issue is the locus of that control. Is it to be absolutist from above downwards? Is it to be democratic from below upwards? Plainly it can be democratic only in the measure in which economic science succeeds in issu utterings not counsel to rulers but precepts to mankind, not specific remedies and plans to increase the power of bureaucracies, but universal laws which men themselves administrate in the personal conduct of their lives. Thus the breaking of the liberal dream of automatic progress provokes a revision of judgement on the old political economists. Their greatness lay not in fostering an amoral devotion to automatism but in developing an economic science and from it issuing universal precepts of proper economic conduct. The automatism is a husk that has withered and fallen, and to cling to it is to fall into the totalitarian abyss. The old science and the old precepts have gone the way of Ptolemy and Newton. But to deny the possibility of a new science and new precepts is, I am convinced, to deny the possibility of the survival of democracy.

2. Method. The method of circulation analysis resembles more the method of arithmetic than the method of botany. It involves a minimum of description and classification, a maximum of inter-connections and functional relations. Perforce, some description and classification are necessary; but they are highly selective and they contain the apparent arbitrariness inherent in all analysis. For analytic thinking uses classes based on similarity only as a spring-board to reach terms defined by the correlations in which they stand. To take the arithmetic illustration, only a few of the integral numbers in the indefinite number series are classes derived from descriptive similarity; by definition, the whole series is a progression in which each successive term is a function of its predecessor. It is this procedure that gives arithmetic its endless possibilities of accurate deduction; and, as has been well argued (*), it is an essentially analogous procedure that underlies all effective theory.

Note:

(*) See for instance, Ernst Cassirer, Substanzbegriff und Funktionsbegriff, Berlin 1910.

On such a methodological model circulation analysis raises a large super-structure of terms and theorems upon a summary classification and a few brief analyses of typical phenomena. Classes of payments quickly become rates of payment standing in the mutual conditioning of a circulation; to this mutual and, so to speak, internal conditioning there is immediately added the external conditioning that arises out of

transfers of money from one circulation to another; in turn this t twofold conditioning in the monetary order is correlated with the conditioning constituted by productive rhythms of goods and services; and from the foregoing dynamic configuration of conditions during a limited interval of time, there is deduced a catalogue of possible types of change in the configuration over a series of intervals. There results a closely knit frame of reference that can envisage any total movement of an economy as a function of variations in rates of payment, and that can define the conditions of desirable movements as well as deduce the causes of break-downs. Through such a frame a reference one can see and express the mechanism to which classical precepts are only partially adapted; and through it again one can infer the fuller adaptation that has to be attained.

However, to set up such a systematic unit of terms and theorems is a logical procedure with norms and criteria of its own. The nature of his task leads the descriptive economist to use, as much as possible, the language of ordinary speech, to be content with resemblances that strike the eye, to move through easy stages of generalization to a nuanced picture of what, in the main, takes place. Again, the statistical economist has his own criteria. He will take advantage of a specialized terminology but, as far as he is concerned, the only justification for a terminology is a proximate possibility of measurements; further, he has no objection to recondite generalizations, but his generalizations resemble not the generalizations of mathematics but those of positive science. Now as the statistical approach differs from the descriptive, the analytic differs from both. Out of endless classificatory possibilities it selects not the one sanctioned by ordinary

speech nor again the one sanctioned by facility of measurement but the one that most rapidly yields terms which can be defined by the functional inter-relations in which they stand. To discover such terms is a lengthy and painful process of trial and error. Experto crede. To justify them, one cannot reproduce the tedious blind efforts that led to them; one can appeal only to the success, be it great or small, with which they serve to account systematically for the phenomena under investigation. Hence it is only fair to issue at once a warning that the reader will have to work through pages, in which parts gradually are assembled, before he will be able to see a whole and pass an equitable judgement upon it.

7. The Exchange Economy. In any economy with a degree of development beyond that of primitive fruit-gathering it is possible to verify the existence of a productive process with basic and surplus stages. The restrictive supposition of an exchange economy is now introduced. It involves property, exchange, prices, money; it postulates a correlation between quantities produced and quantities sold; also it postulates systematic modifications of prices when the productive process accelerates. The latter postulate will be outlined in the next section; this section is devoted to setting down a series of definitions; and as the matter is familiar, treatment will be summary.

a) Prices. If a quantity, q_i, of objects in a class, i, is exchanged for a quantity, q_j, of objects in a class, j, then it is always possible to assign a ratio, p_{ij}, such that

$$q_i = q_j p_{ij} \qquad (2)$$

The ratio so defined is named a price; and the price, p_{ij}, is said to be the price of a unit of "j" measured in units of "i". From the definition it also follows that

$$q_j = q_i p_{ji} \qquad (3)$$

$$q_j = q_k p_{jk} \qquad (4)$$

and the solution of the three equations gives

$$p_{ij} p_{ji} = 1 \qquad (5)$$

$$p_{ij} p_{jk} = p_{ik} \qquad (6)$$

More generally, with respect to "n" classes of objects, there

are possible n(n - 1) different ratios, but of these only (n - 1) are independent ratios; the remaining (n^2 - 2n + 1) are deducible from the independent (n - 1) by the formulae (5) and (6).

b) The Consistency of Prices. By the consistancy of prices is meant, first, that at any one time in a given economy there is to be found but a single price, p_{ij} (or its reciprocal p_{ji}), in all exchanges of classes "i" and "j" and, secondly, that any of the n(n - 1) ratios between quantities in "n" classes is deducible from any (n - 1) independent ratios.

This consistency of prices may be postulated by making it a definition of the objects in the "n" classes. Suppose two exchanges or two sets of exchanges represented by

$$q_i = q_j p_{ij} \tag{7}$$

$$q'_i = q'_j p'_{ij} \tag{8}$$

where

$$p_{ij} < p'_{ij} \tag{9}$$

then one may always write

$$p'_{ij} = p_{ij} + p_{ik} \tag{10}$$

where p_{ik} is the price of some other object or objects. This other object ~~may always be described as lack of consistency.~~ It usually may be described as special services, more agreeable circumstances, pride of purse; as the employment of agents, information, transportation; as the neglect of introducing these three; as lack of foresight, lack of initiative, ignorance, gullibility. As a last resort one can always say that p_{ik} is the price of a lack of consistency.

If equations (7) and (8) do not represent exchanges directly but are deduced from equations representing exchanges between classes "i" and "h", "j" and "h", it is equally possible to introduce a further object, p whose price is p_{ik} and so explain the apparent lack of consistency as before.

In a word it is postulated that the classification of objects exchanged carry analysis far enough that prices be consistent.

c) Money. When in the vast majority of exchanges one of the objects exchanged always belongs to some given class, that class of objects is named the medium of exchange. If further all prices are measured in units of the medium of exchange, that medium is also the common measure of prices. Money is defined as the medium of exchange and the common measure of prices.

When m_i units of money are exchanged for q_i units of the object, i, then it is always possible to assign a ratio, p_i, such that

$$m_i = q_i p_i \qquad (11)$$

where p_i is defined as the monetary price of the object, i. If in an exchange of the object, j, one has

$$m_j = q_j p_j \qquad (12)$$

then on condition that

$$m_i = m_j \qquad (13)$$

it follows that

$$q_i = q_j p_j / p_i \qquad (14)$$

From a comparison of equations (14) and (2) one has

$$p_{ij} = p_j / p_i \qquad (15)$$

which relates the monetary prices of "i" and "j" with the real price, p_{ij}.

d) Demand and Supply. The equation defining real prices involves three variables, q_i, q_j, p_{ij}. Similarly the equation ~~in~~ defining monetary prices involves the three variables, m_i, q_i, p_i. We have now to consider equations involving only two variables, namely, p_i and q_i. They are of the form

$$q_i = f_d(p_i) \tag{16}$$

$$q_i = f_s(p_i) \tag{17}$$

where equation (16) is called the demand function of the ~~object~~ class of objects, i, and equation (17) is called the supply function of the class of objects, i. The subscripts, "d" and "s", denote demand and supply respectively.

A demand function may be defined as resulting from the summation of demand schedules. A demand schedule may be constructed by asking an individual what is the most he is prepared to pay for 1, 2, 3,... units of the class of objects, i. To answer the question the individual concerned has to consider the equation

$$M = p_i q_i + k \tag{18}$$

where M is the quantity of money at his disposal, $p_i q_i$ the sum of money he would have to pay for q_i units costing p_i apiece, and the remainder for other purposes would be the sum, k. If he prefers to pay p_1 for one unit rather than do without the object, then he has a demand schedule with respect to the object. Again if p_2 is small enough, he may prefer to pay $(p_1 + p_2)$ for two units rather than devote his money to other purposes. Similarly, with regard to a third, fourth, fifth, etc., until the point is reached at which any lowering of price fails to

elicit a preference for a further unit. Now by questioning all individuals one may discover how many demand schedules for the class of objects, i, exist and what they are. Further, though the demand schedules make prices a function of quantities, the summation of demand schedules is effected by asking what quantity is sold at given prices. Hence a complete survey yields a function

$$q_i = f_d(p_i) \qquad (19)$$

such that for any price within a given range there is a determinate quantity of the class, i, sold and, as follows from the definition of demand schedules, with quantities increasing as prices decrease. There is not, however, any correspondence of infinitesimal increments.

As the demand schedules make clear, the demand function varies with variations in the circumstances of individuals. A demand schedule for the class of objects, i, becomes stronger as the quantity of money, M, increases; it becomes weaker as the individual prefers to have a greater remainder, k, to devote to other purposes than the purchase of the object, i. There is not then some one demand function but a category of demand functions; at any time some one demand function exists; but it is subject to change as changes emerge in M and k.

Supply schedules yield a supply function as demand schedules yield a demand function. There are however differences. The question to be put is not what is the most you are prepared to pay but what is the least you are ready to accept for 1, 2, 3,... units of the class, i. The ground of this difference is that

for the seller the relevant equation is

$$k = p_i q_i + M \qquad (20)$$

for the greater the quantity he sells and the higher the price the greater the remainder, k. Further, with respect to goods and services produced, there are distinct categories of supply schedules and supply functions. Supply schedules and functions may be constructed with respect to immediate sales, with respect to present production for future sales and so with respect to a series of future dates, and finally with respect to the initiation of new enterprises. Again, when any function is determined with respect to some one of these categories, it is capable of variation in view of unexpected changes of circumstance. However, with respect to immediate sales there always exists some one supply function

$$q_i = f_s(p_i) \qquad (20)$$

and other existent supply functions may be regarded as potential modifications, more or less proximate, of the immediate function. Within a given range, there is a determinate value of q_i for every value of p_i and, in general, as q_i increases, p_i will first decrease and then increase. The latter assertion is true when the quantity supplied is in the vicinity of the production optimum: there is not so little demanded that the use of the best methods is unprofitable, nor again so much that the best methods available are unequal to handling so great a quantity.

~~From the nature of the demand and supply schedules it appears that the demand and supply functions~~ at least in part ~~are independent. Within the regions, in which the two functions exist, there will be at~~ one solution or ~~least one common point if the most that will be paid by the most~~

Granted the existence of both demand and supply functions valid with respect to concrete conditions, there remains the question of solutions. Now if a solution exists, it occurs where the least eager buyer and the least eager seller come to terms. For the postulate of consistency of prices admits only one price; and, while more eager buyers would pay more and more eager sellers take less, still a higher or lower price, emergent in particular cases, could not be generalized. On the supposition of a higher price, less eager buyers could find sellers ready to sell at a lower price. On the supposition of a lower price, less eager sellers could find buyers ready to buy at a higher price. But at the price at which the least eager buyer and seller come to terms, all more eager buyers and sellers exchange at an advantage, and leave on the markets only buyers who will pay less than this price and only sellers who require more than this price.

When, then, there is a solution to a demand and supply function, it determines a price at which the least eager buyer and the least eager seller come to terms, and at this price there is sold the maximum quantity that could be sold on the supposition of consistent prices and the given demand and supply functions.

a) The Nature of Prices. Prices are the marginal comparative valuations of the community. By a valuation is meant any judgement of appreciation on any grounds with respect to any object. By a comparative valuation is meant a decision with respect to alternatives; of which only one is possible of two events, A and B, only one is possible; the comparative valuation takes one and leaves the other; A is preferred to B.

8. Prices. Any exchange involves a ratio between the quantities of the objects exchanged. When x_i units of an object "i" are traded for x_j units of an object "j", then there is always some ratio, say p_{ij}, such that

$$x_i = p_{ij}x_j \qquad (8)$$

The ratio, p_{ij}, is named a price: it is a ratio between quantities of objects exchanged; and it is defined by equation (8) as mu a multiple of the number of units of the second object, "j". When prices are expressed as multiples of the units of some standard object, that object is named a money; the subscripts "j" of equation (8) may then be understood and one may write

$$x_i = p_i x \qquad (9)$$

so that p_i is the price of the object "i" in monetary units.

The price of the same object at the same time is the same in all exchanges of that object. The statement is methodological. It is a definition of "same object" and "same time." Thus, if what seems to be the same object is traded at different prices, the larger price is larger because it includes, besides the price of something-else the object, the price of something else: for instance, between different places, this price/would be the price of information, transportation, the use of agents, and the like; again, in the same locality, this price would be the price of special services, more agreeable circumstances, ignorance, gullibility, pride of purse, and the like. In other cases the apparent divergence is a divergence between clock time and economic time: the new price has not yet become operative at

a more remote market.

The significance of prices is not limited to formal exchanges. This appears at once if equation (8) is differentiated on the supposition that the price, p_{ij}, is a constant. Then,

$$dx_i = p_{ij}dx_j \qquad (10)$$

which the is the same equation and admits the same varieties as equations (7), namely, the marginal comparative valuations that make the quantities of the productive process determinate. Hence constant prices and determinate quantities in process are concomitant: if the one, then also the other. Which is cause and which is effect, is a further question. But one has at once that if either exists, the other also exists. Now the possibility of determinate quantities is not limited to exchange economies. Hence, the ratios that emerge as prices in exchanges are a more universal reality than exchanges.

In an exchange, then, it is necessary to distinguish between two aspects: there is the aspect of free consent; there is also a pragmatic equivalence between different quantities of different objects. The aspect of free consent consists in the fact that an exchange occurs when there is a coincidence of two opposite decisions; one party prefers x_i of "i" to x_j of "j"; the other party prefers the opposite; and the two come to terms. On the other hand, the aspect of pragmatic equivalence lies entirely in the coming to terms, in the objective effect of the exchange. This objective effect may exist regardless of preferences or free consents. There is a pragmatic equivalence between the work and the upkeep of slaves; it exists independently of their

estimated demand; demand schedules are spending ratios determined with respect to hypothetical prices; and actual prices are the solutions, when they exist, of consequent demand and supply functions. They are the solutions at the point at which the least eager buyer and the least eager seller come to terms, and at which the maximum quantity, that could be sold with consistent prices and given conditions, is sold as a matter of fact.

One now may say in what sense prices are the marginal comparative valuations of the community. The statement does not mean that the least eager buyer and seller are the representatives of the community. The least eager buyer may be a monopsonist; the least eager seller a monopolist; and both monopsonists and monopolists need represent no one but themselves. In a first sense, prices are the work of the community in so far as prices are the method chosen by the community for determining its marginal comparative valuations. In a second sense, prices are the work of the community in so far as this method of determining marginal comparative valuations does not make economics a department of politics, operates more through desire than through fear, does not of itself restrict initiative, provides a continuously effective weapon against producing what is not wanted (in this line the genius of technical experts is without limit), and provides a continuous incentive to produce in the most efficient manner precisely what is wanted.

e) The Nature of Prices. Prices are the marginal comparative valuations of the community. By a valuation is meant any judgement of appreciation on any grounds with respect to any object. By a comparative valuation is meant a decision with respect to alternatives: of two events, say A and B, either alone is possible; the comparative valuation decides in favour of one and against the other; it prefers A to B or B to A. By a marginal comparative valuation is meant a decision with respect to alternative quantities. The issue is not, Either A or else B but not both. One may decide in favour of some of A and some of B, but the more one takes of A, the less one can have of B, and vice versa. Thus the marginal comparative valuation has to decide in favour of some pattern, say x of A and y of B, against an indefinite number of other equally possible patterns.

Now any productive process sets a continuous problem of marginal comparative valuation. From a given stock of materials and with a given quantity of labour and management, a variety of goods and services in a greater variety of quantitative ~~patterns~~ patterns may be supplied. But the more there is supplied of any one, the less there can be of one or more others. Hence on the supposition of definite stocks of materials and a definite quantity of labour and management, goods and services are a matter of alternative quantities. But the supposition may be removed. Count labour and management as negative leisure of various kinds: then the greater the supply of goods and services, the less the leisure and vice versa. Again, the greater the present use of materials, the less is either or both present leisure or future available materials. Thus, the whole productive process is a problem of

alternative quantities, and the problem is re-born every instant.

The solution of such a problem necessarily is a set of marginal comparative valuations. These decisions have to be made. In a Robinson Crusoe economy, Robinson makes them. In an exchange economy everyone, according to the measure of his influence on prices, contributes to making them. In a socialist economy the long-term decisions regarding surplus goods and services are made by a planning authority, while the short-term decisions regarding basic goods and services are left to a price mechanism in so far as this is compatible with the decisions of the planning board and with the necessity of maintaining that board's reputation as an excellent institution with a superb personnel. But no matter what the political organization of the economy, the pattern of the productive process has to be determined; and the determination of that pattern is a set of marginal comparative valuations, of decisions with respect to alternative quantities.

Now it is the characteristic of the exchange economy to make not one but two sets of marginal comparative valuations. A first set occurs in supply, in decisions to produce. A second set occurs in demand, in decisions to buy what is produced. Producers are faced with the problem, With how much of materials, labour, management, capital equipment, are how how much of objects in classes, i, j, k,... to be produced? In the manner explained above, the problem is one of alternative quantities. There is needed the solution of a differential equation of the type,

$$0 = A_i dx_i + A_k dx_k + A_j dx_j \ldots\ldots \qquad (22)$$

where x_i, x_j, x_k,... are the quantities to be determined, and the coefficients, A_i, A_j, A_k,... are all positive, so that more of any one quantity means less of some other quantity or quantities. Now ~~the~~ a solution of this equation may be written in the form

$$x_i = x_z r_{iz} \quad (23)$$

$$x_j = x_z r_{jz} \quad (24)$$

$$x_k = x_z r_{kz} \quad (25)$$

and so on, where x_z is some known quantity and r_{iz}, r_{jz}, r_{kz},... aren known ratios. Such ratios resemble prices in one respect and differ from prices in another. Like prices they are ratios between quantities. Unlike prices, they are not ratios emergent pragmatically in exchanges, but ratios ~~emergent~~ emergent pragmatically in decisions to produce. Both prices and what may be termed production ratios are matters of fact: but the matter of fact that is a price is a ratio between quantities exchanged; and the matter of fact that is a production ratio is a ratio between quantities produced.

Production ratios are sufficient to determine what is to be produced in what quantities in the sense that they give a determinate productive process. But the exchange economy endeavours to meet a further issue, namely, Is the pattern of quantities under production the ~~qua~~ pattern of quantities that happens to be wanted? No doubt producers' marginal comparative valuations select the pattern of quantities that producers prefer; but is it also the pattern of quantities that consumers and workers and owners of the sources of raw materials prefer?

This question the exchange economy answers by rewarding the contributions of property, labour, and management with monetary income for the purchase of the goods and services ~~marginal~~ resulting from these contributions. Now income sets another problem of alternative quantities, for the more one buys of any object, i, the less one can buy of other objects, j, k,... Thus there is another differential equation to be solved, say,

$$0 = B_i dm_i + B_j dm_j + B_k dm_k \ldots \quad (26)$$

where the coefficients are again all positive, so that the more one spends on any object the less one can spend on other objects. The solution of this equation sets up another set of marginal comparative valuations, say,

$$m_i = m_z s_{iz} \quad (27)$$

$$m_j = m_z s_{jz} \quad (28)$$

and so on, where m_z is a known quantity, and s_{iz}, s_{jz},... are known ratios, which may be called spending ratios.

Thus, corresponding to the production ratios determined by the decisions of sellers, there are spending ratios determined by the decisions of buyers. If one could postulate a pre-established harmony guaranteeing the continuous congruence of both sets of ratios -- and such a postulate is implicit in the benevolent forms of socialism in which the planning board supplies just what people want -- then there would be no need for the pedestrian trial and error of supply and demand. Without such a pre-established harmony and without authoritarian solutions on what the standard man is and what he is going to want, whether he likes it or not, supply and demand are inevitable. For supply schedules are production ratios determined with respect

f) The Dialectic of Prices. Within the last hundred years economic thought has moved steadily away from the view that fluid prices and competition are the panacea for all economic ills and the guarantee of ever greater benefits. Underlying this change of thought there is the very simple fact that, while the price system is an exquisite mechanism, still it is not a mechanism into which one can put little knowledge and less wisdom and then reasonably expect to receive notable amounts of both. The price system will strike a balance of any present set of preferences; but it will not make the preferences wise, nor will it make the expectations, on which they are based, turn out to be true. On the contrary, it will find the economic mean, so to speak, of wise and stupid, intelligent and foolish preferences; it will weight true and false expectations with the money that backs them; and with a relentless accuracy it will work out the anomaly one may expect a machine so controlled to yield. In the long run one is presented with a dilemma: either eliminate from every strategic post in the economy the unenlightened freedom of choice that works ruin through exchanges and through prices, or else, if you would preserve that freedom, take effective steps to enlighten it. The alternatives are socialism or an enlightenment of insufficiently enlightened self-interest.

By a dialectic of prices is meant, not subscription to the doctrines of the more absurd philosophic systems, but simply the historical see-saw that marches ineptly from the insufficiency of rugged individualism to the insufficiency of rugged collectivism. In a first period fluid prices and competition bring obvious benefits: markets enforce the consistency of prices; first, the economies of the nations, then the nations of the world, are worked into an unified system; the prices of inconsistency tend to be reduced to a minimum, and with them the payments for mere inconsistency. This enforcement of consistency affects not merely actual processes in themselves but also the actual with respect to the potential; the entrepreneur emerges to set up a procession of new and more efficient combinations of production factors; there is an industrial revolution; there are new ideas, new men, new firms, new ways of doing things, new capital equipment, and, as the acceleration works through the surplus stages to reach the basic, eventually a rising standard of living.

But upon the first period superposes a second. Supply and demand under competitive conditions are harsh masters. The $x_i, x_j, x_k, \ldots$ of quantities produced cannot be greater but may be less than the corresponding $q_i, q_j, q_k, \ldots$ of quantities sold. Further, there is the elasticity of prices to effect moderate adjustments.

But upon the first period there superposes a second. Supply and demand are harsh masters. The elasticity of fluid prices will adjust the $x_i, x_j, x_k, \ldots$ of quantities produced to the $q_i, q_j, q_k, \ldots$ of quantities sold. The trial and error of competition with windfall profits and losses will tend to bring

competitive producers to the acceptance of minimum profits and to an ever better knowledge of an ever more elusive equilibrium. Still competitive producers are ever victims of the unforeseen and, what is worse, even when they do foresee, their foresight will not avail them unless they also combine. Why not combine? Is not the doctrine of competition merely an embargo upon one of the more valuable types of new idea, namely, new ideas in organization? The trouble is that that is true. The age of corporations begins. It re-organizes industry. It organizes labour. It reaches out to tame the individualism of small producers of basic materials for world markets, producers of wheat, of cotton, of coffee. It forms cooperatives to link in united fronts of monopolists and monopsonists little sellers and buyers of any description. Issues cease to be merely economic. They are also political in a stretch of legislation that began with the Factory Acts and does not culminate even with Social Security. For such a growth of political interference has its premiss in the inadequacy of competition and fluid prices to meet economic issues. It cannot but continue until it absorbs the whole sphere of economics or, alternatively, until economics find a new charter. Already, But with the increasing generalization of monopoly and monopsony, both competition and fluid prices are becoming dead letters.

If one attempts to break the impasse of government settling the ~~agreements~~ terms upon which monopolists and monopsonists are to agree, and if one proceeds along the existing line of development, one reaches socialism. There is no longer ownership of means of production, but the use and the production of them is dictated to civil servants by the wise and learned men on the planning board. On the supposition that their wisdom and learning

is not over-balanced by fanatical streak, that the liquidation of classes does not prove necessary, that the universal enthusiasm for the system precludes the use of secret police and terrorism, one may expect an economy that is-free presents the advantage of being free from booms and slumps of the old type

1. Sections of a Monetary Circulation.

Though the main body of any analysis consists in relations between terms that themselves are defined by relations, initially it is necessary to assume or describe some objects or points of reference. Here this preliminary work of assumption and description will consist in marking off sections of a monetary circulation, but we would note at once

9. Circuit Velocities. In the preceding section there were defined two monetary circuits, each involving two independently varying rates of payment and an independently varying quantity of money. The basic circuit involved the rate of basic expenditure, DE', and the rate of basic outlay, DO', and during the interval its quantity of money increased by DM'. The surplus circuit involved the rate of surplus expenditure, DE", the rate of surplus outlay, DO", and during the interval its quantity of money increased by DM". Further the whole system of movements became determinate by the addition of another variable, the cross-over difference, DG. For with DE', DO', DM', DE", DO", DM", and DG, there are equations which determine by identity DR', DI', DD', DS', DR", DI", DD", and DS". To increase the implicational compactness of the analysis, this section investigates relations between rates of payment and quantities of money, between DO' and DE' and on the other hand DM', between DO" and DE" and on the other hand DM".

Now the relation between a rate of payment and a quantity of money is the velocity of money. By definition rate of payment is equal to quantity multiplied by velocity; and it is to be noted that velocity of money is not an additional entity but only a conclusion derived from the fact that rates and quantities may vary independently. Even if there were only one dollar in existence, it would be possible for a number of people to spend a hundred dollars a day; to achieve this, everyone would have to spend the dollar as soon as he received it; and however fantastic, the illustration makes obvious that an aggregate rate of payment of several hundred dollars af a day is possible with one dollar.

However, a circuit velocity is not any velocity of money. Its measure lies not in the number of times a given quantity of money changes hands but in the number of times it makes a determined circuit. A basic circuit velocity is a movement of money from basic expenditure, through basic receipts, to basic outlay, through basic income, back to basic expenditure. A surplus circuit velocity means a traversing the similar surplus circuit. How many times money changes hands on the way is a matter of indifference. The one question is how rapidly it gets back to its starting point. Evidently, circuit velocities are much more definite than monetary velocities in general, for they involve not merely ~~exchange but money doing a given quantity of work~~ any exchanges but definite circular series of exchanges.

It is the purpose of this section to show that in the basic and surplus circuits respectively the quantity of money varies with the magnitude of turnovers and the velocity of money with the frequency of turnovers. The consequence of this theorem will be a correlation of the added quantities of money, DM' and DM", with increases in the rates of outlay, DO' and DO", through the mediation of another variable, namely, the production period.

The theorem will be established by generalizing the analysis of an illustration. Suppose two ship-builders, A and B, who each launch a new ship every 15 days. Suppose further that A has 5 ships under construction at once and so completes a ship every 75 days, while B has 10 ships under construction at once and so completes a ship every 150 days. To eliminate irrelevant differences we may suppose also that each ship is sold as soon as it is launched, that all are sold for the same

price, and so that payments to A are at the same rate as payments to B, and again that payments by A (including initial payments of profits to himself) are at the same rate as payments by B. There are, then, two equal volumes of business: the selling price of one ship every 15 days. On the other hand, the magnitude of A's turnover is half the magnitude of B's; and the frequency of A's turnover is twice the frequency of B's. The magnitude of A's is only half of B's, because when A is paid off for a ship, he has been making payments for its construction over a period of 75 days, payments on a second ship for 60 days, on a third for 45 days, on a fourth for 30 days, and on a fifth for 15 days; on the other hand, when B is paid for a ship, he has been making payments on it for 150 days, on a second, third, fourth, fifth, sixth, seventh, eighth, ninth, and tenth for respective periods of 135, 120, 105, 90, 75, 60, 45, 30, and 15 days. Thus, B's need of circulating capital to bridge the gap between payments to him for his ships and payments made by him transitionally and initially is twice as great as A's need; yet A carries on the same volume of business, because he moves money twice as rapidly.

It will emphasize a few points to change the illustration slightly. Let us suppose that A could sell a ship no oftener than once every 16 days. It still remains possible for him to keep his production period at 75 days per ship, but inevitably his turnover period lengthens to 80 days per ship. He can produce as rapidly as ever, but he cannot sell as rapidly; and in an exchange economy production is production for sale. Further, in the limit decreasing sales effect a reduction of turnover magnitude; if sales dropped to one ship every twenty days,

magnitude; as sales dropped from one ship every 15 days to one ship every 16, 17, 18 days, the first ship-builder, A, might lengthen his turnover period in each case; but when sales dropped to one ship every 19 days, then most probably he would revert to a shorter turnover period of 76 days but with 4 instead of 5 ships under construction at once. Thus, at a first approximation only do turnover periods coincide with production periods. Decreasing efficiency of sales makes the turnover period longer than the production period. But in the limit decreasing efficiency of sales restores what is approximately the minimum turnover period with, however, a decreased turnover magnitude. In the opposite case of increasing sales, the inverse theorem holds. As sales advanced to one ship every 14, 13, 12 days, the first ship-builder would have to put 6 ships under construction at once and then 7 under construction; when supplying one ship every 14 days, the turnover period of 6 ships would be 84 days; with one every 13 days, the period would be 78 days; with one every 12 days, there would have to be 7 ships under construction at once and the turnover period would increase to 84 days.

This analysis has now to be generalized. Every entrepreneur carries on at any given time a certain volume of business. In this volume of business there are two components: a quantity of monetary circulating capital; and a frequency of use of the quantity of monetary circulating capital. The quantity of monetary circulating capital varies with variations in the magnitude of his turnovers; and the magnitude of the turnovers varies with two factors, first, the number of items in production at once and, second, the monetary value of each item. The frequency of use of monetary circulating capital also varies with two factors, first, with the period

of time taken to effect the required physical change that is the entrepreneur's contribution to the productive process and, second, with the additional time that may be needed for the entrepreneur to sell his contribution. Obviously, if the entrepreneur effects no physical change, then the additional time for selling is the whole time of his turnover period.

The next step is to proceed from each entrepreneur to all entrepreneurs. To make this advance it is well to use symbols. With respect, then, to the jth turnover or fractional turnover of the ith entrepreneur in the standard interval of time, let r_{ij} be the aggregate of his initial basic payments, r''_{ij} be the aggregate of initial surplus payments, s_{ij} be the aggregate of transitional basic payments, s''_{ij} be the aggregate of transitional surplus payments. It follows that in the jth turnover or fractional turnover of the interval the ith entrepreneur moves a total quantity of money represented by the sum,

$$r_{ij} + r''_{ij} + s_{ij} + s''_{ij} \qquad (20)$$

If he is engaged exclusively in supplying basic goods or services, the second and fourth elements of the sum are zero. If he is engaged exclusively in supplying surplus goods and services, the first and third elements of the sum are zero. In any case the third and fourth elements of the sum are paid transitionally to other entrepreneurs, while the first and second elements are paid initially, that is, to the entrepreneur's own factors of production. The total sum represents the ith entrepreneur's monetary circulating capital in the jth turnover of the interval. It represents monetary circulating capital both in the sense in which monetary circulating capital exists when an entrepreneur begins business, but then it does not

include profits and similar elements in initial payments, and also in the sense in which monetary circulating capital exists in any turnover subsequent to the first, though then it does include profits and similar elements. Again, it represents monetary circulating capital, not as received from other entrepreneurs or from final buyers in payments to the ith entrepreneur, but as paid by the ith entrepreneur either initially or transitionally; thus the sum represented by expression (20) may be greater or less than the sum paid to the ith entrepreneur in the jth turnover, for the given entrepreneur in that turnover may be enlarging or decreasing the scale of his operations, and in doing so he will be drawing upon a positive excess transfer from the redistributive function or contributing to a negative excess transfer.

Further, expression (20) as a sum gives the monetary circulating capital of the ith entrepreneur in the jth turnover of the interval. But the suffix "j" varies from one entrepreneur to another. One entrepreneur may have 52 turnovers in the interval, another 12, another 4, another 1, another merely the fraction of a turnover, or the last part of one turnover and the beginning of another. Still in any case each entrepreneur has a definite number (which may include fractions) of turnovers during the interval; and the suffix "j", considered in all its instances, supplies an initial indication of what this turnover frequency is. Thus, expression (20) as a sum gives a turnover magnitude; turnover frequencies vary not with this sum of money but with the suffixes "j".

Let us now consider the following summations.

$$o_i n_i = \sum_j r_{ij} \tag{21}$$

$$o''_i n''_i = \sum_j r''_{ij} \tag{22}$$

$$DO' = \sum_i o_i n_i = \sum_i \sum_j r_{ij} \tag{23}$$

$$DO'' = \sum_i o''_i n''_i = \sum_i \sum_j r''_{ij} \tag{24}$$

Here, r_{ij} are the ~~xxx~~ initial basic payments of the ith entrepreneur in his jth turnover of the standard interval. The first equation (21) adds up the initial basic payments made by the ith entrepreneur during the whole interval: it is the sum of initial basic payments, r_{i1}, r_{i2}, r_{i3},... to "j" terms where "j" is the number of turnovers or fractional turnovers. This sum is identical with the product, $o_i n_i$, where n_i is the number of turnovers in the interval of the ith entrepreneur and o_i is the average per turnover of his initial basic payments. Note that o_i differs from r_{ij} as an average differs from a series of exact figures: o_i is the average of r_{i1}, r_{i2}, r_{i3},.... which, multiplied by an exact n_i, gives $\sum_j r_{ij}$.

Next, with respect to equation (23), DO' has already been defined as the aggregate of initial basic payments of the interval. Consequently it is equal to the sum of all instances of $o_i n_i$, and so to the double summation of all instances of r_{ij}. Similarly, one may obtain equations (22) and (24) with respect to initial surplus payments during the interval.

So much for initial payments, r_{ij} and r''_{ij}, per turnover. But there are also transitional payments, s_{ij} and s''_{ij}, per turnover, and the immediate task is to grasp their relation to the ~~xxxx~~ initial payments. The question is, Are the transitional payments the same sums of money as the initial payments or are they additional sums of money? In both cases the answer is affirmative. They are the same sums of money in the sense that eventually every transitional payment becomes an initial payment: one entrepreneur pays another who may pay a third and so on, but eventually that payment is used to be divided up among the factors of some entrepreneur in initial payments. But it is also true that transitional payments are additional sums of money in the sense that the quantity of money required to carry on the business of basic supply is

$$\sum_{i} (r_{ij} + s_{ij}) \qquad (25)$$

and the quantity of money required to carry on the business of surplus supply is

$$\sum_{i} (r''_{ij} + s''_{ij}) \qquad (26)$$

the summations being taken with respect to all instances of "i" in one set of and any/contemporaneous instances of "j". For the quantity of money required is the aggregate of monetary circulating capital. Monetary circulating capital includes not merely money to pay for immediate factors of production but also money to pay for the contributions of other entrepreneurial units. Granted that in the proximate future the money for transitional payments will become money for initial payments, it remains that now it is for transitional payments and at the proximate future date

it will have to be replaced by more other money for transitional payments. There would be a relevance to a consideration of future dates if at future dates transitional payments and the requirement of circulating capital to meet them would vanish. But there is no vanishing, and so no relevance.

The same conclusion may be reached by another route. The function of transitional payments is to shorten turnover periods. Were s_{ij} smaller, r_{ij} in other instances of "i" would have to be greater. Suppose a manufacturer produces 1000 products per month each month of the year; suppose he enjoys monopolist advantages and can force dealers to buy the 1000 products each month; suppose the product can be sold only seasonally, so that in May, June, and July 9000 products can be sold but in the rest of the year only 3000, say, 1000 in August and 250 in each of the remaining months. Let the manufacturer sell at $100.00 a product, so that his turnover is $100,000.00 a month. Now the aggregate of dealers accumulate the products at the rate of 750 a month from September to April, and at the end of April they have on their hands 6000 products for which they have paid ~~$6000~~ $600,000.00; at the end of May they have in stock 4000, at the end of June 2000, at the end of July and at the end of August, zero. Thus, their aggregate circulating capital has to reach $600,000.00 at a minimum during April. But if the manufacturer lost his monopoly advantage so that dealers could buy as they pleased, then he could not run his business of a monthly turnover of $100,000.00; he would have to build the warehouse and increase his monetary circulating capital, provided he kept to the same production rates, for the dealers would refuse to foot the interest bill and other

carrying costs. Thus, in calculating aggregate monetary circulating capital, one is calculating the amount of money required to bridge the gap between the aggregate of final sales and the aggregate of initial payments to all factors from sources of raw materials up to sales to final buyers. ~~and, on the other hand, the aggregate of final sales.~~ The burden of these monetary carrying costs will fall entirely on one entrepreneur if there is only one; and in that case there will be no transitional payments. But if there are several entrepreneurs, and so also transitional payments, then the longer periods of monetary carrying costs are divided, and it is this division that transforms what would be circulating capital for initial payments into circulating capital for transitional payments.

Now to put the matter analytically, consider the equation following equation:

$$\sum_i s_{ij} = \sum_i v_i r_{ij} \qquad (27)$$

This states that with respect to any contemporaneous set of turnovers, "j", the aggregate of quantities of money, s_{ij}, required for transitional basic payments is equal to the aggregate of quantities required for initial basic payments, r_{ij}, multiplied by some factor, v_i. This factor, v_i, will vary in the case of each entrepreneur according to the number of times his contribution to the productive process during one of his turnovers, namely r_{ij}, is found to be the property of some other entrepreneur on its way to final sales. Thus, if the ith entrepreneur is a retailer conducting final sales, v_i is zero. If the ith entrepreneur is a wholesaler with the same turnover period as the retailers to whom he sells, v_i

is unity. If the wholesaler's turnover period is twice that of the retailers to whom he sells, v_i is one half. Universally, entrepreneurs at any instant are carrying some multiple of each r_{ij}; they are carrying that multiple because they have made transitional payments for it and have not yet recovered their payments; and the aggregate of quantities of money required for transitional payments at any time is equal to the aggregate of quantities required for initial payments multiplied by that elusive multiple, v_i.

It is to be noted that v_i varies in two ways: relatively and absolutely. It varies relatively inasmuch as the present r_{ij} is greater or less than the earlier instances now on their way to final sales. It varies absolutely inasmuch as greater efficiency of production and of selling or again less efficiency means that there are ~~more~~ fewer or more instances or fractional instances of r_{ij} between the ith entrepreneur and final sales of his contribution.

The next step is to turn to increments of circulating capital. Let us define the increments, dr_{ij}, ds_{ij}, dr''_{ij}, ds''_{ij}, by the difference between the respective types of initial and transitional payments at the beginning and at the end of the standard interval of time. Thus, if initial basic payments of the ith entrepreneur at the beginning of the interval are r_{ij} and at the end of the interval are $r_{ij'}$, then

$$dr_i = r_{ij'} - r_{ij} \tag{28}$$

with parallel definitions for the other three terms. Hence the aggregate increase of basic circulating capital during the interval is

$$\sum_i (dr_i + ds_i) \tag{29}$$

and if one defines a multiplier, u_i, somewhat similar to v_i, by the equation

$$\sum_i ds_i = \sum_i u_i dr_i \qquad (30)$$

one may write as well for the increment of ~~ma~~ basic monetary circulating capital during the interval

$$\sum_i (1 + u_i)dr_i \qquad (31)$$

and similarly for the increment in surplus monetary circulating capital during the interval

$$\sum_i (dr''_i + ds''_i) = \sum_i (1 + u''_i)dr''_i \qquad (32)$$

where all summations are taken with respect to each "i" or each entrepreneur.

Let us now return to DM' and DM" which were defined as the increments during the interval of the quantities of money available within the basic and the surplus circuits respectively [equations (13) and (14)] and were shown to be equal to the increments in basic and surplus supply respectively at the end of the interval [equations (18) and (19)]. Consider, then, whether one may write

$$DM' = \sum_i (1 + u_i)dr_i \qquad (33)$$

$$DM'' = \sum_i (1 + u''_i)dr''_i \qquad (34)$$

so that the increment in the quantity of money in the basic or surplus circuit during the interval is equal to the increment in basic or surplus monetary circulating capital during the same interval. The condition of the truth of these equations (33) and (34), is that either the entrepreneurs have not yet paid out this increment in initial payments at the moment the

interval ends or else, if they have paid it out, the basic increment has already returned from basic demand to basic supply and the surplus increment has already returned from surplus demand to surplus supply. On the other hand, if the increment has been paid out and has not yet returned to the same supply function, then either it has gone to the opposite supply function or else to the redistributive function. For by the definitions of DD' and DD", equations (11) and (12), the quantity of money in either demand function is the same at the end of each interval as it was at the beginning, and consequently the increments in the demand function are zero. Thus, the truth of equations (33) and (34) is a matter of timing the end of the interval or, alternatively, of correcting the right-hand sides of the equations by subtracting any initial payments made towards the end of the interval but, at least for the moment, lost to the circuit by migration to the other circuit or to the redistributive function. In pure theory it is simpler to suppose timing; in practice corrections would have to be the procedure.

Now it may be noticed that nothing has been said about the velocities of money in the demand functions. The reason is that the analysis pins these velocities down to functions of velocities in the supply functions. Velocities in the supply functions are a matter of turnover frequencies: but the turnover on one side generates income by initial payments and on the other side takes in the expenditure of final payments; if the demand functions spend their income in expenditure more rapidly or more slowly, this automatically shortens or lengthens turnover periods; on the other hand, because DD' and DD" merely make good

the difference between DE' and DI', between DE" and DI", quantity variation is shifted from the demand to the redistributive function. This shift of quantity variation to the redistributive function makes income velocities entirely dependent upon turnover frequencies and so eliminates the need of introducing a further pair of variables.

To collect results, the monetary circulation of the basic and surplus circuits per interval is determined by the rates of payment, DE', DO', DE", DO" and the cross-over difference, DG, and it is further determined by changes in quantity of money per interval, DM' and DM". In terms of these rates the other terms of the system may be defined as follows:

$$\begin{aligned} DE' &= DR' \\ DE'' &= DR'' \\ DI' &= DO' + DG \\ DI'' &= DO'' - DG \\ DD' &= DE' - DI' \\ DD'' &= DE'' - DI'' \\ DS' &= DM' - DD' - DG \\ DS'' &= DM'' - DD'' + DG \end{aligned} \tag{35}$$

though another determination is necessary to distinguish between G' and G" since

$$DG = G''DO'' - G'DO' \tag{5}$$

The question raised in this section was whether it was possible to correlate changes in quantity, DM' and DM", with rates of payment, DE', DE", DO', DO". The answer is had by comparing equations (23), (24), (33), (34). To write them out together, we have:

$$DO' = \sum_i o_i n_i = \sum_i \sum_j r_{ij} \qquad (23)$$

$$DO'' = \sum_i o''_i n''_i = \sum_i \sum_j r''_{ij} \qquad (24)$$

$$DM' = \sum_i (1 + u_i) dr_i \qquad (33)$$

$$DM'' = \sum_i (1 + u''_i) dr''_i \qquad (34)$$

The first two equations state that rates of outlay per interval are a single summation of average outlays, o_i and o''_i, per turnover multiplied by turnover frequencies or, again, are a double summation of exact outlays per turnover. The last two equations state that the circuit increments in quantities of money are equal to single summations of increments in outlays per turnover over the interval, dr_i and dr''_i, plus the multiple of these increments, u_i and u''_i, that are increments in transitional payments. Further, the last two equations are subject to a supposition of timing or else to correction for losses to the circuit in question.

Thus the relation of DO' and DM', or again of DO" and DM", underthe supposition or correction required, is as follows. DM' and DM" increase both transitional and initial ~~(outlays)~~ payments with the part going to the increase of initial payments (outlays) dpending on multiples, u_i and u''_i. The quantitative increments/appear in the ~~rems~~ terms of the series, r_{i1}, r_{i2}, r_{i3},... and again in r''_{i1}, r''_{i2}, r''_{i3},... which are summated to give DO' and DO". On the other hand, DO' and DO" vary independently of these increments, for the number of terms in the series are increased or decreased by changes in turnover frequency, which give the velocity components of DO' and DO". Further, changes in turnover frequency not only affect the number of terms in the

series of outlays, r_{ij} and r''_{ij}, but also affect the multipliers, u_i and u''_i. A more rapid turnover frequency decreases these multipliers while a slower turnover frequency increases them: suppose an entrepreneur with a monthly turnover producing an object that a) three months and b) two months later is sold to final buyers; on the first supposition, u_i is 3, on the second, u_i is 2.

Thus, turnover frequency has a double effect: a higher frequency both increases rates of outlay, DO' and DO", without any increment in circulating capital, and further for initial payments and, at the same time, effects a decrease in circulating capital for transitional payments; inversely a lower turnover frequency decreases rates of outlay and increases requirements for circulating capital. Similarly, DM' and DM" have a double effect: they increase both monetary circulating capital both for initial and for transitional payments, the division between the two types of payment depending on the pattern of turnover frequencies.

9. Circulation Trends. Trends are determinate relations between successive intervals. Circulation trends are determinate relations between the rates of payment and of transfer in successive intervals. With respect to any two successive intervals, $\underline{i}$ and $\underline{j}$, let a suffix, $\underline{i}$, added to a rate of payment or transfer denote the rate during the earlier of the two intervals and a suffix, $\underline{j}$, denote the rate during the later interval. Further, let the symbol, D^2, denote the change of rate from one interval to another so that, for example,

$$D^2O' = DO'_j - DO'_i \qquad (15)$$

and similarly in the case of all other rates.

Nine classes of circulation trend are distinguished according as D^2O' and D^2O'' are positive, zero, or negative. The names of the classes are given most simply in a table, as follows.

	D^2O'	D^2O''
Level:	0	0
Basic Expansion:	+	0
Surplus Expansion:	0	+
Compound Expansion:	+	+
Basic Contraction:	-	0
Surplus Contraction:	0	-
Compound Contraction:	-	-
Basic Disequilibrium:	-	+
Surplus Disequilibrium:	+	-

Thus, a circulation level is a series of intervals in which the

rates of basic and of surplus outlay remain constant from one interval to the next. In the circulation expansions one or both rates of outlay is increasing but neither is decreasing. In the circulation contractions, one or both rates is decreasing but neither is increasing. In the disequilibria one rate is increasing and the other decreasing.

The discussion of circulation trends presupposes some correlation of the variables, DO', DO'', DG, DE', DE'', DS', DS'', DD', DD'', all of which, as far as definitions go, are independent. Hence the notion of a normative trend is introduced and the procedure will be to examine, first, normative trends and, secondly, the various possible departures from the normative.

Normative trends are defined as series of intervals in which D^2O' and D^2O'' are as defined in any one of the nine classes and, as well, the following equations are satisfied:

$$0 = DG = DD' = DD'' \tag{16}$$

$$D^2E' = D^2O' = V'D^2S' \tag{17}$$

$$D^2E'' = D^2O'' = V''D^2S'' \tag{18}$$

and there is supposed some initial interval in which

$$0 = DS' = DS'' = DO' - DE' = DO'' - DE'' \tag{19}$$

so that

$$DS'_n = D^2S'_2 + D^2S'_3 + \ldots + D^2S'_n \tag{20}$$

$$DS''_n = D^2S''_2 + D^2S''_3 + \ldots + D^2S''_n \tag{21}$$

Equations (16), (17), (18) hold in each interval of the series of intervals constituting the trend. Equations (19) need be no more than an hypothetical initial interval providing a

definite frame of reference for subsequent changes. Equations (20) and (21) refer to any nth interval. Finally, V' and V" in equations (17) and (18) are velocity coefficients relating changes in DS', DS", which are pure rates of transfer, to changes in DO', DO", DE', DE", which are rates of payment.

Two main questions arise: first, are the normative trends possible, and what are their conditions; secondly, what departures from the normative trends are possible, and what are their conditions. The significance of these questions is that they provide a systematic method of investigating the possible functional relations of the independent variables over a series of intervals.

The possibility of the normative trends seems to follow from an examination of their definition. Their fundamental feature is that in each interval therate of basic outlay equals the rate of basic expenditure and the rate of surplus outlay equals the rate of surplus expenditure. This follows for the first interval from equations (19) and for subsequent intervals from equations (17) and (18). Such equality of basic and surplus respectively outlay and expenditure is no more than the affirmation of continuity in two circuits interval by interval: it is unreal in so far as it disregards the possibility of lags; but to disregard lags is to remain within the limits of theoretical possibility. On the other hand, equalities of outlays and of expenditure affirm a necessary tendency of the circulation: the general condition of a circulation is that entrepreneurs receive back the equivalent of their outlays, so as to be able to repeat them, and similarly that demand receives back in income the equivalent of its expenditure, so as to be able to repeat it.

It is to be noted that the equality of outlay and of expenditure does not exclude profits. Profits are the part of initial payments which entrepreneurs pay to themselves; as far as the equality of outlay and expenditure goes, that part may be as large or as small as you please. Further, it is not necessary that profits be spent in themselves for expenditure to keep pace with outlay; any expenditure of an equivalent sum not derived from current income will enable expenditure to keep pace with outlay even if profits are not spent; finally, the equality of expenditure and outlay does not disappear simply because expenditure is not keeping pace with outlay, for then outlay may keep pace with expenditure and, in fact, that is what happens, since against falling sales entrepreneurs reduce the scale of their operations.

The second feature of the normative trends is that cross-over equilibrium is maintained. The surplus circuit does not gain from the basic nor the basic from the surplus. Though there is a cross-over, still each circuit carries on as though there were none. This is a theoretical possibility and, obviously, a simplifying condition to be introduced as long as one wishes to examine not the inter-actions of the circuits but the process within each circuit.

The third feature of the normative trends is the neutrality of the demand functions. DD' is zero and DD'' is zero. This does not mean that there are no savings. It does mean that present earnings, which are not spent, are balanced by the present spending of past earnings. With respect to the ~~new~~ basic demand function, this involves an equilibrium between the sanguine people who borrow to meet current expenses and the melancholy who put by for rainy days more than they are ever going to spend. With respect

to the surplus demand function, DD" at zero means that expenditure of funds for surplus goods and services, whether derived from current surplus income or from mobilizations of money ~~by investment~~ in the redistributive function

to the surplus demand function, DD" at zero involves an equilibrium between current surplus income that is not spent and, on the other hand, the movement by investment from the redistributive function to the surplus demand function.

Finally, the increase or decrease per interval of rates of outlay and of expenditure are attributed in each circuit to either of two factors

10. Price and Process Indices. To indicate correlations between accelerations of the monetary circuits and accelerations of the productive process, it is necessary to introduce price and process indices. The present section is a summary presentation of some fundamental definitions.

When any quantity, q_i, of any object, i, is exchanged for any quantity, q_j, of any object, j, then it is always possible to determine a ratio, p_{ij}, such that

$$q_i = q_j p_{ij} \tag{19}$$

In any such case, p_{ij} is the price per unit of the object, j, and this price is measured in units of the object, i.

Transformations from prices measured in one unit to prices measured in another unit are ruled by the equations,

$$p_{ij}p_{ji} = 1 \tag{20}$$

$$p_{ij}p_{jk} = p_{ik} \tag{21}$$

as may be shewn readily by writing down the definitions of p_{ji}, p_{jk}, p_{ik}, on tha analogy of equation (19), and then eliminating in two ways the terms, q_i, q_j, q_k.

Money is a medium of exchange and a common measure of prices. A medium of exchange is any object, from Homeric hides through pieces of eight to digits in a banker's ledger, which under given historical conditions regularly appears as at least one term of the vast majority of exchanges. A common measure of prices is had when all prices are expressed in units of the same object.

With respect to equations (20) and (21), if the object, i, is money and if the objects, j and k, are different and not money,

then by equation (20) the price of the object, j, in monetary units is the reciprocal of the price of money in units of the object, j; further by equation (21) the price of the object, k, in units of the object, j, is equal to the monetary price of the object, k, divided by the monetary price of the object, j.

Prices measured in non-monetary units will be termed real prices; they are to be understood whenever a price is denoted by using a double suffix. On the other hand, it is superfluous to have a suffix to denote money in monetary prices, which henceforth will be defined by the equation,

$$m_i = p_i q_i \qquad (22)$$

where q_i units of any object, i, are exchanged for m_i units of money, so that the monetary price of the object, i, is p_i. Then, the price of money in units of the object, i, is the reciprocal, $1/p_i$; and the price of the unit of the object, j, in units of the object, i, is the quotient of the two monetary prices, p_j/p_i, so that equation (21) becomes

$$p_i p_{ij} = p_j \qquad (23)$$

where p_{ij} is a real price and both p_i and p_j are monetary prices.

As appears from equation (23), the real price, p_{ij}, determines no more than a ratio between the monetary prices, p_i and p_j. Hence monetary prices may all change while real prices remain unchanged; further a change of monetary prices may have two components, a change of real prices and a further change of monetary prices. In this case the second component, and in the former case the total change, is a change in the price of money. Inflation is a fall, deflation is a rise in the price of money.

The consistency of prices is the consistency of equations representing simultaneous exchanges in the same economy. If prices are consistent, it is impossible to obtain directly or indirectly from simultaneous exchange equations two different prices, say p_i and p_i', for the same object, i. Now the consistency of prices may be postulated by taking it as a definition of the object, i, so that any apparent inconsistency is attributed to the presence in the exchange of some further object, j. Then, in reality p_i' becomes a mistaken expression for two prices, namely, $(p_i \pm p_j)$. The price, p_j, of the latent object, j, may be the price of special services, of more agreeable circumstances, of pride of purse; it may be the price of information, of the use of agents, of transportation; it may be the ~~the~~ price of exceptional initiative, of foresight, of knowledge, of great acumen; or negatively it may be the price of lack of foresight, lack of initiative, of ignorance, of gullibility. Generically one may say that p_j is the price of a lack of consistency, but it is always possible in any particular case to give a specific content to this generic interpretation; otherwise it would be impossible to account for the difference in prices, and it would be meaningless to assert that prices tend to be consistent.

The latter view, that prices tend to be uniform throughout the economy though actually they never attain uniformity, is the more common one. It offers the advantage that the analysis of exchange is not loaded with latent objects and so in practical discussions it is a preferable mode of statement. The foregoing statement, however, has the theoretical advantage of greater generality and simplicity; and it is presupposed by the affirmation of a tendency to uniformity which is no more than a tendency to reduce latent prices to a minimum.

departures from consistent prices are to be accounted for. Not a little of the hedonism of the old school of economists was derived from efforts to explain what has to be taken for granted.

If prices are consistent, the price of money is consistent. It follows that there is one price of money for the whole economy, and that inflation and deflation are variations in this one price and so affect all prices monetary prices in like manner.

To define price and process indices, let DZ be any definite rate of payment, which in a first interval is DZ_i and in a second interval is DZ_j, where D^2Z_j is the excess of the latter over the former. Let $\underline{z}$ be any object sold in either of these two intervals and paid for in the rate, DZ. Let p_z be the price, q_z the quantity sold in the first interval; and let $(p_z + dp_z)$ be the price and $(q_z + dq_z)$ be the quantity sold in the second interval. Thus, summing with respect to all instances of $\underline{z}$, one has

$$DZ_i = \sum p_z q_z \tag{24}$$

$$DZ_j = \sum (p_z + dp_z)(q_z + dq_z) \tag{25}$$

$$D^2Z_j = DZ_j - DZ_i = \sum (p_z dq_z + q_z dp_z + dp_z dq_z) \tag{26}$$

Let the price indices and process indices of the two intervals be P_i, P_j, DQ_i, DQ_j, and let the definitions of these indices be the solution of the equations

$$P_i DQ_i = DZ_i \tag{27}$$

$$P_j DQ_j = DZ_j \tag{28}$$

$$P_j = P_i + DP_j \tag{29}$$

$$DQ_j = DQ_i + D^2Q_j \tag{30}$$

$$DQ_i DP_j = \sum q_z dp_z \tag{31}$$

$$P_i D^2Q_j = \sum p_z dq_z \tag{32}$$

where, in general, the last two equations must be more approximations.

It follows that the definitions of the indices are in themselves no more than approximations. The reason is not difficulty of obtaining information about p_z, dp_z, q_z, dq_z, nor the fact that the prices may have to be averaged, though both those difficulties also exist. None the less even with perfect information and no averaging, the definitions of the indices remain approximate, unless the four sums of $p_z q_z$, $p_z dq_z$, $q_z dp_z$, $dp_z dq_z$ happen to be a fourfold proportion; for in fact these four sums are by the defining equations made equal to $P_i DQ_i$, $P_i D^2 Q_j$, $DQ_i DP_j$, $DP_j D^2 Q_j$, which are a fourfold proportion; but there is no ground to expect that the four summations will also be in proportion. Hence equations (31) and (32) are only approximate and so the definitions are approximate.

Though for six unknowns there are the six equations (27) to (32) still the fourfold proportion introduces a condition of consistency and so leaves an infinity of solutions possible. Hence solution is by introducing any arbitrary base, say 100, as the value of P_1. This determines DQ_i by (27); thence follow approximate determinations of DP_j and $D^2 Q_j$ by (31) and (32), and of P_j and DQ_j by (29) and (30); a check and adjustment of P_j and DQ_j follows by (28).

The significance of the indices is that it provides a differentiation between increments in prices and increments in quantities sold as the rate of payment moves from DZ_i in one interval to DZ_j in the next. The corrected value of DP_j indicates the price increment, and that of $D^2 Q_j$ indicates the quantity increment.

Thus, the condition that an increment in the rate of payment

is accompanied by a proportionate increment in quantities sold is that DP is zero; on the other hand, the greater is DP, the greater is the inertia of the process of goods and services against circuit accelerations.

The process indices disregard all process change that has no aggregate quantitative manifestation. Increments in one quantity may cancel against increments in other quantities when the two sets are of opposite sign. Similarly, changes in quality do not appear in the indices when the emergence of the new is balanced by disappearance of the old. Hence D^2Q is an increment per interval in the aggregate of price-weighted quantities.

10. The Cross-overs. The distinction between the two circuits involves a cross-over from basic outlay to surplus income and from surplus outlay to basic income. It has been argued that unless these two cancel (DG = 0) the process is apt to be submitted to an expansion of one circuit and an a contraction of the other. We have now to investigate the conditions of their cancelling.

Let all recipients of income, whether basic or surplus, be divided into groups of n_i members each. Let each member of the same group receive approximately the same income per interval, o_i, and devote the same proportion of income, g_i, to expenditure at the basic final market. In any later interval let the situation have changed to the extent that in group, i, numbers have increased by dn_i, income per interval by do_i, and the proportion of income devoted to basic expenditure by dg_i. Then basic income per interval, DI', and the increment of basic income per interval, D^2I', may be expressed as summations with respect to all instances of the groups, i.

$$DI' = \sum g_i o_i n_i \qquad (16)$$

$$D^2I' = \sum(g_i o_i dn_i + o_i n_i dg_i + n_i g_i do_i + g_i do_i dn_i + o_i dn_i dg_i + n_i dg_i do_i + dg_i do_i dn_i) \qquad (17)$$

If DI' is visualized as a rectangular solid, then D^2I' may be visualized as the elements added to expand the rectangualar solid, three plates added to three faces, three bars added along three edges, and a little cube added to the corner.

To estimate the relative importance of the various components of D^2I' one may note the following. As one passes from group to group one finds that as o_i increases, n_i decreases and also g_i decreases: the greater the income, the fewer that receive it and the smaller the proportion of it spent on consumer goods and services. With regard to the increments, dg_i is always quite small since it is a change in a proper fraction, dn_i may be quite large as employment increases of or decreases, and do_i may be extremely large amounting in the aggregate to billions of dollars as an economy moves from the peak of a boom to a slump or from a slump to an all-out war effort. Further, dg_i is usually opposite in sign to do_i: savings increase somewhat more rapidly than aggregate income; however in the highest income brackets changes in income are apt to be cancelled by the opposite changes in dg_i, while as income decreases this tendency is more and more reduced until in the lowest brackets the effect of sax increased rates of savings may be small. Hence, for notable changes in basic income, one has to llok look to the factors $g_i o_i dn_i$ and $n_i g_i do_i$ and, with respect to these two, one may discount/increments to income in groups which already are spending all they intend/to spend on consumer goods and services.

positive

or can manage

Now the condition of cross-over equilibrium ($DG = 0$) may be written from equation (11) in the form

$$DO'/DO'' = (1 - G')/G' \qquad (16)$$

where

$$DI' = (1 - G')(DO' + DO'') \qquad (9)$$

Hence when the ratio, DO'/DO'' is decreasing, G' has to increase, and when DO'/DO'' is increasing, G' hasto decrease to satisfy the

11. Trends. Trends are determinate relations between successive intervals. Two types of trend are considered: process trends and circulation trends. Both have the same general form, namely, on certain suppositions with regard to intervals 1, 2, 3,... i, j,... n, the quantitative process of goods and services or the circulatory process of payments and transfers behaves in such and such a fashion. Nine classes are distinguished in each type of trend; names of the classes and their definitions are to be had in the following table.

Process Trend:	DQ'	DQ''
Circulation Trend:	DM'	DM''
Level:	0	0
Basic Expansion:	+	0
Surplus Expansion:	0	+
Compound Expansion:	+	+
Basic Contraction:	-	0
Surplus Contraction:	0	-
Compound Contraction:	-	-
Basic Disequilibrium:	-	+
Surplus Disequilibrium:	+	-

Thus the nine process trends are defined according as process indices DQ' and DQ'' are zero, positive, or negative. Similarly, the nine circulation trends are defined according as circuit increments in the quantity of money, DM' and DM'', are zero, positive, or negative. In each case the nine classes are complete enumerations with respect to all possible values of

two variables. If over a series of intervals, DQ' is zero and DQ" is positive, the process trend is a surplus expansion. If over a series of intervals, DM' is negative and DM" is positive, the circulation trend is a basic disequilibrium.

To complete the definitions of the process trends, it is necessary to specify the meaning of DQ' and DQ". Let then DQ' be defined by the rates of basic expenditure, DE'_i and DE'_j, in any two successive intervals, i and j. Let DQ" be defined by the rates of surplus expenditure, DE''_i and DE''_j, in the same two successive intervals. Wti With DE'_i, DE''_i, DE'_j, DE''_j determinate with respect to prices and quantities, calculation of DQ' and DQ" proceeds as outlined in equations (83) to (91) in the preceding section.

It is to be observed that process trends are merely functions of aggregate weighted quantities sold at the basic and the surplus final markets. Differences that do not appear in the process indices do not affect the trend. Hence if one quantity increases and another decreases proportionately to the respective weights, indices which regard aggregates are unaffected. Accordingly any amount of qualitative change may be going on without any change of trend. It follows that the process level differs enormously from the neo-classical stationary state. The latter is a pattern of unchanging routines. But the process level, at least in theory, is compatible with an industrial revolution moving along in a strait jacket.

The circulation trends are functions of changes in the quantity of money available in the basic and the surplus circuits. Their general character may be deduced from the circuit equations.

However, it will facilitate the course of such a deduction to set down at once a few general theorems: a) there is a concomitance of variations in initial, transitional, and final payments of uniformly specified types; b) the rates of absorption, DA' and DA", are no more than incidental adjustments; c) with D^2S' at zero over a series of intervals, DS' may be positive, zero, or negative over the same series of intervals; and the same holds for $D^2S"$ and DS"; d) the difference between turnover differences of transitional payments made and turnover differences of transitional payments received is an adjustment factor; e) turnover frequencies are resultant, and not initiating, factors of acceleration.

First, there is a concomitance of variations in initial, transitional, and final payments of uniformly specified types. The ground of the concomitance lies in the definitions of the terms: transitional payments are initial payments in a process of double summation; final payments are initial payments at the end of the double summation. Any one of the three may begin to move out of step, but unless the others follow, then it has to revert to its original level. There will be lags proportionate to the production-and-sales period between different rates. Transitional movements may change their route so as to involve more or fewer transitional payments, and so give greater or less aggregates of transitional payments. But apart from incidental xd differences of such a nature, variations are concomitant. There can be no systematic divergence over a series of intervals with one rate increasing and another zero constant or decreasing.

Secondly, the rates of absorption, DA' and DA", represent no more than incidental adjustments. They were defined by the equations,

$$DA' = DI' + DD' - DE' \qquad (67)$$

$$DA'' = DI'' + DD'' - DE'' \qquad (68)$$

Now DI' and DI" represant money earned per interval; DE' and DE" represent money spent per interval; if DD' and DD" were defined as the excess of money spent and not earned over money earned and not earned per interval, then DA' and DA" would always be zero. It is desirable, however, to define DD' and DD" in terms as of savings, and not to count as savings the money earned at the end of one interval and spent at the beginning of the next. Thus, DA' and DA" represent the excess of the carry-over from this interval to the next over the carry-over from the previous to the present interval. As such, they are not systematic factors in a trend but incidental adjustments.

The third theorem follows immediately from definitions. DS' is a turnover sum, D^2S' is a turnover difference. As long as DS' is the same interval after interval, D^2S' will be zero. The same holds for DS" and D^2S''.

The fourth theorem regards the significance of T' and T" which are defined by the equations,

$$T' = D^2t' - D^2T' \qquad (93)$$

$$T'' = D^2t'' - D^2T'' \qquad (94)$$

Evidently, on the supposition of synchronized turnovers, T' and T" are always zero: transitional payments made are then identical with transitional payments received. Without synchronized turnovers, T' and T" tend to zero as to a statistical average. Hence within intervals they represent factors of adjustment.

Now D^2T and D^2t do not refer to the same turnovers. This may be seen by inspecting the following equations and checking by equations (24) and (36).

$$D^2T = \sum_i \sum_1^n dT_{ij} = \sum_i \sum_1^n (T_{ij} - T_{ij'}) \qquad (53 \ \& \ 22)$$

$$= \sum_i (T_{in} - T_{i0}) \qquad (95)$$

where the constant, K, has been omitted and the limits result from the fact that

$$dT_{i1} = T_{i1} - T_{i0} \qquad (22)$$

On the other hand, since

$$dt_{i1} = t_{i2} - t_{i1} \qquad (32)$$

it follows that

$$D^2t = \sum (t_{in'} - t_{i1}) \qquad (96)$$

where n' is written for (n + 1) and K again is omitted. Hence with respect to three successive intervals, D^2R and D^2T refer to the last turnovers of the first and second, while D^2t, D^2O, and D^2S refer to the first turnovers of the second and third. For turnovers, O and n, are the last turnovers of the first and second intervals, while turnovers, 1 and (n + 1), are the first turnovers of the second and third intervals.

With regard to T' and T", then, on the supposition of synchronized turnovers and a constant acceleration, the difference between turnover differences of transitional payments, D^2T and D^2t, will be always zero. Without synchronized turnovers but with constant acceleration, T' and T" tend to zero as a statistical average. Finally, in so far as the acceleration is changing, T' and T" tend to some positive or negative quantity as statistical averages. However, changes of acceleration are incidental adjustments; they are not trends but intensifications or reversals of trends.

The fifth theorem is that changes in turnover frequency are resultant rather than initiating factors of acceleration. Granted an acceleration is in progress, one may expect a greater efficiency of production and sales to supervene and intensify the acceleration. For with the acceleration in progress, opportunities to introduce improvements multiply and selling is strong. On the other hand, without an acceleration in progress, opportunities to introduce improvements are restricted while the weakness of sales ~~discourages expensive modifications~~ prevents the greater efficiency in selling necessary to convert shorter production periods into shorter turnover periods. Finally, in a deceleration of the process one may expect turnover frequencies to diminish; sales are falling; and reduced rates of production prevent the most efficient use of means of production.

To turn now to the circulation trends. It has been shewn already that

$$DM' - DA' = D^2R' + D^2T' \qquad (71)$$

$$DM'' - DA'' = D^2R'' + D^2T'' \qquad (72)$$

Since the two equations are similar, one may discuss their implications without distinguishing between basic (') and surplus (") rates. On the supposition that DM is positive over a series of intervals, then, since DA is an incidental factor and since D^2R and D^2T keep pace, it follows that both D^2R and D^2T will be positive over the series of intervals; again, if DM is negative, then D^2R and D^2T will be both negative; and if DM is zero, then D^2R and D^2T will average zero.

The conclusions hold no matter what the reason for DM being positive, zero, or negative. It may be any solution of the

equations defining DM' and DM", namely,

$$DM' = DS' + DD' + DG$$

$$DM'' = DS'' + DD'' - DG \qquad (14)$$

and so an acceleration of the circulation may be due to excess transfers to supply, DS, to excess transfers to demand, DD, or to a cross-over difference, DG. The relative importance of the three in effecting accelerations of the circuit may be estimated as follows. With DD', DD", DG each at zero, entrepreneurs are receiving back their aggregate outlays (including the payment of profits to themselves). In such a situation, demand is neutral and prices may be termed normal. On the other hand, with DD', DD", DG above or below zero, there is a strengthening or weakening of aggregate demand independently of supply; demand is not neutral but asks for more or for less; and since this asking is independent of supply, it can effect nothing but an upward or downward movement of prices (unless it is very slight and met out of inventories). Now the upward or downward movement of prices will stimulate at once the whole series of speculative producers to increased or decreased scales of operation and, in the main, such changes in the scale of operations involve excess transfers from or to the redistributive function. In current practice changes in the quantity of money in the circuits are changes in the volume of short-term loans; and short-term loans are made not to purchasers but to producers; they affect not DD' and DD" but DS' and DS". It would seem that in general DM' and DM" depend on DS' and DS", while the role of DD', DD", DG is to act as stimulants to the scale of operations of agents of supply.

By combining equations (65) and (93), (66) and (94), one obtains the correlations of turnover differences in the form,

$$D^2R' + D^2S' = D^2O' + T' \qquad (9\beta)$$

$$D^2R'' + D^2S'' = D^2O'' + T'' \qquad (9\delta)$$

where T' and T'' are adjustment factors discussed above. Since the equations are similar, the argument may disregard the distinction between basic (') and surplus (''). With D^2S at zero over a series of intervals, D^2R and D^2O tend to be equal. But with D^2S at zero, DS is constant interval by interval; however this constant may be positive, zero, or negative; so that D^2R and D^2O tend to be equal whether increasing, averaging zero or decreasing. Hence with D^2S at zero, according as DS is positive, zero, or negative, one has D^2R, D^2T, D^2t, and D^2O similarly positive, averaging zero, or negative. The circulation is expanding, level, or contracting interval after interval. Entrepreneurs are receiving back in final payments (and transitional payments) all that they are spending in initial payments (and transitional payments); no matter how great the aggregate income they are paying to themselves, it keeps coming back. To intensify or reverse any such trend, DD', DD'', DG may provide stimulation, but, in the main, the work will be done by a positive or negative D^2S which effects a change in DS.

So far the argument has dealt with turnover differences. Now to be considered are the turnover sums, DO', DO'', DR', DR''. The first point to be considered is their relation to the turnover differences, D^2R', D^2R'', D^2O', D^2O''. The latter, then, determine the difference between the aggregates of last or first turnovers in successive intervals. This may be seen by inspecting the following

So far the argument has dealt with turnover differences. One now has to ask how do DO', DR', DO", DR" behave when with DM positive, zero, or negative interval by interval over a series of intervals, D^2R, D^2T, D^2t, D^2O are similarly positive, averaging zero, or negative. New Immediately one notes that DO and DR are turnover sums and so vary either from variation in turnover magnitude or from variation in turnover frequency; thus because

$$DO' = \sum_{i} \sum_{1}^{n} i'_{ij} \qquad (46)$$

DO' may be greater either because instances of i'_{ij} are greater or because there are more instances; and there may be more instances of "j" from increasing turnover frequency, or more instances of "i" because additional new units of enterprise exceed liquidated units; and in the latter case the turnover frequency of the additional units may raise or lower the existing average frequency per unit of turnover magnitude.

Now with respect to change of turnover magnitudes, which includes the excess, positive or negative, of new units of enterprise over liquidated units, the utn turnover differences D^2R, D^2O, etc., give information only on the first and last turnovers of each interval. However, we have a source of information with g regard to intermediate turnovers from in the conclusion that, in the main, DM' resulted from DS' and DM" from DS". For DS' and DS" are also turnover sums, double summations of m_{ij}, which is the difference between payments made in the later turnover and payments received in the earlier turnover with respect to all turnovers. For DS to be positive, zero, or negative, interval by interval over a series of intervals, means that the aggregate

of turnover magnitudes during an interval are above, or at, or below the average of the previous interval. Hence DO' and DR', DO" and DR", as far as turnover magnitudes are concerned, have their trend determined as increasing, averaging zero, or decreasing according as DM' and DM" respectively are positive, zero, or negative.

As to the change in turnover frequencies, the change in the general average from the replacement of new firms by old firms by new firms and the emergence of additional new firms cannot be determined generally. On the other hand, as argued previously, there should seem to be a tendency for turnover frequency to increase during an expansion but be hampered and decrease during a contraction. However, the expansions have to be both process and circulation expansions and, similarly, the contractions have to hold in both orders; for turnover frequency is a matter of the efficiency of both production and sales. In situations that are neither double expansions nor double contractions the probabilities of change in turnover frequency are less determinate: a circulation level is less incompatible with change

Granted the existence of both demand and supply functions valid with respect to concrete conditions, there remains the question of solutions. Now no solution exists, there is no exchange, no price, and the quantity sold is zero, if with respect to any determinate quantity the most offered by the most eager buyer is less than the least required by the most eager seller. On the other hand, solutions exist if the most eager buyer offers more than/or at least as much as the most eager seller requires. However, the postulate of consistent prices admits only one solution; further, the same postulate requires the elimination of the prices of all objects distinct from the objects of the class, i, so that the one solution required by the postulate is a minimum price. But a minimum price is had

eager buyer is greater than the least that the most eager seller is ready to take. Further, on the postulate of consistent prices, there will be only one solution or one common point to the two functions. And as prices of ignorance, gullibility, etc., are eliminated, this solution lies in the region in which the least eager buyer and the least eager seller come to terms and so in the region in which the maximum quantity that can be sold is sold.

e) The Nature of Prices. Let the term, valuation, denote any appreciation on any grounds of any object. Let a comparative valuation denote a decision with respect to alternatives: of two events, A and B, only one is possible; the comparative valuation is the decision, A is preferred to B. Let a marginal comparative valuation denote a decision with respect to alternative quantities of alternative objects. The question is, How many units of A and how many of B is preferable to any other combination when the possible numbers, say x and y, are defined by the equation

$$Ax + By = C \qquad (22)$$

As appears from the foregoing account of supply and demand, prices result from the marginal comparative valuations of the community. The demand and supply schedules are solutions of equations (18) and (20) which correspond to equation (22): M is the constant C, Ax is p_1q_1 on each of a series of hypotheses regarding p_1, and k is By

11. Systematic Costs and Profits. Price and process indices provide a method of denoting the concomitance or divergence of the productive process and the monetary circulation. It is now necessary to consider the inter-action of the two and, to begin, it will be well to consider certain general phenomena of particular importance in a "capitalist" economy.

Systematic costs and profits are costs and profits in much the same sense as forced savings are savings, that is, they bear an important relation to costs and profits in individual units of enterprise but they are variables not of any individual unit but of the total situation in any exchange economy. Thus by systematic costs and profits are meant neither the costs and profits of the accountant nor again the windfall profits and losses of the equilibrium analyst. For the accountant costs and profits are a division of payments made with a relation to payments received. For the equilibrium analyst windfall profits and losses are accidental variations in the distribution of the receipts of industry and commerce. On the other hand, systematic costs and profits are a division of aggregate initial payments to factors of production, and the division is based upon the tendency of the products to effect an acceleration of the total economy. The significance of this division is that the resultant acceleration tends to reduce and in the limit to reduce to zero whatever systematic profits may have existed.

The aggregate of initial payments in any interval is the sum $(DO' + DO'')$ which is identical with total income $(DI' + DI'')$. Systematic profits are defined as $K.DO''$ where K has any value in

the range 0 to 1 inclusively, and is defined as the fraction of surplus initial payments made with respect to surplus products which are supplied not to replace or maintain worn out or obsolescent equipment but are increasing the capacity and efficiency of old units of enterprise or fitting out new units.

Now systematic profits have a twofold significance. First, with regard to the monetary circuits, it is plain that to maintain rates of payment at their acquired levels total income has to be spent either in itself or in its equivalent: were DE' + DE" to drop permanently behind DO' + DO", there would result a continuous contraction of the ~~pree~~ circulatory process until it was reduced to zero. However, it makes a notable difference whether total income is being spent for basic products and the maintenance and replacement of capital equipment or, on the other hand, there is over and above that expenditure an element, K.DO", which purchases additional capital equipment. To begin, there is the psychological difference: in the former case people are merely making a living; not matter how high their standard of living may be already and no matter how much they are adding to it, still they are adding only to their enjoyment and not to their ownership of industrial and commercial ~~wealth~~ sources of wealth; this runs counter to current ideas on the "successful man" who can emerge only when there is in income an element, K.DO", which ministers to the increase not only of living standards but as well, over and above all increase of living standards, to the increase of ownership of means of production. Besides this psychological difference, there is a monetary difference: K.DO" yields an equal K.DI"; now there is no necessity that the recipients of K.DI" should also be the

spenders of K.DI"; the continuity of the circuits at their acquired levels is assured as long as DD" remains zero, so that the recipients of K.DI" may devote this part of their income to the purchase of redistributive goods, to real estate or stocks and securities, to augment their financial prestige or to increase their financial power, as long as this subtraction from the circuits is balanced by an equal and opposite movement from the investment market to the purchase of surplus goods and services.

Besides the psychological and the monetary significance of systematic profits, there is also a real significance. The products generating systematic profits fit out new firms and expand old firms. In so far as this constitutes a net increment of capital equipment, by over-balancing the effects of the current rate of liquidation of units of enterprise, the productive process tends to accelerate in long-term style. In so far as the net increment occurs in surplus units, the surplus stage is due for acceleration immediately and the basic stage is due for a still greater acceleration ultimately. In so far as the net increment occurs in basic units, the basic stage is due immediately for an acceleration.

However, these real effects of systematic profits have a repercussion on systematic profits themselves. In the long run, as has been shown, the acceleration of the productive process involves a decrease of K as the portion of surplus activity devoted to maintenance and replacement increases with increasing capital equipment. But besides this long-term effect, there is an immediate effect upon the ratio of systematic profits to total income. If this ratio is denoted by H, then

equipment. However the movement towards this ultimate position is full of interest. Let H be the ratio of systematic profits to total income, so that

$$H = K.DO''/(DO' + DO'')$$

or

$$H/K = DO''/(DO' + DO'') \qquad (33)$$

If now we follow through the emergence and development of a long-term acceleration, we find a first period in which H is increasing, a second period in which H is decreasing, and a third period in which H is zero. As the long-term acceleration begins, DO' is constant but both K and DO'' are increasing; H increases as does the product of K and DO'' in the numerator with a slight drag because of the presence of DO'' in the denominator. Further, this first period lasts as long as DO'' is increasing more rapidly than DO', that is, as long as the efforts of the surplus stage are more on equipping the surplus than the basic stage of the process; for then H is still increasing though less and less rapidly as DO' approaches the rate of acceleration of DO''. However, every increment in the surplus stage stands/point-to-line at least to increments in the basic stage and unless the expansion of the surplus stage is more blundering, sooner or later DO' will begin to increase more rapidly than DO'' while at best K is constant. Then H begins to decrease, and the more successful the expansion of the basic stage, the more rapid the rate of decrease. Finally, K begins to decrease, as the surplus stage has to devote more of its efforts to more maintenance and replacement; and if the long-term acceleration works itself out K returns to zero and H has to reach zero ahead of K.

The single condition to this movement (if we abstract from the favourable balance of foreign trade and from deficit government spending, which will be discussed later) is that there does not supervene a rate of liquidation of old or new firms to eliminate from systematic profits their tendency to accelerate the productive process. Thus there is possible a dynamic situation in which the surplus stage of the process is yielding capital equipment that generates systematic profits but not yielding an aggregate increment of consumer goods that reduces the ratio of systematic profits to total income. This situation is most easily verified in an industrial revolution that is the work of "new" men: because an industrial revolution is in process, the new capital equipment is simply displacing old equipment; because the industrial revolution is the work of "new" men, the displacement of old equipment occurs not as a cost of obsolescence on existing firms but as fresh investment constituting the emergence of new firms. On the other hand, the more industrial and commercial enterprise is in the hands of vast corporations which by their command of talent and resources stand in a virtually impregnable position, the less would seem the possibility of evading the effects of variations in systematic profits by a concomitant rate of liquidations.

Further, it may be noted that it would be absurd for the great corporations to attempt to plan an elimination in the variations of the ratio of systematic profits. For while the planning itself would be possible, perhaps, the objective of the planning would be manifest stupidity: what would be planned would be a steady flow of surplus products that did not yield their increment

in basic products; it would be a matter of devising better machinery and more efficient organization, of effecting both, and then of using them as though they were not better than what already existed. It would be a planned economy in which the idea of the plan was to effect a maximum change in the surplus stage while keeping the basic stage in a relative status quo.

1. Economic Activity.

Economic activity is an ordered and variable process towards a standard of living.

The fundamental fact is the standard of living. In any given area over any period of time there exists some stardard of living, some quantitative rate at which people are obtaining food, clothing, shelter and the implements of amusement, art, education, law, medicine, politics, religion, research and war. Measurement of this standard is not possible immediately, for as yet no yard-stick has been determined. But even without such a determination, we can and do know that there is such a thing as a standard of living, that it is quantitative now standing at a higher level and now at a lower, and that it is a rate, not "goods and services" but a "flow of goods and services," not a "so much" but a "so much every so often."

To effect the standard of living continuous human effort is required. For between these potentialities of nature, whether physical, chemical, vegetal, animal or human, and on the other hand the standard of living, there is a gap to be bridged. Even South Sea islanders have some effort to make. Such an effort is termed economic activity. It may be anything from digging ditches to signing slips of paper, from tending machines to clipping coupons. But it exists ~~invariab~~ always, for the world's work is never done. Like the standard of living itself, it is not a "so much" but a "so much every so often." It is an endless series of repeated routines. It is a flow, not indeed

A Method of Independent Circulation Analysis.

Circulation analysis is a set of definitions, postulates, and deductions relevant to a monetary circulation. Such a systematic construction of terms and theorems might be worked out within a wider theoretical context. Thus, its concepts might be derived from the concepts of value theory; its postulates might be modifications of postulates regarding value; its deductions might be special cases of the more general deductions concerning scarce objects with alternative uses. Such a procedure offers the obvious advantages of theoretical unity; analytic apparatus is all in one large, nicely articulated, and agreeably complicated piece. However, like the armour of Saul, it is apt to be too cumbrous for David. Just as one can study Euclidean geometry without the slightest suspicion that it is a particular case of a more general geometry, so also one can attempt an independent circulation analysis in which the formation of concepts, the choice of postulates, and the seriation of deductions are not dictated by the higher exigences of value theory but by the more immediate and more germane considerations of the monetary circulation itself. In that fashion one would obtain an independent analytic tool which, from its greater compactness and simplicity, would perhaps prove more efficient in the solution of certain types of problem. No doubt, once such an independent tool were constructed and found successful, theorists would be troubled by profound questions regarding the possible developments that might result from the mutual inter-action of equilibrium and circulation analysis. But such thoughts cannot occupy us here. It is enough that we attempt to indicate by concrete example a method of independent circulation analysis in the belief that it offers special advantages in handling some economic issues.

1. Frame of Reference. The productive process of an exchange economy operating in a closed area, offers the most favourable starting-point for a study of a monetary circulation. For a circulation is not so much a rotational movement as an aggregate of instantaneous events, namely payments, which stand in circular series of relationships; and while a productive process is rectilinear rather than circular, it does provide, by the technical dependence of each successive stage of production upon previous stages, a correlative and almost palpable ground for series of relationships between payments. Thus, each element of stage of the material process has its proper outlay, payments of immediate factors of production in wages and salaries, rents and royalties, interest and dividends, depreciation charges and undistributed profits. Next there is the building up of prices as these elements are united materially into a growing product and simultaneously the outlays upon the single elements are added into a growing volume of transitional payments from subsequent to previous units of activity. At the end of these lines of production are the aggregate finished products, the gross receipts of industry and commerce, and the gross aggregate expenditure of final buyers.

All such payments form a class by themselves. They stand in a net-work that is congruent with the technical net-work of the productive process. They recur with the recurrence of its routines. In the main they vary with variations in the volume of these routines. But, above all, their connection with production is immediate: they emerge not through repercussions or as responses to external stimulus but are, so to speak, the immanent manifestation of the productive process as a process of value. For in an exchange

economy production is not a merely technical affair of designing, assembly, processing, and distribution; intrinsically it is an economic affair, an expression of preference and choice, and so not merely production, as some technical experts seem to fancy, but production for sale, production in view of and at every instant adapted to payments.

Payments, then, forming a net-work congruent with the net-work of the productive process, shall be termed operative, and from among them two boundary classes are selected: the aggregate receipts that are also the expenditure of final buyers; and the aggregate outlays that are also the income of factors of production. Let DR and DE be the aggregate sums of money that in a given interval are receipts and expenditure; let DO and DI be the aggregate sums of money that in the same interval are outlay and income; so that by definition DR = DE and DO = DI. Further, let us say that money held in reserve for expenditure is in the demand function of the economy; and that money held in reserve for outlay, or on its way to outlay through transitional payments, is in the supply function. Note, however, that it is not assumed that DO is identical with DR, or that DE is identical with DI. Such an assumption, in general, would be contrary to fact. Not only are there other exits and entrances to the supply and demand functions besides outlay and expenditure, receipts and income; but even if these other lines of communication did not exist, it would not be clear that all the receipts of an interval become outlay in the same interval, or that all the income of an interval becomes expenditure in the same interval.

Besides the operative payments described above there are in any economy other, redistributive payments. Such payments we define negatively; they form a remainder class of all payments that are not operative, that do not stand in the net-work congruent with the net-work of the productive process. The existence of this remainder class follows from the fact that property is a broader category than the current supply of goods and services. There are things never produced, such as an economy's endowment of natural resources. There are other things that, though produced, already have become the property of final buyers. Most of these may be sold, and the payments for them will be termed redistributive; for on the one hand such payments change titles to ownership, redistribute property, but on the other they are not operative, not intrinsic to the forward movement of industry and commerce. It is to be observed that such payments remain redistributive even when they mark the re-entry of property into the productive process; to re-enter the process is one thing; to be in the process is another; the former gives no more than one single payment; the latter is a source of income; the junk dealer receives an income but people selling junk to junk dealers receive single redistributive payments.

The demand and supply functions were defined above as sums of money held in reserve for expenditure and outlay; the redistribution function will be defined by all other sums of money. Thus, this function is not only the seat, so to speak, of money held in reserve for redistributive payments; it is also the seat of all idle money, and also of all money mobilized for general

purposes. This general mobilization is the most important feature in the redistribution function inasmuch as finance may be defined as the art of procuring money for any purpose. However, with regard to financial operations it is necessary to distinguish between the service rendered and the commodity procured: payments for the services of financiers are final operative payments that appear in aggregate expenditure; on the other hand, transfers of the commodity in which the financier deals are redistributive payments. While this distinction may be applied readily enough to the transactions of bankers, brokers, underwriters, a more complex instance appears in insurance; here payments of praemiums have to be divided into a final operative payment for services, proportionate to the outlay of the insurance company in wages, salaries, rents, dividends, and a redistributive payment, proportionate to the company's redistributive payments of awards; and, of course, this application of the distinction is only a first approximation that attends simply to the essential business of insurance of collecting praemiums and paying awards; but it suffices for present purposes.

The next step is to introduce two more rates, DD and DS, similar to DE, DR, DO, DI. In the given interval DD is the net transfer of sums of money from the redistributional function to the demand function, while DS is the net transfer from the redistributional function to the supply function. Since DD and DS are net transfers they may be positive, zero, or negative; an excess in favour of the demand or supply functions is counted positive; an excess in favour of the redistributional function is counted negative. DS consists in movements to and from circulating capital: thus when entrepreneurs increase their volume of business by

selling securities or contracting short term loans, DS is positive; when on the other hand they decrease their volume of business, to purchase securities or pay off loans, DS is negative. As DS is the balance of monetary movements from redistribution to supply, so DD is the balance from redistribution to demand. In the given interval some income will be diverted from expenditure to the redistribution function; it goes to savings, insurance, the liquidation of debts, the purchase of securities or of other redistributional property such as second-hand motor-cars, private homes, farms, factories. In the same interval there is also an opposite movement: savings of earlier intervals are now spent; property is sold to meet current demands; payments of debts or awards of insurance companies go to education, medical fees, and so forth. The excess of the latter movement over the former in the given interval is the quantity, DD.

A three-point circulatory system has now been defined. At any instant sums of money are either in the supply function, the demand function, or the remainder redistribution function. If at the beginning of an interval there is a sum, S, in supply, D, in demand, and R in redistribution, then at the end of that interval one finds in supply the sum (S + DR + DS - DO), in demand the sum (D + DI + DD - DE), and in redistribution the sum (R - DS - DD). Adding these three, one obtains (S + D + R) since DR and DE, DI and DO, are identical pairs. It follows that the circulation exists by definition.

However, this three-point system has now to be enlarged into a five-point system by a sub-division of the supply function into basic supply and surplus supply and by a sub-division of the demand function into basic demand and surplus demand. The transformation

DD' as the net transfer to basic demand, and DD" as the net transfer to surplus demand, all in the given interval. At this point it will be well to collect results by drawing a diagramme, say, a baseball diamond with the redistribution function in the pitcher's box, basic demand at home base, basic supply at first, surplus demand at second, and surplus supply at third. DD', DS', DD", DS" may be denoted by arrows pointing from the pitcher's box to the bases; DE' by an arrow from home base to first, and DE" by an arrow from second base to third. But before DO' and DO" can be represented on the diagramme, one has to settle their relations to DI' and DI".

DI' is defined as the quantity of income entering basic demand in the given interval, and DI" as the quantity of income entering surplus demand in the same interval. It will be convenient to maintain the identity of aggregate outlay and income so that

$$DO' + DO" = DI' + DI" \qquad (1)$$

and hence all income will be supposed to enter the demand functions at least for an instant; if its real destination is the redistribution function, that fact can be represented by negative values of DD' and DD". Next, it cannot be supposed that all basic outlay becomes basic income and all surplus outlay becomes surplus income. At least some basic outlay becomes surplus income, namely, the depreciation charges that purchase maintenance of capital equipment from surplus supply. Again, at least some surplus outlay becomes basic income, namely, wages paid to labour which are spent for consumer goods and services. Thus, there is a cross-over at which part of the basic circuit of expenditure, receipts, outlay, pours into surplus income and simultaneously part of the surplus circuit

turns upon a distinction between final buyers who are consumers and final buyers who are producers. It will be recalled that a final buyer is one who makes a final operative payment and, indeed, it is easy enough to think of consumers as final buyers. However, one must not do so for the wrong reasons. A consumer is not a final buyer because he consumes the object bought and so precludes any subsequent sale; a consumer is a final buyer because he is not a middle-man buying only to sell again. Thus, durable consumer goods just as much as food or fuel enter into final sales; the fact that motor-cars, private homes, and so forth may be sold again and often are sold again is a fact of redistributive and not of operative exchange; the consumer is the final buyer because his payment was the last in the series of operative payments. Still greater difficulty seems to attend the conception of producers as final buyers. Here three distinct confusions seem to arise, coalesce, and so cover over one another's insufficiency. Let us attack them in detail. First, most of a producer's payments are transitional or initial: payments of wages are initial; payments for raw materials that are processed are transitional; but payments for the factory and the machinery with which the processing is done are final, the last of the series of operative payments made upon factory and machinery. Second, it is true that factory and machinery may later be sold: but such a sale is not operative but redistributive; one does not expect producers to sell their factories just as one expects them to sell what they make in their factories; only construction companies make a business of selling factories and their final buyers are the producers who pay for them. Third, it is again true that producers in purchasing and maintaining capital equipment hope to get their money back;

but this hope is not a hope of re-selling capital equipment but a hope of profits from continued ownership; you may have traveled enough on a railway to have paid for a mile of track; but no question arises of the railway company ceding you the ~~x~~ ownership of a mile of track, because there was no question of your buying that; what you bought was transportation.

Final buyers, then, fall into two classes, consumers and producers. It follows that we can distinguish between the basic expenditure of consumers and the surplus expenditure of producers. Similarly, we can distinguish between the basic receipts from the sale of consumer goods and services and the surplus receipts from the sale of producer goods and services. Further, we can distinguish between basic outlay, the reward of factors in the supply of consumer goods and services, and surplus outlay, the reward of factors in the supply of ~~xx~~ producer goods and services. Let DE' be the rate of basic expenditure, DE" the rate of surplus expenditure, DR' (= DE') the rate of basic receipts, DR" (= DE") the rate of surplus receipts, DO' the rate of basic outlay, DO" the rate of surplus outlay, all in any given interval. Next, it follows that the basic demand function is set up by sums of money in reserve for basic expenditure, the surplus demand function is set up by sums of money in reserve for surplus expenditure, the basic supply function is set up by sums of money in reserve for basic outlay, and the surplus supply function is set up by sums of money in reserve for surplus outlay. The five-point frame of reference has been defined. Finally, as DS and DD were defined as net transfers from redistribution to supply and demand, we may now define DS' as the net transfer to basic supply from the redistribution function, DS" as the net transfer to surplus supply

of expenditure, receipts, outlay pours into a basic income.

Let G' be the fraction of basic outlay that becomes surplus income in the given interval, and G" be the fraction of surplus outlay that becomes basic income in the same interval. Then,

$$DI' = (1 - G')DO' + G''DO'' \qquad (2)$$

$$DI'' = (1 - G'')DO'' + G'DO' \qquad (3)$$

The diagramme may now be completed: (1 - G')DO' marking an arrow from first base to home; G'DO' marking an arrow from first base to second; (1 - G")DO" marking an arrow from ~~three~~ third base to second; and G"DO" marking an arrow from third base to home.

At times, it will be convendant to deal simply with the difference between the two cross-over rates, G'DO' and G"DO"; let the difference between them in the given interval be the quantity, DG, so that

$$DG = G''DO'' - G'DO' \qquad (4)$$

When this difference is zero, the cross-overs will be said to be in equilibrium; hence one has

$$DG = 0 \qquad (5)$$

or alternatively in terms of G' and G"

$$G'/G'' = DO''/DO' \qquad (6)$$

either of which may be taken as the condition of cross-over equilibrium. It is to be noticed that equation (4) makes it possible to write equations (2) and (3) more simply, viz.,

$$DI' = DO' + DG \qquad (7)$$

$$DI'' = DO'' - DG \qquad (8)$$

Finally, let us compare the aggregate sums of money in the five functions at the beginning and at the end of any given interval. ~~A~~ If at the beginning these sums are respectively R, D', S', D", S", then at the end of the interval the sums will be

respectively

$$R - DD' - DS' - DD'' - DS''$$

$$D' + DI' + DD' - DE'$$

$$S' + DR' + DS' - DO'$$

$$D'' + DI'' + DD'' - DE''$$

$$S'' + DR'' + DS'' - DO''$$

On adding these one obtains the initial sums, R, D', S', D'', S'', since DE' and DR', DE'' and DR'', (DO' + DO'') and (DI' + DI'') are all equal by definition. Since the redistribution function is the home of finance, it is to be noted that unlike the initial sums, D', S', D'', S'', the sum R is a variable: it increases with the production (or import) of gold, with increases in the fiduciary issue, with increases in bank credit.

Before advancing further with the analysis, it will be well to review and consolidate. A circulation is an aggregate of instantaneous events, namely payments, which stand in circular series of relationships. Payments have been arranged in five classes: redistributive, basic expenditure, basic outlay, surplus expenditure, surplus outlay. Corresponding to each class of payments there has been posited a monetary function of money held in reserve for payments of the class; thus there is a redistribution function, a basic demand function, a basic supply function, a surplus demand function, and a surplus supply function. These five functions supply the points of reference of the frame. Transfers of money from one function to another take place at given rates, so much money per interval; nine symbols have been selected and defined to represent these rates of transfer, namely, DD', DS', DD'', DS'', DE', DE'', DO', DO'', and DG; another six symbols

are also used, namely, DR', DR", DI', DI", G' and G" but all of these are determinate when the first nine are determinate, with the sole exception that G' and G" cannot be separated.

This frame of reference pays no attention to monetary operations within the given five functions. Thus in the supply functions money moves in complex fashion from receipts to outlay with increases or decreases of monetary circulating capital effected by DS' and DS"; similarly, within the redistribution function there are the manifold transfers of finanaial operations. Now the frame of reference neither denies the existence nor refuses to acknowledge any importance in all such movements within functions. It simply preseinds from them. It expresses a view-point that sees the monetary circulation as fundamentally a matter of a basic circuit of expenditure, receipts, outlay and income concerned with consumer goods and services, of a similar surplus circuit of expenditure, receipts, outlay, and income, with a cross-over in which these two circuits mingle, and with a central area of redistribution which not only is a remainder function gathering together the odds and ends that do not fit into the definition of operative payments but also a function of general monetary mobilization that conditions and so controls accele rations in the basic and surplus circuits. A justification of the value of this view-point can be had, of course, only from the interpretations and definitions of general economic phenomena to which it leads.

2. Normative Phases. The frame of reference that has been devised views the circulation in the cross-section of a single interval and not in the process over several intervals. To attain the latter, much more important view-point, the notion of normative phases is introduced. A phase is defined as a series of intervals in which the difference between the first interval and the second is also found between the second and the third, between the third and the fourth, and so on throughout the series. Thus, a phase is a period of uniform and cumulative change. Phases are said to be normative when, first, the systematic variation from interval to interval is defined in terms of variation of basic and surplus outlay and, second, certain simplifying conditions are posited with regard to cross-over equilibrium, movements from the redistribution function to basic and/surplus demand, and lags between income and expenditure. Thus, normative phases are types of moving frames of reference from which more complicated movements of the circulation can be studied.

Let the suffixes, 1, 2, be added to the terms of the frame of reference, namely, DO', DO", G', G", DI', DI", etc., when these terms are used to denote with reference to any two successive intervals; for example, DI''_1 and DI''_2 are the values of DI", the rate of surplus income, in any two successive intervals.

Next, let D^2O' and D^2O'' be the increments of basic and surplus outlay determined by the comparison of two successive intervals, so that

$$D^2O' = DO'_2 - DO'_1 \qquad (9)$$

$$D^2O'' = DO''_2 - DO''_1 \qquad (10)$$

Similarly, one may define D^2I', D^2I'', D^2E', D^2E'', etc. However,

with regard to G' and G", the fractions of outlay that cross over to the opposite type of income, it will be most convenient to write

$$DH = G''_2/G'_2 - G''_1/G'_1 \qquad (11)$$

so that DH is the relative change in the cross-over fractions.

Now the first element in the definition of the normative phases is a systematic variation of basic and surplus outlay. But D^2O' and D^2O'', the increments of outlay, may be positive, or zero, or negative. On this simple head one obtains nine types of phase; their names and definitions are given in the following table; to these which is added a third column under the rubric, DH, which is not immediately relevant.

Phase:	D^2O'	D^2O''	DH
Static Phase:	0	0	0
Basic Expansion:	+	0	+
Surplus Expansion:	0	+	-
Compound Expansion:	+	+	
Basic Contraction:	-	0	-
Surplus Contraction:	0	-	+
Compound Contraction:	-	-	
Basic Disequilibrium:	-	+	-
Surplus Disequilibrium:	+	-	+

The principle of the nomenclature is simple: six of the nine phases are expansions or contractions; expansions when outlay in increasing and contractions when outlay is decreasing; these expansions and contractions are divided into basic, surplus, and compound according as basic outlay, surplus outlay or both

are increasing (expansion) or decreasing (contraction). When one type of outlay is increasing and the other decreasing, there is said to be a disequilibrium; a disequilibrium is named basic when basic outlay is the weak sister, surplus when surplus outlay is. There remains only the static phase in which both basic and surplus outlay are constant.

The second element in the definition of normative phases is a set of simplying conditions. Their effect will be to make the other variables of the basic circuit, DI', DD', DE', DS' dependent on DO', and similarly the other variables of the surplus circuit, DI", DD", DE", DS" dependent on DO". Thus, the definitions of the phases in terms of the variation of DO' and DO" become definitions in terms of a systematic variation of all the variables.

The first of these simplyfying conditions is a postulate of cross-over equilibrium. It will be recalled that the rate of cross-over difference, DG, is defined by

$$DG = G''DO'' - G'DO' \qquad (4)$$

so that when cross-over equilibrium makes this difference zero

$$DO'/DO'' = G''/G' \qquad (6)$$

Hence, in the static phase, when DO' and DO" are both constant, cross-over equilibrium is satisfied by a constant ratio of the cross-over fractions; G"/G' is the same ratio, interval after interval, and so DH is zero. On the other hand, when one of the pair, DO' and DO", is varying while the other is constant, the ratio, G"/G', has to be constantly undergoing adaptation if cross-over equilibrium is to be maintained; thus, in the basic expansion, the surplus contraction, and the surplus disequilibrium, DH has to be increasing, while in the surplus expansion, the basic contraction, and the basic disequilibrium, DH has to be

decreasing. Finally, in the compound expansion and the compound contraction, when both rates of outlay are varying in the same direction, one cannot conclude immediately whether DH has to be positive or zero or negative to give cross-over equilibrium; in these two cases a blank was left in the third column, DH, of the table of names and definitions of the phases.

The postulate of cross-over equilibrium may be stated more concretely as follows. G" is the fraction of surplus outlay going to basic demand: thus, G"DO" includes nearly all the wages in surplus outlay and a notable proportion of salaries, rents, royalties, dividends; everyone has to live, to purchase consumer goods and services. Similarly, (1 - G') is the fraction of basic outlay going to basic demand; it is similar xxximxchanx in character to G" and indeed, since people do not regulate their spending according as their income is from basic or from surplus production, one may expect that

$$G'' = 1 - G' \qquad (12)$$

Hence one may eliminate either G' or G" from the condition of cross-over equilibrium, writing

$$DO'/DO'' = (1 - G')/G' = G''/(1 - G'') \qquad (13)$$

The meaning of the condition now becomes clear. While people do not regulate their spending according to the origin of their income, they do have to regulate it according to the proportion of consumer goods and services in total production. If the production of consumer goods and services involves an outlay that is four times as great as that involved in the production of producer goods and services, then four fifths of total income have to move to the consumer market; otherwise cross-over equilibrium fails. On the assumption expressed in

equation (12) above, four fifths of total income move to the consumer market when G" is 80% and G' is 20%. In the static phase these percentages would remain unchanged. But in the expansions and the contractions these percentages have to be changing constantly. If a basic expansion makes DO'/DO" equal to 5, then G" has to advance from 80% to 83.$\dot{3}$% while G' has to recede from 20% to 16.$\dot{6}$%. On the other hand, if a surplus expansion makes DO'/DO" equal to 3, then G" has to recede from 80% to 75% while G' advances from 20% to 25%. Of course, when we say that G' or G" have to undergo certain modifications, we speak of no objective and necessary cause; we merely enuntiate the consequent of the hypothesis of cross-over equilibrium. With regard to the verifiability of that hypothesis in actual economic history, we are inclined to be very sceptical. Under the profit criterion there is a marked bias in favour of a large G' and a small G", so that surplus expansions are prolonged and basic expansions short-lived.

The second of the simplifying conditions defining normative phases is an equilibrium between the thrifty and the spendthrifts, between the melancholy who put their money earnings aside in anticipation of future rainy days and, on the other hand, the sanguine who cannot fancy the future being as bad as the present and so spend what they have, and what they can borrow, with an open hand. In precise form this second postulate is that

$$0 = DD' = DD'' \qquad (14)$$

Movements from the redistribution function to basic demand are cancelling, interval by interval, with movements from basic demand to the redistribution function. Similarly, movements between surplus demand and the redistribution function result in a cancellation. Such movements may be as great or as small as you please; they may vary enormously or not at all; the postulate is

satisfied as long as the aggregate result is, in each case, a cancellation. Obviously, this is very much a simplifying condition. At a single stroke it brushes aside all the complications of the equation between savings and investment until such time as these questions can be discussed profitably.

The third of the simplifying conditions is that basic expenditure keeps pace with basic outlay, and that surplus expenditure keeps pace with surplus outlay. The possibility of this "keeping pace" has been secured by the two previous postulates. Cross-over equilibrium makes basic income exactly equal to basic outlay, and surplus income exactly equal to surplus outlay. For

$$DI' = DO' + DG \qquad (7)$$

$$DI'' = DO'' - DG \qquad (8)$$

and cross-over equilibrium means that DG is zero. Next, the equilibrium between the sanguine and the melancholy prevents this income from running off to the redistribution function to the depletion of the demand functions as also it prevents the demand functions from becoming clamorous because of excess releases from the redistribution function. Thus, the possibility of expenditure keeping pace with outlay has been provided for. The postulate is that expenditure not merely can but actually does keep pace. And, as is clear enough, this postulate is implicit in the idea of types of phase initiated by outlay. For suppose that outlay increased and expenditure did not follow; plainly enough, entrepreneurs would take the hint and desist from their expansion; and if they did desist, the phase would change, for the phase in question would be defined by increasing outlay. Thus, this third simplifying condition is not so much an additional postulate as an implication of our method of procedure. It remains,

however, that some attempt be made to declare more precisely what is meant by expenditure "keeping pace." In the first place, it does not mean that in each interval

$$DE' = DO'$$

$$DE'' = DO''$$

Such a postulate would disregard entirely the fact of a production period, that, form example, in an expansion outlay begins to increase, and keeps increasing, considerably in advance of the arrival of the increment of goods and services on the final markets. Equality of expenditure and outlay interval by interval would mean a rise of price levels to enable the increased income to be spent when the increment of goods and services was not yet on sale; similarly, it would mean a drop of price levels to enable decreased income to clear the market before the market began to suffer a curtailment of supply. Thus, the postulate of continuity, of "keeping pace," has to be put in the form of such equations as

$$DE'_j = DO'_i \qquad (15)$$

$$DE''_j = DO''_i \qquad (16)$$

where the suffixes "i" and "j" refer to two different intervals, and the time between these intervals is equal to the weighted average production period of the goods and services undergoing increase or diminution. In this precise form, the postulate of continuity is not particularly realistic; but it may be not be out of place to observe that there is no necessity of realism at this point of the inquiry. The function of theory is to construct ideal lines from which one can approximate systematically towards the real lines; our present concern is to obtain clear and definite ideal lines.

The fourth and last of the simplifying conditions has to do with the velocity of money in the basic and the surplus circuits. The postulate is framed as a conditioned correlation, namely, that D^2O' and DS', and similarly D^2O'' and DS'', are simultaneously positive, zero, or negative, except in so far as this is prevented by the already posited postulate of continuity which also regards monetary velocities in the circuits. The correlation itself amounts to saying that when DS' or DS'', the net transfers from redistribution to the demand functions, are positive, then the decrease in velocity will not be so great as to cause D^2O' or D^2O'' to be negative; again, when DS' or DS'' is negative, the increase in velocity will not be so great as to enable D^2O' or D^2O'' to be positive. In the main these suppositions are plausible enough: industry and commerce generally are not brisked when there is a contraction of short term loans, nor are they slackening when short term loans are expanding. What is not plausible is the exact correlation of a zero D^2O' with a zero DS', and a zero D^2O'' with a zero DS''. But while this is not plausible, it remains a convenient assumption for the moment. So much for the correlation of the net transfers to the supply functions with the increments in the rates of outlay. It is has been said that this correlation is supposed only in so far as it does not conflict with the postulate of continuity, with the postulate that there is a lag, proportionate to production periods, between ~~income and expenditure~~ changes in the rates of income and of expenditure. The possible conflict becomes apparent as soon as one attempts to envisage the process of an expansion or contraction.

Let us say that DS' is some positive quantity, k, over a series of intervals. The immediate effect is an increment of basic outlay of, say, k' per interval, where k' is a function of k and of the velocity in the basic circuit. Thus, in the first interval DO' becomes, say, (m' + k'). In the second interval, DS' again transfers k and this makes possible, we may suppose, another addition of k' to the rate of basic outlay. But the question arises whether this k' is to be added to m' or to (m' + k'). ~~If the rate of expenditure~~ If increases in the rate of expenditure do not lag behind increases in the rate of income, one would be inclined to say that in the second interval DO' is (m' + 2k'). But ~~as~~ when one has ~~prop~~ postulated lags proportionate to production periods, one has to choose the other alternative. Thus, the effect of a net transfer of k per interval will raise DO' from m' to (m' + k') in a first interval, and so give a postive D^2O', then for a series of intervals equal to the lag between increments of income and increments of expenditure, it will maintain DO' at (m' + k'), and so give a zero D^2O' when DS' is positive; finally, only at the end of this lag will DO' set forth on its full expansion with increasing receipts combining with the net transfers from redistribution to basic supply to give the series, m' + k', m' + 2k', m' + 3k', m' + 4k',.... and so make D^2O' equal to k' interval by interval. Thus, there is a real conflict between the present velocity postulate and the previous postulate of continuity; accordingly, we have made the ~~ex~~ velocity postulate conditioned, so that the postulate of continuity prevails. Except for lags in D^2E' and D^2E'', a positive or negative value of DS' or DS'' gives a corresponding positive or negative value of D^2O' or D^2O''.

So much for the idea, the names, the definitions, and the simplifying conditions of the normative phases. Under the defined conditions any phase can be had by controlling DS' and DS". According as these are positive, zero, or negative, D^2O' and $D^2O"$ will be positive, zero or negative, in virtue of the conditioned velocity postulate. According as D^2O' and $D^2O"$ are positive, zero, or negative, it follows by definition that DO' and DO" are increasing, constant, or decreasing. By the postulate of cross-over equilibrium, DI' is always equal to DO' and DI" is always equal to DO". By the postulated equilibrium between the thrifty and the spendthrifts initiative is removed from the demand functions, and by the postulate of continuity the rates of expenditure, DE' and DE", keep pace in due time with the rates of outlay, DO' and DO". Thus increased outlay in due time returns to the supply functions to join with present increments and give the cumulative effect of the expansions; similarly, decreased outlay in due time is manifested in decreased receipts, and this negative joined with the negative action of a minus DS' or minus DS" gives the cumulative effect of the contractions. A positive DS' maintained over a series of intervals will make the basic circuit bigger and bigger; a negative DS' will make it smaller and smaller; a zero DS' will leave it constant. Similarly, DS" controls the surplus circuit. Thus, the idea of normative phases has enabled us to take our static frame of reference and transform it into nine types of dynamic frames of reference.

3. The Cycle of the Normative Phases. We must now revert to our point of departure. The division of payments into redistributive and operative, expenditure-receipts and outlay-income, basic and surplus, was based upon relations between the payments and the productive process. Then the process was forgotten: two circuits of expenditure, receipts, outlay, income were set up, and conditions were defined under which the ~~acceleration of the~~ acceleration of the circuits was made dependent upon movements of money between the redistribution function and the ~~demand~~ basic and the surplus supply functions. This gave the nine normative phases, but we have now to inquire into the relation between such phases and the productive process. The result of this inquiry will be twofold: it will reveal an analogy of productive phases parallel to the monetary phases already defined; and it will arrange all such phases into a series, into the unity of a cycle.

Let us suppose a complete list made of all types and qualities of goods and services sold at the basic final market in either or both of two successive intervals. Let the prices and quantities of the first of these intervals be

$$p_1, p_2, p_3, \cdots p_n$$

$$q_1, q_2, q_3, \cdots q_n$$

and let the increments of these prices and quantities emerging in the second interval be

$$dp_1, dp_2, dp_3, \cdots dp_n$$

$$dq_1, dq_2, dq_3, \cdots dq_n$$

so that one can write with complete accuracy

$$DE'_j \doteq \sum p_i q_i \qquad (17)$$

$$DE'_k = \sum (p_i + dp_i)(q_i + dq_i) \qquad (18)$$

$$D^2E' = \sum (q_i dp_i + p_i dq_i + dp_i dq_i) \qquad (19)$$

where the third equation results from the subtraction of the first from the second, the suffixes "j" and "k" refer to any two successive intervals, and the summations are taken by giving "i" successively all values from "1" to "n."

On inspection of the third equation, (19), it is apparent that the increment of basic expenditure, D^2E', consists of three elements: the first depends entirely upon price increments; the second depends entirely upon quantity increments; and the third is a mixture of both, a product of price increments and quantity increments. ~~Fortunately, however, our problem is not a problem of arriving at exact or nearly exact quantity numerical values.~~ Immediately there arises the problem of determining to what extent D^2E' results from price change and to what extent it arises from quantity change. To meet it we define terms and distinguish cases. Let DP' be the average increment of prices and DQ' be the average increment of quantities; and let us suppose that these increments are added to a price index, P', and a quantity index, Q'. However, before these definitions can be made more precise, two cases have to be distinguished: market continuity, when the product of price increments by quantity increments, $dp_i dq_i$, is relatively small and so may be neglected in an approximate estimate; and market discontinuity, when this product is not relatively small and so may not be neglected.

In the case of market continuity the relations between P', DP', Q', DQ' are defined by the equations:

$$P'Q' = \sum p_i q_i \tag{20}$$

$$P'DQ' = \sum p_i dq_i \tag{21}$$

$$Q'DP' = \sum q_i dp_i \tag{22}$$

By assigning any numerical value for P', the price index, one can at once determine numerical values for Q', DQ', and DP'. Further, with regard to a series of intervals, one can choose price and quantity indices

$$P'_1, P'_2, P'_3, \ldots P'_n$$

$$Q'_1, Q'_2, Q'_3, \ldots Q'_n$$

that satisfy the exact series of equations

$$P'_j Q'_j = DE'_j \tag{23}$$

and fall within the limits determined by the approximate equations

$$P'_k = P'_j + DP'_j \tag{24}$$

$$Q'_k = Q'_j + DQ'_j \tag{25}$$

In this manner, there will be only one purely arbitrary number in the double series of indices, say P'_1, the price index of the first interval of the series.

In the case of market discontinuity, DP' and DQ' will cease to be algebraic symbols and become mere symbolic abbreviations, that is, one gives up the problem of assigning numerical values to DP' and DQ' and becomes content with determining on the whole whether or not there is upward or downward price or quantity change, whether DP' and DQ' are positive, zero, or negative. In general such determination offers little difficulty. For if there is market discontinuity, then $\sum dp_i dq_i$ is large; and from an inspection of

the terms in the summation one can tell whether this is the result of increasing or decreasing prices, of increasing or decreasing quantities. In any such instance one may conclude that DP' and DQ' are positive or negative though one cannot say what numerical increments are to be added to the price index, P', and the quantity index, Q'.

There remains an ambiguity, namely, the case of the emergence of new types or new qualities of goods and services. If we suppose that in the previous interval, when their quantities were zero, their prices were also zero, then the total receipts from these goods and services appear in the summation, $\sum dp_i dq_i$. If, on the other hand, we project their prices backwards from the second interval to the first, then the same total receipts appear in the summation $\sum p_i dq_i$ for then the price increments, dp_i, are zero instead of the initial prices, p_i, being zero. A balance of considerations seems to favour the latter procedure. Accordingly, it is here assumed, though one must bear in mind its implication, namely, that there is a case when DQ' is zero yet a qualitative acceleration is going forward because new types and qualities of goods and services are displacing older types and qualities.

So much for the definitions of DP' and DQ' and, in the case of market continuity, of the indices, P' and Q'. In like manner we suppose defined P", Q", DP", DQ", which have the same meaning with regard to final sales of surplus goods and services as P', Q', DP', DQ' have with regard to final sales of basic goods and services.

So far attention has been directed to the analysis of the increments per interval of final expenditure, D^2E' and D^2E''. We have now to turn to the factors in these increments, and first to the indices of quantity change, DQ' and DQ''. It has been argued that, in general, it is possible to tell whether these indices are positive, zero, or negative. Hence it follows that one may define nine quantity phases in parallel fashion to the nine ~~circular~~ circulation phases already examined. Thus, a quantity static phase is a series of intervals in which both DQ' and DQ'' remain zero; a quantity basic expansion is a series of intervals in which DQ' is positive and DQ'' zero; a quantity surplus expansion is a series of intervals in which DQ' is zero and DQ'' positive; and similarly there is a quantity compound expansion (+, +), a quantity basic contraction (-, 0), a quantity surplus contraction (0, -), a quantity basic disequilibrium (-, +), and a quantity surplus disequilibrium (+, -).

In the short run such quantity phases may result from mere variations in the use of existing capital capacity; thus, the present war has witnessed a great increase in the use of railroads with very slight addition to railway capital equipment. In a still shorter run quantity phases may result from the mere depletion or piling up of inventories or stocks of goods, from the lengthening or shortening of hours of labour, and so on. But In such instances there is no correlation between basic and surplus quantity variations. But in the long run, and especially in the very long run, such a correlation exists. It is that surplus production is the accelerator of basic production. In other words the correspondence between the two is not a point-to-point but a point-to-line correspondence: a new ship is not needed for every trip across the seas, nor a

new shoe-factory for every new new pair of shoes; one ship yields a flow of voyages, one factory yields a flow of shoes, and so a series or flow or stream of surplus goods and services corresponds to a series of series, a flow of flows, a stream of streams of basic goods and services. Now such a correspondence, if it is to be expressed not in terms of expectations of the future but in terms of present fact, is a correspondence of accelerator to accelerated. Thus, with regard to any given pattern of combinations of production factors, there is some long term average quantity of surplus production necessary for the maintenance and the renewal of both surplus and basic means of production. When Q" stands at that average, then the accelerator of the system is merely overcoming what may be termed the system's friction. There results a quantity static phase: both DQ' and DQ" are zero. If the system is to move into a long term expansion, this movement has to begin with a surplus quantity acceleration: surplus production has not merely to maintain or renew existing capital equipment but has to reach a level at which it turns out new units of production and maintains or renews a greater number of existing units; this gives the quantity surplus expansion; DQ" is positive but DQ' as yet remains zero. The quantity surplus expansion has its most conspicuous instances in industrial revolutions and five-year plans in which standards of living do not improve while a national industrial equipment is wholly transformed; indeed, in such a movement it may happen that standards of living may deteriorate, and then one has the closely allied basic disequilibrium in which DQ" is positive and DQ' negative. Eventually, however, this increasing the means of producing the means of production reaches its goal and turns to increasing basic products. The new units now emerging are not surplus but basic; thus DQ" stands at

zero while DQ' is positive; there is the quantity basic expansion, the general rise in standard of living that is the normal objective of the previous surplus expansion. It may very well happen that the standard of living begins to rise before the increase of the surplus process comes to a halt: the phase then is the quantity compound expansion with both DQ" and DQ' positive. Again, it may happen that the increase of the surplus process was over-estimated, and so the basic expansion will be interrupted momentarily by a surplus disequilibrium in which DQ" is negative, with Q" moving down to a lower average level, while DQ' remains positive. Finally, unless this transforming process is immediately followed by another, the basic expansion eventually reaches its term, and a new static phase on a notably higher level than the initial static phase results.

Thus, the quantity phases have an inner logic of their own. They are not merely a list of possible dynamic configurations but they naturally fall into a series, into a cyclic process, that moves from a static phase through surplus expansion, basic disequilibrium, compound expansion, surplus disequilibrium, and basic expansion, towards a new static phase in which a higher standard of living is attained permanently. Now this cycle has two features. In the first place, it is grounded in the nature of things. A higher standard of living in an economic community is, generally, both a qualitative and and a quantitative improvement of the flow of basic goods and services. To attain that improvement, the community has to set about transforming its pattern of combinations of production factors. Such a transformation postulates at the outset an

increase of surplus production and so a surplus expansion with perhaps a basic disequilibrium. However, once this condition is fulfilled, there follows the increase of the standard of living in a compound and then a basic expansion with perhaps a period of surplus disequilibrium. Finally, the higher standard of living reaches its peak, the maximum possible within the transformed economy, and once it is attained there is no more to be done than maintain it. The second feature is that this cyclic process, grounded in the nature of things, does not coincide in all respects with the familiar trade cycles. The latter are marked by basic and surplus and compound contractions, while no mention of contractions was made in the logical scheme by which an economic community moves systematically from a lower to a higher standard of living. Thus it is necessary to distinguish between pure cycles, which omit contractions, and perturbed cycles in which the upward movement of the pure cycle is cut short by a general contraction. Generally there is no objection to the pure cycle which yields an improvement of living standards; equally generally there is vehement objection to the trade cycle which begins with the movements of the pure cycle but ends up with something very different. However, the pure cycle is for the moment the mere suggestion of a possible theoretic construction; later we shall return to it, but the present point is simply the observation that quantity phases are phases of a process.

The analysis of D^2E' and D^2E'' revealed not only quantity factors of acceleration, DQ' and DQ'', but also price factors, DP' and DP''. The significance of the latter is that they mark a divergence between the circulation phases, defined in terms of D^2O' and D^2O'', and the quantity phases, defined in terms of DQ' and DQ''.

Accordingly, it is not worth while to set up a further group of nine price-level phases, since DP' and DP" simply indicate what might be described metaphorically as the inertia of the quantity process of goods and services in its response to accelerations initiated in the circulatory process of payments. Rapid increases or decreases in the circulatory process have not a proportionate effect in the quantity process but are in part absorbed by positive or negative price increments. Thus booms are notoriously inflationary and slumps deflationary. Hence DP' and DP" are best taken as indices of divergence between circulatory and quantity phases.

The main analytic apparatus is now complete. Two acceleration systems have been defined: a circulatory system consisting of two connected circuits that are accelerated by an external redistribution function; a quantity system of two parts in which one part is the long-term accelerator of the other. In each of these acceleration systems nine phases of a cyclic process have been defined in parallel fashion, with postulates determining the normative phases of the circulatory system, and an inner logic or ground in the nature of things indicated as the normative or pure cycle of the quantity process. Finally, indices of price increments serve as markers of the divergence between the two systems.

4. The Effect of Net Transfers. The basic circuit is connected with the redistributional function by two routes, net transfers to supply, DS', and net transfers to demand, DD'. Similarly, the surplus circuit is connected by two routes, net transfers to supply, DS", and net transfers to demand, DD". In defining the normative phases, the correlation of these net transfers with the rates of the circuits was evaded by the introduction of postulates. It is now necessary to determine, in so far as possible, what the correlation is.

The existence of the problem is apparent. For instance, DO' is a rate, so much money going to basic outlay every so often. The increase of this rate may result from an increase in the quantity or from an increase in the velocity of money, so that ~~kk~~ one might write

$$D^2O' = m.dv + v.dm + dv.dm \qquad (26)$$

where "m" is a quantity of money and "dm" its finite increment, and "v" is a velocity of money and "dv" its finite increment. Evidently, unless one knows the conditions under which money changes its velocity in the circuits, one cannot tell when a net transfer, DS', is needed to effect an increase of DO'. Further, unless one knows some correlation between DS', and which is an increment of circulating capital, and dm, which is an increment in the quantity of money in outlay, one is still in the dark about the relations between DS' and D^2O'.

A general solution of the problem is not as difficult as might appear. We have to deal not with the quantity and velocity of money in all and any payments but only with the quantity and velocity in operative payments. But operative payments have

been defined as standing in a net-work congruent with the net-work of the productive process; it follows that we have to deal with quantities of money congruent with the values emerging in the productive process, and with velocities of money congruent with the velocities of the productive process. In fact we shall be able to deal with the more precise ideas of turnover size and turnover frequency instead of the ill-defined ideas of quantity of money and velocity of money.

Perhaps the first step will best be an illustration of this correlation. Suppose that two ship-builders, A and B, each launch a new ship every 15 days, that A has 5 ships under construction at once while B has 10, and so that A completes another ship every 75 days while B requires 150 days. To avoid irrelevant differences, we may suppose that all ships are similar in all respects, that they are sold as soon as they are launched for the same selling price, and so that total receipts and the aggregates of costs and profits are the same in both instances. There are then two ~~identi~~ equal volumes of business: each receives the selling price of one ship every fifteen days; and each proceeds to make aggregates of initial and transitional payments at the same rate. However, this identity of volumes of business does not involve an identity of quantities and velocities of money. A's turnover is an aggregate of receipts and payments on 5 ships, while B's turnover is an aggregate of receipts and payments on 10 ships. When A sells a ship, he has been making payments on it for 75 days, on a second ship for 60 days, on a third for 45 days, on a fourth for 30 days, on a fifth for 15 days. But when B sells a ship, he has making ma payments on it for 150 days, on a second for 135 days, and on

a third to a tenth ship for periods of 120, 105, 90, 75, 60, 45, 30, and 15 days respectively. Thus, A's volume of business is a matter of 5 ships every 75 days, while B's is a matter of 10 ships every 150 days. The two volumes are equal, but A moves money twice as rapidly as B, yet moves only half as much as B.

The difference between turnover size and turnover frequency has been put with exaggerated clarity. It remains that the same distinction can be made with regard to every entrepreneur in basic or in surplus supply. Each one is performing a certain number of services or contributing to the supply of a certain number of products at once. Such performance or contribution takes a certain amount of time. But once this time has elapsed, the entrepreneur proceeds to a new batch of services or products. Thus entrepreneurial activities fall into series of repeated routines. Further, each of these routines form financial unities: receipts come in for the goods or services supplied; transitional payments are made to other entrepreneurs for their contribution to the supply; initial payments are made to the immediate factors; and the aggregate of transactions regarding that batch of goods or services is closed. Thus, the production period hasits correlative in the monetary order, namely, the turnover period; and similarly the value of the goods processed or the services rendered in the production period has its monetary correlative in turnover size.

Certain clarifications are in order. The turnover period is not necessarily identical with the production period, for the turnover period is the period of both production and sale. If the first ship-builder, A, could sell a ship no oftener than once every sixteen days, his production period might remain 75 days

but his turnover period would lengthen to 80 days. The production period sets a lower limit to the turnover period, but turnover periods lengthen when sales do not keep pace; and in the limit decreasing sales lead to a reduction of turnover size. Thus, if A could sell one ship only once every 19 days, he might deliberate between having 5 ships in construction at once with a turnover period of 95 days, or reducing his construction to 4 ships at once with a turnover period of 75 days.

Turnover size will be measured by the transitional and initial payments arising from the turnover. When the entrepreneur's operations are constant, turnover size will also be equal to the receipts from the turnover. When however the entrepreneur is increasing or decreasing the scale of his operations, recipts differ from turnover size, and this difference involves a net transfer to or from or to the redistribution function. Thus if decreasing sales led the first ship-builder to have only four ships under construction at once, his active circulating capital would decrease by one fifth; the receipts for five ships are not needed to meet the initial and transitional payments on four. Later, if increasing sales encourage a return to a turnover of 5 ships at once, then circulating capital that had gone off to the redistribution function has to return; receipts from four ships do not suffice to meet the transitional and initial payments on five. It may be noted, finally, that we make provision ka further on for the complication caused by increasing or decreasing inventories, that is, stocks of goods kept on hand to meet sudden increases in demand.

Let us now systematize the results obtained. With regard to all the entrepreneurs in basic supply during a given interval,

let r_{ij} be the initial payments of the ith ~~turnover~~ entrepreneur in his jth turnover or fractional turnover during that interval, and let s_{ij} be the corresponding transitional payments. Then, the aggregate initial payments of basic supply during the ~~xxx~~ interval, which is the definition of DO', may also be expressed as a double summation of r_{ij}, ~~namelyy~~ so that

$$DO' = \sum\sum r_{ij} \tag{27}$$

and similarly the volume of transitional payments, which may be termed DT', is another double summation, namely,

$$DT' = \sum\sum s_{ij} \tag{28}$$

Next, if we define turnover frequency as the number of turnovers of a given entrepreneur in a given interval, and observe that this number may be a fraction, proper or improper, it will be possible to find average values for the initial payments and other average values for the transitional payments in the successive turnovers of each entrepreneur during any given interval. This makes it possible to replace the double summations by single summations so that

$$DO' = \sum r_i n_i \tag{29}$$

$$DT' = \sum s_i n_i$$

where n_i is the turnover frequency of the ith entrepreneur in the given interval and the summations are taken with respect to all entrepreneurs in basic supply.

If now two successive intervals are compared, and it is found that the ith entrepreneur increases his initial payments

by dr_i, his transitional payments by ds_i, and his turnover frequency by dn_i, then the increments in DO' and DT' will be

$$D^2O' = \sum (r_i dn_i + n_i dr_i + dn_i dr_i) \qquad (31)$$

$$D^2T' = \sum (s_i dn_i + n_i ds_i + dn_i ds_i) \qquad (32)$$

where again the summations are taken with regard to all entrepreneurs in basic supply. Equation (31) gives a correlation between changes in velocity of money and changes in the rate of basic outlay. Basic outlay can increase, through the increase of monetary velocity, to the extent that turnover frequencies can increase; and turnover frequencies increase in two ways; first, by the elimination of lagging sales so that turnover periods are reduced to the size of production periods; second, by the introduction of more rapid methods of production, provided that these more rapid methods are accompanied by an increased efficiency in sales. This correlation is far from being a model of precision, but at least it takes variations in DO' from changing monetary velocity out of an obscure region of pure indetermination. Changing methods of production periods are observable phenomena; so also are brisker and slower sales; without either of these we cannot suppose that DO' varies from changes in monetary velocity; and with these one can suppose no more than a limited and proportionate change of monetary velocity. Other changes in DO' have to be attributed to net transfers, DS'.

There remains the question, To what extent does DS' effect an increase or decrease in DO'? For evidently DS' does not effect solely the quantity increments, dr_i, but also the

quantity increments, ds_i. Consider the equation,

$$\sum s_i n_i = \sum v_i r_i n_i \qquad (33)$$

in which the left-hand side gives the volume of transitional payments in the interval while the ~~left-hand~~ right-hand side multiplies by a "v_i" the volume of initial payments. What is the multiplier, v_i? At a first approximation it is the number of times the product per interval of the ith entrepreneur is sold transitionally during the interval. Thus, when the ith entrepreneur deals immediately with consumers, v_i is zero. When his product per interval is sold once transitionally during the interval and once finally, then v_i is one. When three quarters of his product is sold four times and one quarter is sold five times, exclusive of final sales, then v_i is (4x3/4 + 5x1/4) or 4X25 four and one quarter. But this gives only the first approximation to v_i. At a second approximation one has to take into account that, particularly in the more distant transitional sales, it is not the product of the present interval but the product of previous intervals that is being sold transitionally. These products may differ in quantity and in price from the present interval's $r_i n_i$, but it remains that there is some numerical proportion between the payments they involve. Thus a further correction can be introduced into the calculation of the transitional velocities, v_i, and it must be introduced to satisfy equation (33). It will be convenient to term the latter type of variation of v_i its conventional variation, while variation in the number of transitional sales will be called its independent variation.

So much for the general functional relation between initial and transitional payments, equation (33). If we suppose that turnover frequencies are constant, then the relation between increments dr_i, ds_i, dv_i is given by

$$\sum (r_i dv_i + v_i dr_i + dr_i dv_i) = \sum ds_i \qquad (34)$$

The immediate effect of increments in outlay per turnover, dr_i, is offset by the opposite increments in transitional velocities, dv_i, according to the convention of the preceding paragraph; hence increments in transitional payments per turnover, ds_i, do not appear at once, for the transitional buyers do not increase their payments when the ith entrepreneur increases his outlays but when they purchase his increased products. On the other hand, as soon as these purchases begin, the convention works in the opposite direction, for the increments, dv_i, now have the opposite sign, and through dr_i may have returned to zero, r_i is standing on its new level. Next, if one turns from these short-term effects, one may suppose that v_i remains constant so that dv_i is zero; this gives the logg-term correlation

$$\sum v_i dr_i = \sum ds_i \qquad (35)$$

which holds in a successful acceleration of the whole process. Now the net transfer from or to the redistributional function, DS', is the proximate source of increments in circulating capital needed for both transitional and initial payments. Hence, when turnover frequencies are constant and accelerations are successful, i.e. the increased product is sold all along the line, then

DS' = 0

$$DS' = \sum (dr_i + ds_i) \tag{36}$$

which with equation (35) gives

$$DS' = \sum (1 + v_i)dr_i \tag{37}$$

Thus, the net transfer, DS', is equal to the increments in active monetary circulating capital; and these increments are equal to the increments in outlay per turnover, plus multiples of the latter depending on the number of transitional sales.

One must be content merely to mention the possibility of independent variations of v_i. These emerge in changes in the structure of transitional payments when, for example, a merger eliminates or the break-up of a large corporation into smaller units creates a proprietary barrier that involves transitional payments. If the aggregate of outlays remains the same, one may expect the second term of the left-hand side of equation (34) to summate to zero, positive instances of dr_i cancelling against negative instances. The same holds for the third term. Hence one would get

$$\sum ds_i = \sum r_i dv_i \tag{38}$$

so that the merger, in which dv_i is negative, would give negative increments in active monetary circulating capital devoted to transitional payments. The break-up of a corporation would have the opposite effect. However, such structural changes affect not only transitional payments but also turnover frequencies; the length of the turnover periods determines the quantity of money required for outlay payments in each turnover and so the new pattern of instances of dr_i and ds_i. Thus while circulating capital needed for transitional payments decreases in one respect, this may be offset by other requirements.

Equations (36) and (37) tacitly assume that the increased quantities of money involved in increased turnovers are derived exclusively from the net transfer to basic supply, DS'. This tacit assumption has now to be corrected: the quantity of money in the basic circuit may increase or decrease in three ways, by a net transfer to basic supply, by a net transfer to basic demand, and by a cross-over disequilibrium. Let DM' be this increase in quantity of money in a given interval, so that

$$DM' = DS' + DD' + DG \qquad (39)$$

where the parallel equation for the surplus circuit would be

$$DM'' = DS'' + DD'' - DG \qquad (40)$$

since a positive DG empties the surplus circuit in favour of the basic while a negative DG empties the basic circuit in favour of the surplus (cf. equation 4 above).

The next question is whether DM' may replace DS' in equations (36) and (37). The answer involves some determination of the concept of income velocities. Let us write

$$DD' = DE' - DI' \qquad (41)$$

so that savings are in the redistribution function: when people spend less than they earn, the difference gives a negative DD'; and when they spend more than they are earn, they are drawing on savings in the redistribution function and effecting a positive DD'. The effect of this equation, (41), is to eliminate the concept of income velocities. There is a rate of income, DO'; there is a rate of expenditure, DE'; but between these two there is no rate but only a quantity of money which DD' increases or diminishes in a manner that equilibrates the two terminal rates, DO' and DE'. This device assimilates the analysis of velocity

in basic demand to the analysis already given for ~~sa~~ basic supply: for in supply there were posited no velocities of money between payments, but simply rates of payment with quantities of money between them and DS' as the proximate source of variations in these quantities.

A further effect of equation (41), more relevant to the present issue, appears on recalling that

$$DI' = DO' + DG \qquad (7)$$

so that

$$DE' = DO' + DG + DD' \qquad (42)$$

so that basic expenditure equals basic outlay plus the cross-over difference plus the net transfer to basic demand. But basic expenditure is also basic receipts, that is, the receipts of the entrepreneurs who deal immediately with consumers. Now such receipts over an interval may be equal to or greater than or less than the payments, initial and transitional, of these entrepreneurs over the interval. Let us assume that at the end of each interval these entrepreneurs make up their books, transfer a positive difference to the redistribution function to give a ~~neg~~ negative element in DS', or make good a negative difference by effecting a positive transfer from the redistribution function. Then,

$$DM' = \sum (ds_i + dr_i) \qquad (43)$$

and on the suppositions of equation (37)

$$DM' = \sum (1 + v_i) dr_i \qquad (44)$$

The foregoing results may be put more precisely. It will be recalled that $r_i n_i$ and $s_i n_i$ are approximate average figures over the interval and that dr_i, ds_i, dn_i are increments found by comparing the averages of two intervals. However, there is another notation, already mentioned, that makes r_{ij} the exact initial payments and s_{ij} the exact transitional payments of the ith entrepreneur in his jth turnover or fractional turnover during the interval. Let us define dr_{ij} and ds_{ij} as the increments emerging from the comparison of two successive turnovers, figures being taken from complete and not from fractional turnovers. Further, let DM' be the quantitative increment of money in the circuit during the same interval and not, as hitherto, the increment in the second of two intervals under comparison. Then, on the suppositions of equations (42) and (43)

$$DM' = \sum\sum (dr_{ij} + ds_{ij}) \qquad (45)$$

and since it is always possible to find numbers, u_{ij}, such that

$$\sum\sum ds_{ij} = \sum\sum u_{ij} dr_{ij} \qquad (46)$$

one can also write

$$DM' = \sum\sum (1 + u_{ij}) dr_{ij} \qquad (47)$$

where all summations are taken first with respect to turnovers "j" and then with respect to entrepreneurs "i." In this notation varying velocities, i.e. turnover frequencies, appear in the number of terms in the summations with respect to "j".

expansion. Finally, the cycle initiatied by the movement of the process from an initial static phase through a surplus expansion, a basic disequilibrium, a compound expansion, a surplus disequilibrium, and a basic expansion, may be said to have a normative goal in the attainment of a new static phase on a higher level. But to advance steadily towards that goal, to avoid the interruptions of basic, surplus, compound contractions, the agents of the economy have to adapt their preference schedules and correct their expectations to each of the successive phases. For the cycle has an objective logic of its own and its successive phases postulate different preferences and different expectations. On the other hand, to believe and act up to the belief that the preferences and expectations proper to, say, a surplusa expansion are equally legitimate and satisfactory pragmatically in a basic expansion or a static phase, that is to invite a type of disaster which by its frequent recurrence has become familiar.

This brings us to the second difference between a Robinson Crusoe and a large scale exchange conomy. The latter is a monetary economy, and the use of the medium of exchange can act as a screen that hides from view the objective necessity of changing preferences and expectations in accordance with change in productive phases. When Robinson is clearing a new field, he is incapable of the illusion that that activity enables him to have more to eat here and now. When Robinson is reaping greater harvests from more more numerous fields, he is ~~incapble~~ incapable of the illusion that the corn he will not care to eat can be transmogrified into the capital equipment of, say, a powder plant or another cleared field. But the multitudinous Robinsons of the exchange economy are rewarded with money whether they clear fields of

The argument now moves forward to a fresh topic. Monetary velocities in basic supply have been shown to be a function of turnover frequencies, and turnover frequencies a function of the efficiency of sales and the length of production periods. The division of net transfers, DS', between monetary/circulating capital for initial payments and for transitional payments, have been made a function of transitional velocities, v_1, which depend on the number of times the product of a given entrepreneur is sold transitionally in the standard interval. It remains that we complete the circuit with a consideration of income velocities and of additions to income by net transfers, DD'.

As a matter of convenience let us divide entrepreneurs into three classes: an initial group, E_i, which makes no transitional payments; a group of middlemen, E_j, whose transitional payments form the total receipts of the initial group; and a final group, E_k, whose total receipts are basic expenditure, DE', and whose transitional payments are the total receipts of the middlemen. There are two conventional elements in this structure of basic supply. The first conventional element lies in the manner of the description: we speak of groups of entrepreneurs when really we have no interest in entrepreneurs; we are studying not entrepreneurs but payments in their circular relationships, and, in fact, the entrepreneurs in the three groups are mere figure-heads; what they stand for are sets of payments and receipts, and really what is under discussion are such sets. The second conventional element lies not in the nomenclature but in the structure itself. No real structure of basic supply admits the elegant simplicity of the above description; there are

endless complications, and these complications are not constant but shifting. But whatever the complications and their changes, there is one constant feature, namely, the balancing of ledgers, the equality of receipts and payments. The complexities of interdependence represented by the balancing of ledgers are not the object of our study, but that balancing itself. Hence we are content to study such balancing under the simple conditions of three groups of entrepreneurial figure-heads.

Let us say that the volume of payments per interval of the initial group, E_i, is $\sum r_i n_i$, of the group of middlemen, E_j, is $\sum (r_j n_j + s_j n_j)$, and of the final group, E_k, is $\sum (r_k n_k + s_k n_k)$, where "r" denotes initial payments, "s" transitional payments, "n" turnover frequencies, and the three summations are taken with respect to all instances of "i", "j", and "k" respectively. Then since initial payments are identical with outlay we have

$$DO' = \sum (r_i n_i + r_j n_j + r_k n_k) \qquad (39)$$

On the further assumption that in each case payments of the interval equal receipts of the interval, we have

$$DE' = \sum (r_k n_k + s_k n_k) \qquad (40)$$

$$\sum s_k n_k = \sum (r_j n_j + s_j n_j) \qquad (41)$$

$$\sum s_j n_j = \sum r_i n_i \qquad (42)$$

Equation (40) states that the receipts of the final group are equal to their initial and transitional payments. Equation (41) states that the receipts of the group of middlemen are equal to their initial and transitional payments. Equation (42) states

that the receipts of the initial group are equal to their initial payments, which, ex hypothesi, are their sole payments. But the transitional payments of the final group are identical with the receipts of the middlemen, and the transitional payments of the middlemen are identical with the receipts of the initial group. Hence the summation of $s_k n_k$ appears in both (40) and (41), and the summation of $s_j n_j$ appears in both (41) and (42). On the elimination of these summations, it appears that DO' and DE' are equal, for both equal the summation of the rates of initial payments per interval. Let us further suppose cross-over equilibrium, so that DG equals zero, and DO' equals DI'. It then appears that the condition, and, inversely, the consequent of entrepreneurial receipts equaling entrepreneurial payments is that the expenditure of basic demand, DE', equals the income of basic demand, DI'.

Now it is important to distinguish two different aspects of equations (39) to (42). Under a certain aspect these equations express a truism: if entrepreneurial receipts and payments equate, then they equate not only among entrepreneurs but also between entrepreneurs and the third party, demand. But under another aspect the same equations, so far from expressing a necessary truth, express an almost unattainable ideal, namely, a dynamic equilibrium to which any actual process continually attempts to approximate by varying prices and changing quantities of supply. To study the truism is to study book-keeping, to study the art of double entry, and to learn the magic of the variable items, profit and loss, which perforce make the books balance. To study the ideal is to study equilibrium analysis. The book-keepers are wise after the event. But if the entrepreneurs are to be wise, they have to be wise before the event, for their

payments precede their receipts, and the receipts may equal the payments but they may also be greater or less, to give the entrepreneur a windfall profit or loss. Such justification or condemnation of payments by receipts the book-keeper records but the entrepreneur has to anticipate, and the grounds of his anticipations, their effects upon his decisions, and the interaction of all decisions, form the staple topic of equilibrium analysis. Now the view-point of the present discussion is neither that of the book-keeper nor that of the equilibrium analyst. Equations (39) to (42) are regarded not as a set of facts recorded by book-keepers, nor as an ideal which entrepreneurs strive yet ever fail to attain, but as a first approximation to the law of the circulation in the basic circuit. The first approximation to the law of projectiles is the parabola: one might, if one chose, consider projectiles as aiming at/or tending towards the ideal of the parabola yet ever being frustrated by wind-resistance; one might elaborately describe the trajectory of the projectile as an indefinite series of parabolas, each one in succession the goal of its tendency only to be deserted because adverse circumstance set it on another track. In such a description of trajectories there is to be found at least a superficial resemblance with the statement that an economy is tending towards equilibrium at ever instant, ~~thro~~ though towards a different equilibrium at every ~~different~~ successive instant. But whatever the resemblance, and however deep and significant the difference, we here propose to ~~study~~ take a circuit in equilibrium as a first approximation to the law of the circuit and examine first the implications of this law and then the second approximations that are relevant to our inquiry.

If we suppose that equations (39) to (42) represent any first interval and that in a second interval, in which cross-over equilibrium is assumed, the increments in the terms are D^2O', D^2I', D^2E', D^2R', dr_i, dr_j, dr_k, dn_i, dn_j, dn_k, ds_j, ds_k, then the conditions that the acceleration is a success, i.e., that the acceleration has extended round the circuit in accordance with the first approximation to the law of the circuit, are to found in the following equations.

$$D^2O' = D^2I' = \sum (dr_i n_i + dr_j n_j + dr_k n_k + r_i dn_i + r_j dn_j + r_k dn_k) \quad (43)$$ //

$$D^2E' = D^2R' = \sum (dr_k n_k + r_k dn_k + ds_k n_k + s_k dn_k) \quad (44)$$

$$\sum (ds_k n_k + s_k dn_k) = \sum (dr_j n_j + r_j dn_j + ds_j n_j + s_j dn_j) \quad (45)$$

$$\sum (ds_j n_j + s_j dn_j) = \sum (dr_i n_i + r_i dn_i) \quad (46)$$

These equations are derived by substituting in equations (39) to (42) a $(r_i + dr_i)$ for an r_i, etc., etc., multiplying out the expressions, neglecting the products of two increments, such as $dr_i dn_i$, and eliminating through (39) to (42) the products containing no increments, such as $r_i n_i$. The initial substitution implies that the increments are such as to satisfy the conditions defined by the initial equations. The final elimination separates the donditions of an equilibrium circulation from the conditions of ~~a~~ successful acceleration of a circulation. The neglect of the products of two increments makes equations (43) to (46) approximate except when one is considering pure frequency accelerations (when all instances of dr and ds are zero) or pure quantity accelerations (when all instances of dn are zero).

The significance of equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circuit, a meaning for the already indicated distinction between a pure quantity acceleration and a pure frequency acceleration, a basis of discussion for abortive accelerations, and finally a means of contrasting such circulatory success or failure with the success or failure of acceleration of the productive t rhythms.

Our first task is to complete our inspection of the circuit. There remains the question of quantities of money and velocities of money in basic demand. Now if we define DD' by the equation

$$DD' = DE' - DI' \qquad (47)$$

there seems no lack of plausibility. The equation means that savings are in the redistribution function, so that when people in basic demand are spending more than they earn, they are transfering savings to basic demand to make up the difference; again when they are spending less than they earn, they are creating savings and so transfering money from basic demand to the redistribution function. However, this equation has a further implication, namely, that income velocities, in the aggregate, cease to be an added variable in the system. By giving DD' the above precise meaning, one eliminates such suppositions as that DD' adds to the quantity of money in basic demand merely to decrease the velocity of money there or, inversely, that it subtracts from the quantity of money there merely to increase its velocity. DD' has been tied down to an exclusive role of quantity acceleration, and income velocities in the aggregate are determinate when DI' and DE' are determinate. To put the same point differently,

the concept of income velocities has been eliminated; there is a rate of income, DI'; there is a rate of expenditure, DE'; but between the two there is no rate, but only a quantity of money, which DD' has the function of increasing or decreasing. This last statement assimilates the analysis of the basic demand function to that of the basic supply function; for in supply we consider only rates of payment, turnover sizes and frequencies, and between such rates we do not posit virtual monetary velocities but only quantities of money in reserve, quantities which DS' augments or diminishes.

If now we revert ot to equations (43) to (46), we observe that the successful acceleration of the circuit involves two types of increment, quantity increments, dr_i, dr_j, dr_k, ds_j, ds_k, and frequency increments, dn_i, dn_j, dn_k; reference to D^2O', D^2I' D^2E', D^2R', is omitted since they are but other names for the same realities. Now with respect to the quantity increments, the question arises, To what extent are they due to DS' and to what extent are they due to DD'? In other words, the quantity increments show that there is more money in circulation, say, DM', where DM' is defined by the equation,

$$DM' = \sum (dr_i + dr_j + dr_k + ds_j + ds_k) \qquad (48)$$

or on the suppositions of equation (37)

$$DM' = \sum [(1 + v_i)dr_i + (1 + v_j)dr_j + dr_k] \qquad (49)$$

the summations being taken with respect to all instances of "i," "j," and "k," that is, with respect to all members of the three groups of entrepreneurs. Now when we were engaged in the study

of basic supply, it was natural to consider this total quantity increment as the work of the net transfer, DS'; but now evidently such a total increment in the whole circuit may to some extent be the work of DD', so that

$$DM' = DS' + DD' \qquad (50)$$

Which, then, of the two net transfers has the preponderant role in increasing or decreasing the quantity of money in the circuit?

First, with reggrd to increases, it should seem that the role of DD' can be little more than initial stimulation. People may spend more than they earn by drawing on savings, but they cannot do so to any great extent. To give DD' a preponderant role would make quantity accelerations of the basic circuit both small and short-lived. On the other hand, our society has developed vast mechanisms to provide entrepreneurs with the means of making large and sustained additions to the quantity of money in the circuit. It practised mercantilism to obtain more gold when money was gold. It developed banking and bills of exchange. It replaced a gold currency, first, with a gold standard fiduciary issue and, later, with what is to all practical purposes a new kind of money, money-of-account. These developments did not take place to enable consumers to spend more than they earn, but to enable entrepreneurs to increase the size of their turnovers. Consumers cannot pay interest on their consumption or even on the increments in their consumption. But entrepreneurs can pay interest on the size of their turnovers. There is a second argument. It is that in an economy in which supply is responsive to demand, any positive action of DD' would immediately stimulate entrepreneurial

expansion. The middlemen and the initial group would begin increased turnovers before they began to receive increased receipts. The increased initial payments in the turnovers would mean an increased rate of income, so that to maintain a positive DD' a further immediate increase in the rate of expenditure would be necessary. There is a third argument which follows out of the second, namely, that at the beginning of an expansion outlays are increasing more rapidly than goods at the final market, so that the increase in income is larger than the immediately available increase of objects on which income may be spent; hence unless DD' is negative and people are spending less than they are earning, prices will rise to give a positive DP' and make excessive expenditure equal to insufficient goods. But as soon as a negative DD' is needed to prevent price inflation, a positive DD' would only accentuate the price inflation; and while this positive action on the part of DD' would reduce to some extent the need of positive action on the part of DS', it also would increase that need/if the acceleration of the supply of goods and services continues; for the rising price level would be communicated back from the final market through all the transitional markets and at the same time a demand for increased wages would arise, so that the whole basis of calculation of initial and transitional payments is raised.

The three arguments tend to show that quantity acceleration of the circuit through the action of DD' is unnecess impossible, inasmuch as people cannot, to any great extent, spend more than they earn, unnecessary, inasmuch as entrepreneurs will effect the quantity acceleration in response to any real stimulus, inadvisable, inasmuch as such

The significance of the equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circulation, a meaning for a distinction between pure quantity and pure frequency accelerations, a basis of discussion for abortive or non-successful accelerations, and finally a means of contrasting circulatory success, so defined, and the success of accelerations of the productive process.

The first step is a consideration of the general conditions of such an acceleration. In so far as there are increments in the quantity of initial or transitional payments per turnover, the net transfers, DS' and DD', have been active. For variations in income velocities can be eliminated from the discussion by defining DD' by the equation

$$DD' = DE' - DI' \qquad (47)$$

This is not arbitrary

in terms of known, i.e. already assigned variables.

This procedure is not arbitrary even though the argument used involves an arbitrary control over the precise meaning to be attributed to "money in the demand function" and "net transfers during an interval." The absence of pure arbitrariness may be shown by establishing the same result in another way. The assigned variables determine the quantities and rates of initial payments;per turnover; they also determine the quantities and rates of final sales;per turnover; but the initial payments are income and the final sales are expenditure. The quantities and velocities in basic demand begin from income and end at expenditure; changes in the quantities are determinate by when DD' is determinate; but if we know the rate of flow at either end and the quantity in between, we also know the velocity in between.

The significance of the equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circuit, a meaning for a distinction between pure frequency acceleration and pure quantity acceleration, as indicated above, a basis of discussion for abortive accelerations, and finally a means of constrasting circulatory success, so defined, with the success of an acceleration of productive rhythms.

The first step is a consideration of the general conditions of the successful acceleration and, to begin, there is the question whether income velocities are a further variable or whether they are determinate when the already assigned variables are determinate. Two considerations favour the second alternative. The first is that income velocities are periods of time between initial payments and final sales; but the assigned variables determine the quantity of initial payments per turnover and the number of turnovers per interval; they also define the quantities of money involved and the rapidity of turnovers of final sales. ~~It xremainsxthus~~ It follows that the aggregate of income velocities have some determinate index. Again, if a circular flow is successful, then the elements of the flow are not piling up at one point to leave a vacuum at another

throughout industry

activity ~~does more to raise prices than anything else.~~ is apt to raise price levels and so multiply the need for more money No one of the three arguments is peremptory, but the three combined tend to limit positive action by DD' to a stimulation of the quantity acceleration and to leave to DS' the main part of the work of providing the increased quantities of money for the increased turnovers.

~~Stimulating action on the part of DD' is not, of course,~~ The immediate stimulation of basic supply necessary. ~~Its origin~~ we shall find later in cross-over disequilibrium. Meanwhile it will be interesting to note a correlation between the rise in prices, DP', and the rate of acceleration, DQ', when DD' is zero. For any rate rate of acceleration of goods and services, DQ', there is an excess of goods in production over goods at the final market; the former is increasing ~~incomes~~ outlays and so also incomes while the increase in goods for expenditure lags; corresponding to this lag

Mutatis mutandis, the same arguments that hold at the beginning of an acceleration hold throughout it during.

For any rate of acceleration of goods & services, DQ', ~~there is an~~ the lag ~~of~~ goods in production are in advance of goods on the final market, and ~~income~~ increasing income is in advance of the possible basic expenditure at initial prices. This lag is a function of DQ', increasing with its increase, & decreasing with it decrease; and corresponding to the size of this lag there is some slight advance in prices, DP', that clears the basic market ~~is~~ DD' at zero. When the lag changes DP' and the change in prices is necessary if DD' is still zero. But ~~of~~ when the acceleration DQ' eventually becomes zero, a positive DD' is for a time desirable, namely, to prevent the ~~first~~ negative DP' implied