Total
1. pp 11 11
2. pp 10 21
3. pp 9 30

# A Method of Independent Circulation Analysis.

Circulation analysis is a set of definitions, postulates, and deductions relevant to a monetary circulation. Such a systematic construction of terms and theorems might be worked out within a wider theoretical context. Thus, its concepts might be derived from the concepts of value theory; its postulates might be modifications of postulates regarding value; its deductions might be special cases of the more general deductions concerning scarce objects with alternative uses. Such a procedure offers the obvious advantages of theoretical unity; analytic apparatus is all in one large, nicely articulated, and agreeably complicated piece. However, like the armour of Saul, it is apt to be too cumbrous for David. Just as one can study Euclidean geometry without the slightest suspicion that it is a particular case of a more general geometry, so also one can attempt an independent circulation analysis in which the formation of concepts, the choice of postulates, and the seriation of deductions are dictated not by the higher exigences of value theory but by the more immediate and more germane considerations of the monetary circulation itself. In that fashion one would obtain an independent analytic tool which, from its greater compactness and simplicity, would perhaps prove more efficient in the solution of certain types of problem. No doubt, once such an independent tool were constructed and found successful, theorists would be troubled by profound questions regarding the developments that might result from the mutual inter-action of equilibrium and circulation analysis. But such thoughts cannot occupy us here. It is enough that we attempt to indicate by concrete example a method of independent circulation analysis in the belief that it offers special advantages in handling some economic issues.

1. Frame of Reference. The productive process of an exchange economy operating in a closed area, offers the most favourable starting-point for a study of a monetary circulation. For a circulation is not so much a rotational movement as an aggregate of instantaneous events, namely payments, which stand in circular series of relationships; and while a productive process is rectilinear rather than circular, it does provide, by the technical dependence of each successive stage of production upon previous stages, a correlative and almost palpable ground for series of relationships between payments. Thus, each element of stage of the material process has its proper outlay, payments of immediate factors of production in wages and salaries, rents and royalties, interest and dividends, depreciation charges and undistributed profits. Next there is the building up of prices as these elements are united materially into a growing product and simultaneously the outlays upon the single elements are added into a growing volume of transitional payments from subsequent to previous units of activity. At the end of these lines of production are the finished products, the aggregate receipts of industry and commerce, and the aggregate expenditure of final buyers.

All such payments form a class by themselves. They stand in a net-work that is congruent with the technical net-work of the productive process. They recur with the recurrence of its routines. In the main they vary with variations in the volume of these routines. But, above all, their connection with production is immediate: they emerge not through repercussions or as responses to external stimulus but are, so to speak, the immanent manifestation of the productive process as a process of value. For in an exchange economy production is not a merely technical affair of designing, assembly, processing, and distribution; intrinsically it is an economic affair, an expression of preference and choice, and so not merely production, as some technical experts seem to fancy, but production for sale, production in view of and at every instant adapted to payments.

Payments, then, forming a net-work congruent with the net-work of the productive process, shall be termed operative, and from among them two boundary classes are selected: the aggregate receipts that are also the expenditure of final buyers; and the aggregate outlays that are also the income of factors of production. Let DR and DE be the aggregate sums of money that in a given interval are receipts and expenditure; let DO and DI be the aggregate sums of money that in the same interval are outlay and income; so that by definition DR = DE and DO = DI. Further, let us say that money held in reserve for expenditure is in the demand function of the economy; and that money held in reserve for outlay, or on its way to outlay through transitional payments, is in the supply function. Note, however, that it is not assumed that DO is identical with DR, or that DE is identical with DI. Such an assumption, in general, would be contrary to fact. Not only are there other exits and entrances to the supply and demand functions besides outlay and expenditure, receipts and income; but even if these other lines of communication did not exist, it would not be clear that all the receipts of an interval become outlay in the same interval, or that all the income of an interval becomes expenditure in the same interval.

Besides the operative payments described above there are in any economy other, redistributive payments. Such payments we define negatively; they form a remainder class of all payments that are not operative, that do not stand in the net-work congruent with the net-work of the productive process. The existence of this remainder class follows from the fact that property is a broader category than the current supply of goods and services. There are things never produced, such as an economy's endowment of natural resources. There are other things that, though produced, already have become the property of final buyers. Most of these may be sold, and the payments for them will be termed redistributive; for on the one hand such payments change titles to ownership, redistribute property, but on the other they are not operative, not intrinsic to the forward movement of industry and commerce. It is to be observed that such payments remain redistributive even when they mark the re-entry of property into the productive process; to re-enter the process is one thing; to be in the process is another; the former gives no more than one single payment; the latter is a source of income; the junk dealer receives an income but people selling junk to junk dealers receive single redistributive payments.

The demand and supply functions were defined above as sums of money held in reserve for expenditure and outlay; the redistribution function will be defined by all other sums of money. Thus, this function is not only the seat, so to speak, of money held in reserve for redistributive payments; it is also the seat of all idle money, and also of all money mobilized for general purposes. This general mobilization is the most important feature in the redistribution function ~~for~~ inasmuch as finance may be defined as the art of procuring money for any purpose. However, with regard to financial operations it is necessary to distinguish between the service rendered and the commodity procured: payments for the services of financiers ~~and~~ are final operative payments that appear in aggregate expenditure; on the other hand, transfers of the commodity in which the financier deals are redistributive payments. ~~and~~ While this distinction may be applied readily enough to the transactions of bankers, brokers, underwriters, ~~insurance-offers~~ a more complex instance appears in insurance; here payments of praemiums have to be divided into a final operative payment for services, proportionate to the outlay of the insurance company in wages, salaries, rents, dividends, and a redistributive payment, proportionate to the company's redistributive payments of awards; and, of course, this application of the distinction is only a first approximation that attends simply to the essential business of insurance of collecting praemiums and paying awards; but it suffices for present purposes.

The next step is to introduce two more rates, DD and DS, similar to DE, DR, DO, DI. In the given interval DD is the net transfer of sums of money from the redistributional function to the demand function, while DS is the net transfer from the redistributional function to the supply function. Since DD and DS are net transfers they may be positive, zero, or negative; an excess in favour of the demand or supply functions is counted positive; an excess in favour of the redistributional function is counted negative. DS consists in movements to and from circulating capital: thus when entrepreneurs increase their volume of business by selling securities or contracting short term loans, DS is positive; when on the other hand they decrease their volume of business, to purchase securities or pay off loans, DS is negative. As DS is the balance of monetary movements from redistribution to supply, so DD is the balance from redistribution to demand. In the given interval some income will be diverted from expenditure to the redistribution function; it goes to savings, the redistributive part in insurance, the liquidation of debts, the purchase of securities or of other redistributional property such as second-hand motor-cars, private homes, farms, factories. In the same interval there is also an opposite movement: savings of earlier intervals are now spent; property is sold to meet current demands; payments of debts or awards of insurance companies go to education, medical fees, and so forth. The excess of the latter movement over the former in the given interval is the quantity, DD.

A three-point circulatory system has now been defined. At any instant sums of money are either in the supply function, the demand function, or the remainder redistribution function. If at the beginning of an interval there is a sum, S, in supply, D, in demand, and R in redistribution, then at the end of that interval one finds in supply the sum (S + DR + DS - DO), in demand the sum (D + DI + DD - DE), and in redistribution the sum (R - DS - DD). Adding these three, one obtains (S + D + R) since DR and DE, DI and DO, are identical ~~equal~~ pairs. It follows that the circulation exists by definition.

However, this three-point system has now to be enlarged into a five-point system by a sub-division of the supply function into basic supply and surplus supply and by a sub-division of the demand function into basic demand and surplus demand. The transformation turns upon a distinction between final buyers who are consumers and final buyers who are producers. It willx be recalled that a final buyer is one who makes ~~the~~ a final operative payment and, indeed, it is easy enough to think of consumers as final buyers. However, one must not do so for the wrong reasons. A consumer is not a final buyer because he consumes the object bought and so precludes any subsequent sale; a consumer is a final buyer because he is not a middle-man buying only to sell again. Thus, durable consumer goods just as much as food or fuel enter into final sales; but the fact that motor-cars, private homes, and so forth may be sold again and often are sold again is a fact of ~~a~~ redistributive and not of operative exchange; the consumer is the final buyer because his payment was the last in the series of operative payments. Still greater difficulty seems to attend the conception of producers as final buyers. Here three distinct confusions seem to arise, coalesce, and so cover over one another's insufficiency. Let us attack them in detail. First, most of a producer's payments are transitional or initial: payments of wages are initial; payments for raw materials that are processed are transitional; but payments for the factory and the machinery with which the processing is done are final, the last of the series of operative payments made upon factory and machinery. Second, it is true that factory and machinery may later be sold: but such a sale is not operative but redistributive; one does not expect producers to sell their factories just as one expects them to sell what they make in their factories; only construction companies make a business of selling factories and their final buyers are the producers who pay for them. Third, it is again true that producers in purchasing and maintaining capital equipment hope to get their money back;

but this hope is not a hope of re-selling capital equipment but a hope of profits from continued ownership; you may have traveled enough on a railway to have paid for a mile of track; but no question arises of the railway company ceding you the ownership of a mile of track, because there was no question of your buying that; what you bought was transportation.

Final buyers, then, fall into two classes, consumers and producers. It follows that we can distinguish between the basic expenditure of consumers and the surplus expenditure of producers. Similarly, we can distinguish between the basic receipts from the sale of consumer goods and services and the surplus receipts from the sale of producer goods and services. Further, we can distinguish between basic outlay, the reward of factors in the supply of consumer goods and services, and surplus outlay, the reward of factors in the supply of producer goods and services. Let $DE'$ be the rate of basic expenditure, $DE''$ the rate of surplus expenditure, $DR'$ ( $= DE'$) the rate of basic receipts, $DR''$ ( $= DE''$) the rate of surplus receipts, $DO'$ the rate of basic outlay, $DO''$ the rate of surplus outlay, all in any given interval. Next, it follows that the basic demand function is set up by sums of money in reserve for basic expenditure, the surplus demand function is set up by sums of money in reserve for surplus expenditure, the basic supply function is set up by sums of money in reserve for basic outlay, and the surplus supply function is set up by sums of money in reserve for surplus outlay. The five-point frame of reference has been defined. Finally, as DS and DD were defined as net transfers from redistribution to supply and demand, we may now define $DS'$ as the net transfer to basic supply from the redistribution function, $DS''$ as the net transfer to surplus supply

DD' as the net transfer to basic demand, and DD" as the net transfer to surplus demand, all in the given interval. At this point it will be well to collect results by drawing a diagramme, say, a baseball diamond with the redistribution function in the pitcher's box, basic demand at home base, basic supply at first, surplus demand at second, and surplus supply at third. DD', DS', DD", DS" may be denoted by arrows pointing from the pitcher's box to the bases; DE' by an arrow from home base to first, and DE" by an arrow from second base to third. But before DO' and DO" can be represented on the diagramme, one has to settle their relations to DI' and DI".

DI' is defined as the quantity of income entering basic demand in the given interval, and DI" as the quantity of income entering surplus demand in the same interval. It will be convenient to maintain the identity of aggregate outlay and income so that

$$DO' + DO'' = DI' + DI'' \qquad (1)$$

and hence all income will be supposed to enter the demand functions at least for an instant; if its real destination is the redistribution function, that fact can be represented by negative values of DD' and DD". Next, it cannot be supposed that all basic outlay becomes basic income and all surplus outlay becomes surplus income. At least some basic outlay becomes surplus income, namely, the depreciation charges that purchase maintenance of capital equipment from surplus supply. Again, at least some surplus outlay becomes basic income, namely, wages paid to labour which are spent for consumer goods and services. Thus, there is a cross-over at which part of the basic circuit of expenditure, receipts, outlay, pours into surplus income and simultaneously part of the surplus circuit of expenditure, receipts, outlay pours into a basic income. Let $G'$ be the fraction of basic outlay that becomes surplus income in the given interval, and $G''$ be the fraction of surplus outlay that becomes basic income in the same interval. Then,

$$DI' = (1 - G')DO' + G''DO'' \qquad (2)$$

$$DI'' = (1 - G'')DO'' + G'DO' \qquad (3)$$

The diagramme may now be completed: $(1 - G')DO'$ marking an arrow from first base to home; $G'DO'$ marking an arrow from first base to second; $(1 - G'')DO''$ marking an arrow from third base to second; and $G''DO''$ marking an arrow from third base to home.

At times, it will be convenient to deal simply with the difference between the two cross-over rates, $G'DO'$ and $G''DO''$; let the difference between them in the given interval be the quantity, $DG$, so that

$$DG = G''DO'' - G'DO' \qquad (4)$$

When this difference is zero, the cross-overs will be said to be in equilibrium; hence one has

$$DG = 0 \qquad (5)$$

or alternatively in terms of $G'$ and $G''$,

$$G'/G'' = DO''/DO' \qquad (6)$$

either of which may be taken as the condition of cross-over equilibrium. It is to be noticed that equation (4) makes it possible to write equations (2) and (3) more simply, viz.,

$$DI' = DO' + DG \qquad (7)$$

$$DI'' = DO'' - DG \qquad (8)$$

Finally, let us compare the aggregate sums of money in the five functions at the beginning and at the end of any given interval. If at the beginning these sums are respectively R, D', S', D'', S'', then at the end of the interval the sums will be respectively

$$R - DD' - DS' - DD'' - DS''$$

$$D' + DI' + DD' - DE'$$

$$S' + DR' + DS' - DO'$$

$$D'' + DI'' + DD'' - DE''$$

$$S'' + DR'' + DS'' - DO''$$

On adding these one obtains the initial sums, $R$, $D'$, $S'$, $D''$, $S''$, since $DE'$ and $DR'$, $DE''$ and $DR''$, $(DO' + DO'')$ and $(DI' + DI'')$ are all equal by definition. Since the redistribution function is the home of finance, it is to be noted that unlike the initial sums, $D'$, $S'$, $D''$, $S''$, the sum $R$ is a variable: it increases with the production (or import) of gold, with increases in the fiduciary issue, with increases in bank credit.

Before advancing further with the analysis, it will be well to review and consolidate. A circulation is an aggregate of instantaneous events, namely payments, which stand in circular series of relationships. Payments have been arranged in five classes: redistributive, basic expenditure, basic outlay, surplus expenditure, surplus outlay. Corresponding to each class of payments there has been posited a monetary function of money held in reserve for payments of the class; thus there is a redistribution function, a basic demand function, a basic supply function, a surplus demand function, and a surplus supply function. These five functions supply the points of reference of the frame. Transfers of money from one function to another take place at given rates, so much money per interval; nine symbols have been selected and defined to represent these rates of transfer, namely, $DD'$, $DS'$, $DD''$, $DS''$, $DE'$, $DE''$, $DO'$, $DO''$, and $DG$; another six symbols are also used, namely, DR', DR", DI', DI", G' and G" but all of these ~~can be obtained~~ are determinate when the first nine are determinate, with the sole exception that G' and G" cannot be separated.

This frame of reference pays no attention to monetary operations within the given five functions. Thus in the supply functions money moves in complex fashion from receipts to outlay with increases or decreases of monetary circulating capital effected by DS' and DS"; similarly, within the redistribution function there are the manifold transfers of financial operations. Now the frame of reference neither denies the existence nor refuses to acknowledge any importance in all such movements within functions. It simply prescinds from them. It expresses a view-point that sees the monetary circulation as fundamentally a matter of a basic circuit of expenditure, receipts, outlay and income concerned with consumer goods and services, of a similar surplus circuit of expenditure, receipts, outlay, and income, with a cross-over in which these two circuits mingle, and with a central area of redistribution which not only is a remainder function gathering together the odds and ends that do not fit into the definition of operative payments but also a function of general monetary mobilization that conditions and so controls accelerations in the basic and surplus circuits. A justification of the value of this view-point can ~~be~~ be had, of course, only from the interpretations and definitions of ~~general~~ economic phenomena to which it leads.

2. <u>Normative Phases.</u> The frame of reference that has been devised views the circulation in the cross-section of a single interval and not in the process over several intervals. To attain the latter, much more important view-point, the notion of normative phases is introduced. A phase is defined as a series of intervals in which the difference between the first interval and the second is also found between the second and the third, between the third and the fourth, and so on throughout the series. Thus, a phase is a period of uniform and cumulative change. Phases are said to be normative when, first, the systematic variation from interval to interval is defined in terms of variation of basic and surplus outlay and, second, certain simplifying conditions are posited with regard to cross-over equilibrium, movements from the redistribution function to basic and/to surplus demand, and lags between income and expenditure. Thus, normative phases are types of moving frames of reference from which more complicated movements of the circulation can be studied.

Let the suffixes, 1, 2, be added to the terms of the frame of reference, namely, $DO'$, $DO''$, $G'$, $G''$, $DI'$, $DI''$, etc., when these terms are used ~~to denote~~ with reference to any two successive intervals; for example, $DI''_1$ and $DI''_2$ are the values of $DI''$, the rate of surplus income, in any two successive intervals.

Next, let $D^2O'$ and $D^2O''$ be the increments of basic and surplus outlay determined by the comparison of two successive intervals, so that

$$D^2O' = DO'_2 - DO'_1 \qquad (9)$$

$$D^2O'' = DO''_2 - DO''_1 \qquad (10)$$

Similarly, one may define $D^2I'$, $D^2I''$, $D^2E'$, $D^2E''$, etc. However, with regard to G' and G'', the fractions of outlay that cross over to the opposite type of income, it will be most convenient to write

$$DH = G''_2/G'_2 - G''_1/G'_1 \qquad (11)$$

so that DH is the relative change in the cross-over fractions.

Now the first element in the definition of the normative phases is a systematic variation of basic and surplus outlay. But $D^2O'$ and $D^2O''$, the increments of outlay, may be positive, or zero, or negative. On this simple head one obtains nine types of phase; their names and definitions are given in the following table; to these is added a third column under the rubric, DH, which is not immediately relevant.

| Phase: | $D^2O'$ | $D^2O''$ | DH |
|---|---|---|---|
| Static Phase: | 0 | 0 | 0 |
| Basic Expansion: | + | 0 | + |
| Surplus Expansion: | 0 | + | - |
| Compound Expansion: | + | + | |
| Basic Contraction: | - | 0 | - |
| Surplus Contraction: | 0 | - | + |
| Compound Contraction: | - | - | |
| Basic Disequilibrium: | - | + | - |
| Surplus Disequilibrium: | + | - | + |

The principle of the nomenclature is simple: six of the nine phases are expansions or contractions; expansions when outlay in increasing and contractions when outlay is decreasing; these expansions and contractions are divided into basic, surplus, and compound according as basic outlay, surplus outlay or both are increasing (expansion) or decreasing (contraction). When one type of outlay is increasing and the other decreasing, there is said to be a disequilibrium; a disequilibrium is named basic when basic outlay is the weak sister, surplus when surplus outlay is. There remains only the static phase in which both basic and surplus outlay are constant.

The second element in the definition of normative phases is a set of simplying conditions. Their effect will be to make the other variables of the basic circuit, DI', DD', DE', DS' dependent on DO', and similarly the other variables of the surplus circuit, DI", DD", DE", DS" dependent on DO". Thus, the definitions of the phases in terms of the variation of DO' and DO" become definitions in terms of a systematic variation of all the variables.

The first of these simplifying conditions is a postulate of cross-over equilibrium. It will be recalled that the rate of cross-over difference, DG, is defined by

$$DG = G''DO'' - G'DO' \quad (4)$$

so that when cross-over equilibrium makes this difference zero

$$DO'/DO'' = G''/G' \quad (6)$$

Hence, in the static phase, when DO' and DO" are both constant, cross-over equilibrium is satisfied by a constant ratio of the cross-over fractions; G"/G' is the same ratio, interval after interval, and so DH is zero. On the other hand, when one of the pair, DO' and DO", is varying while the other is constant, the ratio, G"/G', has to be constantly undergoing adaptation if cross-over equilibrium is to be maintained; thus, in the basic expansion, the surplus contraction, and the surplus disequilibrium, DH has to be increasing, while in the surplus expansion, the basic contraction, and the basic disequilibrium, DH has to be decreasing. Finally, in the compound expansion and the compound contraction, when both rates of outlay are varying in the same direction, one cannot conclude immediately whether DH has to be positive or zero or negative to give cross-over equilibrium; in these two cases a blank was left in the third column, DH, of the table of names and definitions of the phases.

The postulate of cross-over equilibrium may be stated more concretely as follows. $G''$ is the fraction of surplus outlay going to basic demand: thus, $G''DO''$ includes nearly all the wages in surplus outlay and a notable proportion of salaries, rents, royalties, dividends; everyone has to live, to purchase consumer goods and services. Similarly, $(1 - G')$ is the fraction of basic outlay going to basic demand; it is similar ~~axxinxchmmx~~ in character to $G''$ and indeed, since people do not regulate their spending according as their income is from basic or from surplus production, one may expect that

$$G'' = 1 - G' \qquad (12)$$

Hence one may eliminate either $G'$ or $G''$ from the condition of cross-over equilibrium, writing

$$DO'/DO'' = (1 - G')/G' = G''/(1 - G'') \qquad (13)$$

The meaning of the condition now becomes clear. While people do not regulate their spending according to the origin of their income, they do have to regulate it according to the proportion of consumer goods and services in total production. If the production of consumer goods and services ~~is yielding~~ involves an outlay that is four times as great as that involved in the production of producer goods and services, then four fifths of total income have to move to the consumer market; otherwise cross-over equilibrium fails. On the assumption expressed in equation (12) above, four fifths of total income move to the consumer market when $G''$ is 80% and $G'$ is 20%. In the static phase these percentages would remain unchanged. But in the expansions and the contractions these percentages have to be changing constantly. If a basic expansion makes $DO'/DO''$ equal to 5, then $G''$ has to advance from 80% to 83.$\dot{3}$% while $G'$ has to recede from 20% to 16.$\dot{6}$%. On the other hand, if a surplus expansion makes $DO'/DO''$ equal to 3, then $G''$ has to recede from 80% to 75% while $G'$ advances from 20% to 25%. Of course, when we say that $G'$ or $G''$ have to undergo certain modifications, we speak of no objective and necessary cause; we merely enuntiate the consequent of the hypothesis of cross-over equilibrium. With regard to the verifiability of that hypothesis in actual economic history, we are inclined to be very sceptical. Under the profit criterion there is a marked bias in favour of a large $G'$ and a small $G''$, so that surplus expansions are prolonged and basic expansions short-lived.

The second of the simplifying conditions defining normative phases is an equilibrium between the thrifty and the spendthrifts, between the melancholy who put their earnings aside in anticipation of future rainy days and, on the other hand, the sanguine who cannot fancy the future being as bad as the present and so spend what they have, and what they can borrow, with an open hand. In precise form this second postulate is that

$$O = DD' = DD'' \qquad (14)$$

Movements from the redistribution function to basic demand are cancelling, interval by interval, with movements from basic demand to the redistribution function. Similarly, movements between surplus demand and the redistribution function result in a cancellation. Such movements may be as great or as small as you please; they may vary enormously or not at all; the postulate is satisfied as long as the aggregate result is, in each case, a cancellation. Obviously, this is very much a simplifying condition. At a single stroke ~~tx~~ it brushes aside all the ~~comp~~ complications of the equation between savings and investment until such time as these questions can be ~~prx~~ discussed profitably.

The third of the simplifying conditions is that basic expenditure keeps pace with basic outlay, and that surplus expenditure keeps pace with surplus outlay. The possibility of this "keeping pace" has been secured by the two previous postulates. Cross-over equilibrium makes basic income exactly equal to basic outlay, and surplus income exactly equal to surplus outlay. For

$$DI' = DO' + DG \qquad (7)$$

$$DI'' = DO'' - DG \qquad (8)$$

and cross-over equilibrium means that DG is zero. Next, the equilibrium between the sanguine and the melancholy prevents this income from running off to the redistribution function to the depletion of the demand functions as also it prevents the demand functions from becoming clamorous because of excess releases from the redistribution function. Thus, the possibility of expenditure keeping pace with outlay has been provided for. The postulate is that expenditure not merely can but actually does keep pace. And, as is clear enough, this postulate is implicit in the idea of types of phase initiated by outlay. For suppose that outlay increased and expenditure did not follow; plainly enough, entrepreneurs would take the hint and desist from their expansion; and if they did desist, the phase would change, for the phase ~~ix~~ in question would be defined by increasing outlay. Thus, this third simplifying condition is not so much an additional postulate as an implication of our method of procedure. It remains, however, that some attempt be made to declare more precisely what is meant by expenditure "keeping pace." In the first place, it does not mean that in each interval

$$DE' = DO'$$

$$DE'' = DO''$$

Such a postulate would disregard entirely the fact of a production period, that, form example, in an expansion outlay begins to increase, and keeps increasing, considerably in advance of the arrival of the increment of goods and services on the final markets. Equality of expenditure and outlay interval by interval would mean a rise of price levels to enable the increased income to be spent when the increment of goods and services was not yet on sale; similarly, it would mean a drop of price levels to enable decreased income to clear the market before the market bagan to suffer a curtailment of supply. Thus, the postulate of continuity, of "keeping pace," has to be put in the form of such equations as

$$DE'_j = DO'_i \qquad (15)$$

$$DE''_j = DO''_i \qquad (16)$$

where the suffixes "i" and "j" refer to two different intervals, and the time between these intervals is equal to the weighted average production period of the goods and services undergoing increase or diminution. In this precise form, the postulate of continuity is not particularly realistic; but it may not be out of place to observe that there is no necessity of realism at this point of the inquiry. The function of theory is to construct ideal lines from which one can approximate systematically towards the real lines; our present concern is to obtain clear and definite ideal lines.

The fourth and last of the simplifying conditions has to do with the velocity of money in the basic and the surplus circuits. The postulate is framed as a conditioned correlation, namely, that $D^2O'$ and $DS'$, and similarly $D^2O''$ and $DS''$, are simultaneously positive, zero, or negative, except in so far as this is prevented by the already posited postulate of continuity which also regards monetary velocities in the circuits. The correlation itself amounts to saying that when $DS'$ or $DS''$, the net transfers from redistribution to the demand functions, are positive, then the decrease in velocity will not be so great as to cause $D^2O'$ or $D^2O''$ to be negative; again, when $DS'$ or $DS''$ is negative, the increase in velocity will not be so great as to enable $D^2O'$ or $D^2O''$ to be positive. In the main these suppositions are plausible enough: industry and commerce generally are not brisker when there is a contraction of short term loans, nor are they slackening when short term loans are expanding. What is not plausible is the exact correlation of a zero $D^2O'$ with a zero $DS'$, and a zero $D^2O''$ with a zero $DS''$. But while this is not plausible, it remains a convenient assumption for the moment. So much for the correlation of the net transfers to the supply functions with the increments in the rates of outlay. It is has been said that this correlation is supposed only in so far as it does not conflict with the postulate of continuity, with the postulate that there is a lag, proportionate to production periods, between ~~income and expenditure~~ changes in the rates of income and of expenditure. The possible conflict becomes apparent as soon as one attempts to envisage the process of an expansion or contraction.

Let us say that DS' is some positive quantity, k, over a series of intervals. The immediate effect is an increment of basic outlay of, say, k' per interval, where k' is a function of k and of the velocity in the basic circuit. Thus, in the first interval DO' becomes, say, (m' + k'). In the second interval, DS' again transfers k and this makes possible, we may suppose, another addition of k' to the rate of basic outlay. But the question arises whether this k' is to be added to m' or to (m' + k'). ~~If the rate of expenditure~~ If increases in the rate of expenditure do not lag behind increases in the rate of income, one would be inclined to say that in the second interval DO' is (m' + 2k'). But ~~ax~~ when one has ~~prop~~ postulated lags proportionate to production periods, one has to choose the other alternative. Thus, the effect of a net transfer of k per interval will raise DO' from m' to (m' + k') in a first interval, and so give a postive $D^2O'$, then for a series of intervals equal to the lag between increments of income and increments of expenditure, it will maintain DO' at (m' + k'), and so give a zero $D^2O'$ when DS' is positive; finally, only at the end of this lag will DO' set forth on its full expansion with increasing receipts combining with the net transfers from redistribution to basic supply to give the series, m' + k', m' + 2k', m' + 3k', m' + 4k',.... and so make $D^2O'$ equal to k' interval by interval. Thus, there is a real conflict between the present velocity postulate and the previous postulate of continuity; accordingly, we have made the ~~past~~ velocity postulate conditioned, so that the postulate of continuity prevails. Except for lags in $D^2E'$ and $D^2E''$, a positive or negative value of DS' or DS" gives a corresponding positive or negative value of $D^2O'$ or $D^2O''$.

So much for the idea, the names, the definitions, and the simplifying conditions of the normative phases. Under the defined conditions any phase can be had by controlling DS' and DS". According as these are positive, zero, or negative, $D^2O'$ and $D^2O''$ will be positive, zero or negative, in virtue of the ~~according to the~~ conditioned velocity postulate. According as $D^2O'$ and $D^2O''$ are positive, zero, or negative, it follows by definition that DO' and DO" are increasing, constant, or decreasing. By the postulate of cross-over equilibrium, DI' is always equal to DO' and DI" is always equal to DO". By the postulated equilibrium between the thrifty and the spendthrifts initiative is removed from the demand functions, and by the postulate of continuity the rates of expenditure, DE' and DE", keep pace in due time with the rates of outlay, DO' and DO". Thus increased outlay in due time returns to the supply functions to join with present increments and give the cumulative effect of the expansions; similarly, decreased outlay in due time is manifested in decreased receipts, and this negative joined with the negative action of a minus DS' or minus DS" gives the cumulative effect of the contractions. A positive DS' maintained over a series of intervals will make the basic circuit bigger and bigger; a negative DS' will make it smaller and smaller; a zero DS' will leave it constant. Similarly, DS" controls the surplus circuit. Thus, the idea of normative phases has enabled us to take our static frame of reference and transform it into nine types of dynamic frames of reference.

3. The Cycle of the Normative Phases. We must now revert to our point of departure. The division of payments into redistributive and operative, expenditure-receipts and outlay-income, basic and surplus, was based upon relations between the payments and the productive process. Then the process was forgotten: two circuits of expenditure, receipts, outlay, income were set up, and conditions were defined under which the ~~acceleration of the~~ acceleration of the circuits was made dependent upon movements of money between the redistribution function and the ~~demand~~ basic and the surplus supply functions. This gave the nine normative phases, but we have now to inquire into the relation between such phases and the productive process. The result of this inquiry will be twofold: it will reveal an analogy of productive phases parallel to the monetary phases already defined; and it will arrange all such phases into a series, into the unity of a cycle.

Let us suppose a complete list made of all types and qualities of goods and services sold at the basic final market in either or both of two successive intervals. Let the prices and quantities of the first of these intervals be

$$p_1, p_2, p_3, \ldots \quad p_n$$

$$q_1, q_2, q_3, \ldots \quad q_n$$

and let the increments of these prices and quantities emerging in the second interval be

$$dp_1, dp_2, dp_3, \ldots \quad dp_n$$

$$dq_1, dq_2, dq_3, \ldots \quad dq_n$$

so that one can write with complete accuracy

$$DE'_j = \sum p_i q_i \qquad (17)$$

$$DE'_k = \sum (p_i + dp_i)(q_i + dq_i) \qquad (18)$$

$$D^2E' = \sum (q_i dp_i + p_i dq_i + dp_i dq_i) \qquad (19)$$

where the third equation results from the subtraction of the first from the second, the suffixes "j" and "k" refer to any two successive intervals, and the summations are taken by giving "i" successively all values from "1" to "n."

On inspection of the third equation, (19), it is apparent that the increment of basic expenditure, $D^2E'$, consists of three elements: the first depends entirely upon price increments; the second depends entirely upon quantity increments; and the third is a mixture of both, a product of price increments and quantity increments. ~~Fortunately, however, our problem is not a problem of arriving at exact or nearly exact quantity numerical values.~~ Immediately there arises the problem of determining to what extent $D^2E'$ results from price change and to what extent it arises from quantity change. To meet it we define terms and distinguish cases. Let $DP'$ be the average increment of prices and $DQ'$ be the average increment of quantities; and let us suppose that these increments are added to a price index, $P'$, and a quantity index, $Q'$. However, before these definitions can be made more precise, two cases have to be distinguished: market continuity, when the product of price increments by quantity increments, $dp_i dq_i$, is relatively small and so may be neglected in an approximate estimate; and market discontinuity, when this product is not relatively small and so may not be neglected.

In the case of market continuity the relations between P', DP', Q', DQ' are defined by the equations:

$$P'Q' = \sum p_i q_i \qquad (20)$$

$$P'DQ' = \sum p_i dq_i \qquad (21)$$

$$Q'DP' = \sum q_i dp_i \qquad (22)$$

By assigning any numerical value for P', the price index, one can at once determine numerical values for Q', DQ', and DP'. Further, with regard to a series of intervals, one can ~~mark~~ choose price and quantity indices

$$P'_1,\ P'_2,\ P'_3,\ldots\ P'_n$$
$$Q'_1,\ Q'_2,\ Q'_3,\ldots\ Q'_n$$

that satisfy the exact series of equations

$$P'_j Q'_j = DE'_j \qquad (23)$$

and fall within the limits determined by the approximate equations

$$P'_k = P'_j + DP'_j \qquad (24)$$

$$Q'_k \cong Q'_j + DQ'_j \qquad (25)$$

In this manner, there will be only one purely arbitrary number in the double series of indices, say $P'_1$, the price index of the first interval of the series.

In the case of market discontinuity, DP' and DQ' will cease to be algebraic symbols and become mere symbolic abbreviations, that is, one gives up the problem of assigning numerical values to DP' and DQ' and becomes content with determining on the whole whether or not there is upward or downward price or quantity change, whether DP' and DQ' are positive, zero, or negative. In general such determination offers little difficulty. For if there is market discontinuity, then $\sum dp_i dq_i$ is large; and from an inspection of the terms in the summation one can tell whether this is the result of increasing or decreasing prices, of increasing or decreasing quantities. In any such instance one may conclude that DP' and DQ' are positive or negative though one cannot say what numerical increments are to be added to the price index, P', and the quantity index, Q'.

There remains an ambiguity, namely, the case of the emergence of new types or new qualities of goods and services. If we suppose that in the previous interval, when their quantities were zero, their prices were also zero, then the total receipts from these goods and services appear in the summation, $\sum dp_i dq_i$. If, on the other hand, we project their prices backwards from the second interval to the first, then the same total receipts appear in the summation $\sum p_i dq_i$ for then the price increments, $dp_i$, are zero instead of the initial prices, $p_i$, being zero. A balance of considerations seems to favour the latter procedure. Accordingly, it is here assumed, though one must bear in mind its implication, namely, that there is a case when DQ' is zero yet a qualitative acceleration is going forward because new types and qualities of goods and services are displacing older types and qualities.

So much for the definitions of DP' and DQ' and, in the case of market continuity, of the indices, P' and Q'. In like manner we suppose defined P", Q", DP", DQ", which have the same meaning with regard to final sales of surplus goods and services as P', Q', DP', DQ' have with regard to final sales of basic goods and services.

So far attention has been directed to the analysis of the increments per interval of final expenditure, $D^2E'$ and $D^2E''$. We have now to turn to the factors in these increments, and first to the indices of quantity change, $DQ'$ and $DQ''$. It has been argued that, in general, it is possible to tell whether these indices are positive, zero, or negative. Hence it followed that one may define nine quantity phases in parallel fashion to the nine ~~circultas~~ circulation phases already examined. Thus, a quantity static phase is a series of intervals in which both $DQ'$ and $DQ''$ remain zero; a quantity basic expansion is a series of intervals in which $DQ'$ is positive and $DQ''$ zero; a quantity surplus expansion is a series of intervals in which $DQ'$ is zero and $DQ''$ positive; and similarly there is a quantity compound expansion (+, +), a quantity basic contraction (-, 0), a quantity surplus contraction (0, -), a quantity basic disequilibrium (-, +), and a quantity surplus disequilibrium (+, -).

In the short run such quantity phases may result from mere variations in the use of existing capital capacity; thus, the present war has witnessed a great increase in the use of railroads with very slight addition to railway capital equipment. In a still shorter run quantity phases may result from the mere depletion or piling up of inventories or stocks of goods, from the lengthening or shortening of hours of labour, and so on. ~~But~~ In such instances there is no correlation between basic and surplus quantity variations. But in the long run, and especially in the very long run, such a correlation exists. It is that surplus production is the accelerator of basic production. In other words the correspendence between the two is not a point-to-point but a point-to-line correspondence: a new ship is not needed for every trip across the seas, nor a new shoe-factory for every new new pair of shoes; one ship yields a flow of voyages, one factory yields a flow of shoes, and so a series or flow or stream of surplus goods and services corresponds to a series of series, a flow of flows, a stream of streams of basic goods and services. Now such a correspondence, if it is to be expressed not in terms of expectations of the future but in terms of present fact, is a correspondence of accelerator to accelerated. Thus, with regard to any given pattern of combinations of production factors, there is some long term average quantity of surplus production necessary for the maintenance and the renewal of both surplus and basic means of production. When Q" stands at that average, then the accelerator of the system is merely overcoming what may be termed the system's friction. There results a quantity static phase: both DQ' and DQ" are zero. If the system is to move into a long term expansion, this movement has to begin with a surplus quantity acceleration: surplus production has not merely to maintain or renew existing capital equipment but has to reach a level at which it turns out new units of production and maintains or renews a greater number of existing units; this gives the quantity surplus expansion; DQ" is positive but DQ' as yet remains zero. The quantity surplus expansion has its most conspicuous instances in industrial revolutions and five-year plans in which standards of living do not improve while a national industrial equipment is wholly transformed; indeed, in such a movement it may happen that standards of living may deteriorate, and then one has the closely allied basic disequilibrium in which DQ" is positive and DQ' negative. Eventually, however, this increasing the means of producing the means of production reaches its goal and turns to increasing basic products. The new units now emerging are not surplus but basic; thus DQ" stands at zero while $DQ'$ is positive; there is the quantity basic expansion, the general rise in standard of living that is the normal objective of the previous surplus expansion. It may very well happen that the standard of living begins to rise before the increase of the surplus process comes to a halt: the phase then is the quantity compound expansion with both $DQ''$ and $DQ'$ positive. Again, it may happen that the increase of the surplus process was over-estimated, and so the basic expansion will be interrupted momentarily by a surplus disequilibrium in which $DQ''$ is negative, with $Q''$ moving down to a lower average level, while $DQ'$ remains positive. Finally, unless this transforming process is immediately followed by another, the basic expansion eventually reaches its term, and a new static phase on a notably higher level than the initial static phase results.

Thus, the quantity phases have an inner logic of their own. They are not merely a list of possible dynamic configurations but they naturally fall into a series, into a cyclic process, that moves from a static phase through surplus expansion, basic disequilibrium, compound expansion, surplus disequilibrium, and basic expansion, towards a new static phase in which a higher standard of living is attained permanently. Now this cycle has two features. In the first place, it is grounded in the nature of things. A higher standard of living in an economic community is, generally, both a qualitative and and a quantitative improvement of the flow of basic goods and services. To attain that improvement, the community has to set about transforming its pattern of combinations of production factors. Such a transformation postulates at the outset an increase of surplus production and so a surplus expansion with perhaps a basic disequilibrium. However, once this condition is fulfilled, there follows the increase of the standard of living in a compound and then a basic expansion with perhaps a period of surplus disequilibrium. Finally, the higher standard of living reaches its peak, the maximum possible within the transformed economy, and once it is attained there is no more to be done than maintain it. The second feature is that this cyclic process, grounded in the nature of things, does not coincide in all respects with the familiar trade cycles. The latter are marked by basic and surplus and compound contractions, while no mention of contractions was made in the logical scheme by which an economic community moves systematically from a lower to a higher standard of living. Thus it is necessary to distinguish between pure cycles, which omit contractions, and perturbed cycles in which the upward movement of the pure cycle is cut short by a general contraction. Generally there is no objection to the pure cycle which yields an improvement of living standards; equally generally there is vehement objection to the trade cycle which begins with the movements of the pure cycle but ends up with something very different. However, the pure cycle is for the moment the mere suggestion of a possible theoretic construction; later we shall return to it, but the present point is simply the observation that quantity phases are phases of a process.

The analysis of $D^2E'$ and $D^2E''$ revealed not only quantity factors of acceleration, $DQ'$ and $DQ''$, but also price factors, $DP'$ and $DP''$. The significance of the latter is that they mark a divergence between the circulation phases, defined in terms of $D^2O'$ and $D^2O''$, and the quantity phases, defined in terms of $DQ'$ and $DQ''$.

Accordingly, it is not worth while to set up a further group of nine price-level phases, since DP' and DP" simply indicate what might be described metaphorically as the inertia of the quantity process of goods and services in its response to accelerations initiated in the circulatory process of payments. Rapid increases or decreases in the circulatory process have not a proportionate effect in the quantity process but are in part absorbed by positive or negative price increments. Thus booms are notoriously inflationary and slumps deflationary. Hence DP' and DP" are best taken as indices of divergence between circulatory and quantity phases.

The main analytic apparatus is now complete. Two acceleration systems have been defined: a circulatory system consisting of two connected circuits that are accelerated by an external redistribution function; a quantity system of two parts in which one part is the long-term accelerator of the other. In each of these acceleration systems nine phases of a cyclic process have been defined in parallel fashion, with postulates determining the normative phases of the circulatory system, and an inner logic or ground in the nature of things indicated as the normative or pure cycle of the quantity process. Finally, indices of price increments serve as markers of the divergence between the two systems.

4. <u>The Effect of Net Transfers.</u> The basic circuit is connected with the redistributional function by two routes, net transfers to supply, DS', and net transfers to demand, DD'. Similarly, the surplus circuit is connected by two routes, net transfers to supply, DS", and net transfers to demand, DD". In defining the normative phases, the correlation of these net transfers with the rates of the circuits was evaded by the introduction of postulates. It is now necessary to determine, in so far as possible, what the correlation is.

The existence of the problem is apparent. For instance, DO' is a rate, so much money going to basic outlay every so often. The increase of this rate may result from an increase in the quantity or from an increase in the velocity of money, so that one might write

$$D^2O' = m.dv + v.dm + dv.dm \qquad (26)$$

where "m" is a quantity of money and "dm" its finite increment, and "v" is a velocity of money and "dv" its finite increment. Evidently, unless one knows the conditions under which money changes its velocity in the circuits, one cannot tell when a net transfer, DS', is needed to effect an increase of DO'. Further, unless one knows some correlation between DS', which is an increment of circulating capital, and dm, which is an increment in the quantity of money in outlay, one is still in the dark about the relations between DS' and $D^2O'$.

A general solution of the problem is not as difficult as might appear. We have to deal not with the quantity and velocity of money in all and any payments but only with the quantity and velocity in operative payments. But operative payments have been defined as standing in a net-work congruent with the net-work of the productive process; it follows that we have to deal with quantities of money congruent with the values emerging in the productive process, and with velocities of money congruent with the velocities of the productive process. In fact we shall be able to deal with the more precise ideas of turnover size and turnover frequency instead of the ill-defined ideas of quantity of money and velocity of money.

Perhaps the first step will best be an illustration of this correlation. Suppose that two ship-builders, A and B, each launch a new ship every 15 days, that A has 5 ships under construction at once while B has 10, and so that A completes another ship every 75 days while B requires 150 days. To avoid irrelevant differences, we may suppose that all ships are similar in all respects, that they are sold as soon as they are launched for the same selling price, and so that total receipts and the aggregates of costs and profits are the same in both instances. There are then two identd equal volumes of business: each receives the selling price of one ship every fifteen days; and each proceeds to make aggregates of initial and transitional payments at the same rate. However, this identity of volumes of business does not involve an identity of quantities and velocities of money. A's turnover is an aggregate of receipts and payments on 5 ships, while B's turnover is an aggregate of receipts and payments on 10 ships. When A sells a ship, he has been making payments on it for 75 days, on a second ship for 60 days, on a third for 45 days, on a fourth for 30 days, on a fifth for 15 days. But when B sells a ship, he has making ma payments on it for 150 days, on a second for 135 days, and on a third to a tenth ship for periods of 120, 105, 90, 75, 60, 45, 30, and 15 days respectively. Thus, A's volume of business is a matter of 5 ships every 75 days, while B's is a matter of 10 ships every 150 days. The two volumes are equal, but A moves money twise as rapidly as B, yet moves only half as much as B.

The difference between turnover size and turnover frequency has been put with exaggerated clarity. It remains that the same distinction can be made with regard to every entrepreneur in basic or in surplus supply. Each one is p rforming a certain number of services or contributing to the supply of a certain number of products at once. Such performance or contribution takes a certain amount of time. But once this time has elapsed, the entrepreneur proceeds to a new batch of services or products. Thus entrepreneurial activities fall into series of repeated routines. Further, each of these routines form financial unities: receipts come in for the goods or services supplied; transitional payments are made to other entrepreneurs for their contribution to the supply; initial payments are made to the immediate factors; and the aggregate of transactions regarding that batch of goods or services is closed. Thus, the production period hasits correlative in the monetary order, namely, the turnover period; and similarly the value of the goods processed or the services rendered in the production period has its monetary correlative in turnover size.

Certain clarifications are in order. The turnover period is not necessarily identical with the production period, for the turnover period is the period of both production and sale. If the first ship-builder, A, could sell a ship no oftener than once every sixteen days, his production period might remain 75 days but his turnover period would lengthen to 80 days. The production period sets a lower limit to the turnover period, but turnover periods lengthen when sales do not keep pace; and in the limit decreasing sales lead to a reduction of turnover size. Thus, if A could sell one ship only once every 19 days, he might deliberate between having 5 ships in construction at once with a turnover period of 95 days, or reducing his construction to 4 ships at once with a turnover period of 75 days.

Turnover size will be measured by the transitional and initial payments arising from the turnover. When the entrepreneur's operations are constant, turnover size will also be equal to the receipts from the turnover. When however the entrepreneur is increasing or decreasing the scale of his operations, recipts differ from turnover size, and this difference involves a net transfer from or to the redistribution function. Thus if decreasing sales led the first ship-builder to have only four ships under construction at once, his active circulating capital would decrease by one fifth; the receipts for five ships are not needed to meet the initial and transitional payments on four. Later, if increasing sales encourage a return to a turnover of 5 ships at once, then circulating capital that had gone off to the redistribution function has to return; receipts from four ships do not suffice to meet the transitional and initial payments on five. It may be noted, finally, that we make provision further on for the complication caused by increasing or decreasing inventories, that is, stocks of goods kept on hand to meet sudden increases in demand.

Let us now systematize the results obtained. With regard to all the entrepreneurs in basic supply during a given interval, let $r_{ij}$ be the initial payments of the ith ~~turnover~~ entrepreneur in his jth turnover or fractional turnover during that interval, and let $s_{ij}$ be the corresponding transitional payments. Then, the aggregate initial payments of basic supply during the ~~tax~~ interval, which is the definition of DO', may also be expressed as a double summation of $r_{ij}$, ~~namely~~ so that

$$DO' = \sum \sum r_{ij} \tag{27}$$

and similarly the volume of transitional payments, which may be termed DT', is another double summation, namely,

$$DT' = \sum \sum s_{ij} \tag{28}$$

Next, if we define turnover frequency as the number of turnovers of a given entrepreneur in a given interval, and observe that this number may be a fraction, proper or improper, it will be possible to find average values for the initial payments and other average values for the transitional payments in the successive turnovers of each entrepreneur during any given interval. This makes it possible to replace the double summations by single summations so that

$$DO' = \sum r_i n_i \tag{29}$$

$$DT' = \sum s_i n_i$$

where $n_i$ is the turnover frequency of the ith entrepreneur in the given interval and the summations are taken with respect to all entrepreneurs in basic supply.

If now two successive intervals are compared, and it is found that the ith entrepreneur increases his initial payments by $dr_i$, his transitional payments by $ds_i$, and his turnover frequency by $dn_i$, then the increments in DO' and DT' will be

$$D^2O' = \sum (r_i dn_i + n_i dr_i + dn_i dr_i) \qquad (31)$$

$$D^2T' = \sum (s_i dn_i + n_i ds_i + dn_i ds_i) \qquad (32)$$

where again the summations are taken with regard to all entrepreneurs in basic supply. Equation (31) gives a correlation between changes in velocity of money and changes in the rate of basic outlay. Basic outlay can increase, through the increase of monetary velocity, to the extent that turnover frequencies can increase; and turnover frequencies increase in two ways; first, by the elimination of lagging sales so that turnover periods are reduced to the size of production periods; second, by the introduction of more rapid methods of production, provided that these more rapid methods are accompanied by an increased efficiency in sales. This correlation is far from being a model of precision, but at least it takes variations in DO' from changing monetary velocity out of an obscure region of pure indetermination. Changing ~~methods of~~ production periods are observable phenomena; so also are brisker and slower sales; without either of these we cannot suppose that DO' varies from changes in monetary velocity; and with these one can suppose no more than a limited and proportionate change of monetary velocity. Other changes in DO' have to be attributed to net transfers, DS'.

There remains the ~~ne~~ question, To what extent does DS' effect an increase or decrease in DO'? For evidently DS' does not effect solely the quantity increments, $dr_i$, but also the quantity increments, $ds_i$. Consider the equation,

$$\sum s_i n_i \quad = \quad \sum v_i r_i n_i \tag{33}$$

in which the left-hand side gives the volume of transitional payments in the interval while the right-hand side multiplies by a "$v_i$" the volume of initial payments. What is the multiplier, $v_i$? At a first approximation it is the number of times the product per interval of the _i_th entrepreneur is sold transitionally during the interval. Thus, when the _i_th entrepreneur deals immediately with consumers, $v_i$ is zero. When his product per interval is sold once transitionally during the interval and once finally, then $v_i$ is one. When three quarters of his product is sold four times and one quarter is sold five times, exclusive of final sales, then $v_i$ is $(4x3/4 + 5x1/4)$ or four and one quarter. But this gives only the first approximation to $v_i$. At a second approximation one has to take into account that, particularly in the more distant transitional sales, it is not the product of the present interval but the product of previous intervals that is being sold transitionally. These products may differ in quantity and in price from the present interval's $r_i n_i$, but it remains that there is some numerical proportion between the payments they involve. Thus a further correction can be introduced into the calculation of the transitional velocities, $v_i$, and it must be introduced to satisfy equation (33). It will be convenient to term the latter type of variation of $v_i$ its conventional variation, while variation in the number of transitional sales will be called its independent variation.

So much for the general functional relation between initial and transitional payments, equation (33). If we suppose that turnover frequencies are constant, then the relation between increments $dr_i$, $ds_i$, $dv_i$ is given by

$$\sum (r_i dv_i + v_i dr_i + dr_i dv_i) = \sum ds_i \qquad (34)$$

The immediate effect of increments in outlay per turnover, $dr_i$, is offset by the opposite increments in transitional velocities, $dv_i$, according to the convention of the preceding paragraph; hence increments in transitional payments per turnover, $ds_i$, do not appear at once, for the transitional buyers do not increase their payments when the ith entrepreneur increases his outlays but when they purchase his increased products. On the other hand, as soon as these purchases begin, the convention works in the opposite direction, for the increments, $dv_i$, now have the opposite sign, and through $dr_i$ may have returned to zero, $r_i$ is standing on its new level. Next, if one turns from these short-term effects, one may suppose that $v_i$ remains constant so that $dv_i$ is zero; this gives the logg-term correlation

$$\sum v_i dr_i = \sum ds_i \qquad (35)$$

which holds in a successful acceleration of the whole process. Now the net transfer from or to the redistributional function, DS', is the proximate source of increments in circulating capital needed for both transitional and initial payments. Hence, when turnover frequencies are constant and accelerations are successful, i.e. the increased product is sold all along the line, then

DS

$$DS' = \sum (dr_i + ds_i) \tag{36}$$

which with equation (35) gives

$$DS' = \sum (1 + v_i) dr_i \tag{37}$$

Thus, the net transfer, DS', is equal to the increments in active monetary circulating capital; and these increments are equal to the increments in outlay per turnover, plus multiples of the latter depending on the number of transitional sales.

One must be content merely to mention the possibility of independent variations of $v_i$. These emerge in changes in the structure of transitional payments when, for example, a merger eliminates or the break-up of a large corporation into smaller units creates a proprietary barrier that involves transitional payments. If the aggregate of outlays remains the same, one may expect the second term of the left-hand side of equation (34) to summate to zero, positive instances of $dr_i$ cancelling against negative instances. The same holds for the third term. Hence one would get

$$\sum ds_i = \sum r_i dv_i \tag{38}$$

so that the merger, in which $dv_i$ is negative, would give negative increments in active monetary circulating capital devoted to transitional payments. The break-up of a corporation would have the opposite effect. However, such structural changes affect not only transitional payments but also turnover frequencies; the length of the turnover ~~intervals~~ periods determines the quantity of money required for ~~outlay~~ payments in each turnover and so the new pattern of instances of $dr_i$ and $ds_i$. Thus while circulating capital needed for transitional payments decreases in one respect, this may be offset by other requirements.

Equations (36) and (37) tacitly assume that the increased quantities of money involved in increased turnovers are derived exclusively from the net transfer to basic supply, DS'. This tacit assumption has now to be corrected: the quantity of money in the basic circuit may increase or decrease in three ways, by a net transfer to basic supply, by a net transfer to basic demand, and by a cross-over disequilibrium. Let DM' be this increase in quantity of money in a given interval, so that

$$DM' = DS' + DD' + DG \qquad (39)$$

where the parallel equation for the surplus circuit would be

$$DM'' = DS'' + DD'' - DG \qquad (40)$$

since a positive DG empties the surplus circuit in favour of the basic while a negative DG empties the basic circuit in favour of the surplus (cf. equation 4 above).

The next question is whether DM' may replace DS' in equations (36) and (37). The answer involves some determination of the concept of income velocities. Let us write

$$DD' = DE' - DI' \qquad (41)$$

so that savings are in the redistribution function: when people spend less than they earn, the difference gives a negative DD'; and when they spend more than they are earn, they are drawing on savings in the redistribution function and effecting a positive DD'. The effect of this equation, (41), is to eliminate the concept of income velocities. There is a rate of income, DO'; there is a rate of expenditure, DE'; but between these two there is no rate but only a quantity of money which DD' increases or diminishes in a manner that equilibrates the two terminal rates, DO' and DE'. This device assimilates the analysis of velocity in basic demand to the analysis already given for ~~sa~~ basic supply: for in supply there were posited no velocities of money between payments, but simply rates of payment with quantities of money between them and DS' as the proximate source of variations in these quantities.

A further effect of equation (41), more relevant to the present issue, appears on recalling that

$$DI' = DO' + DG \tag{7}$$

so that

$$DE' = DO' + DG + DD' \tag{42}$$

so that basic expenditure equals basic outlay plus the cross-over difference plus the net transfer to basic demand. But basic expenditure is also basic receipts, that is, the receipts of the entrepreneurs who deal immediately with consumers. Now such receipts over an interval may be equal to or greater than or less than the payments, initial and transitional, of these entrepreneurs over the interval. Let us assume that at the end of each interval these entrepreneurs make up their books, transfer a positive difference to the redistribution function to give a ~~nea~~ negative element in DS', or make good a negative difference by effecting a positive transfer from the redistribution function. Then,

$$DM' = \sum (ds_i + dr_i) \tag{43}$$

and on the suppositions of equation (37)

$$DM' = \sum (1 + v_i)dr_i \tag{44}$$

The foregoing results may be put more precisely. It will be recalled that $r_i n_i$ and $s_i n_i$ are approximate average figures over the interval and that $dr_i$, $ds_i$, $dn_i$ are increments found by comparing the averages of two intervals. However, there is another notation, already mentioned, that makes $r_{ij}$ the exact initial payments and $s_{ij}$ the exact transitional payments of the _i_th entrepreneur in his _j_th turnover or fractional turnover during the interval. Let us define $dr_{ij}$ and $ds_{ij}$ as the increments emerging from the comparison of two successive turnovers, figures being taken from complete and not from fractional turnovers. Further, let DM' be the quantitative increment of money in the circuit during the same interval and not, as hitherto, the increment in the second of two intervals under comparison. Then, on the suppositions of equations (42) and (43)

$$DM' = \sum\sum (dr_{ij} + ds_{ij}) \qquad (45)$$

and since it is always possible to find numbers, $u_{ij}$, such that

$$\sum\sum ds_{ij} = \sum\sum u_{ij} dr_{ij} \qquad (46)$$

one can also write

$$DM' = \sum\sum (1 + u_{ij}) dr_{ij} \qquad (47)$$

where all summations are taken first with respect to turnovers "j" and then with respect to entrepreneurs "i." In this notation varying velocities, i.e. turnover frequencies, appear in the number of terms in the summations with respect to "j".

expansion. Finally, the cycle initiatied by the movement of the process from an initial static phase through a surplus expansion, a basic disequilibrium, a compound expansion, a surplus disequilibrium, and a basic expansion, may be said to have a normative goal in the attainment of a new static phase on a higher level. But to advance steadily towards that goal, to avoid the interruptions of basic, surplus, compound contractions, the agents of the economy have to adapt their preference schedules and correct their expectations to each of the successive phases. For the cycle has an objective logic of its own and its successive phases postulate different preferences and different expectations. On the other hand, to believe and act up to the belief that the preferences and expectations proper to, say, a surplus expansion are equally legitimate and satisfactory pragmatically in a basic expansion or a static phase, that is to invite a type of disaster which by its frequent recurrence has become familiar.

This brings us to the second difference between a Robinson Crusoe and a large scale exchangee conomy. The latter is a monetary economy, and the use of the medium of exchange can act as a screen that hides from view the objective necessity of changing preferences and expectations in accordance with change in productive phases. When Robinson is clearing a new field, he is incapable of the illusion that that activity enables him to have more to eat here and now. When Robinson is reaping greater harvests from more more numerous fields, he is ~~incapble~~ incapable of the illusion that the corn he will not care to eat can be transmogrified into the capital equipment of, say, a powder plant or another cleared field. But the multitudinous Robinsons of the exchange economy are rewarded with money whether they clear fields or

The argument now moves forward to a fresh topic. Monetary velocities in basic supply have been shown to be a function of turnover frequencies, and turnover frequencies a function of the efficiency of sales and the length of production periods. The division of net transfers, DS', between monetary circulating capital for initial payments and for transitional payments, have been made a function of transitional velocities, $v_i$, which depend on the number of times the product of a given entrepreneur is sold transitionally in the standard interval. It remains that we complete the circuit with a consideration of income velocities and of additions to income by net transfers, DD'.

As a matter of convenience let us divide entrepreneurs into three classes: an initial group, $E_i$, which makes no transitional payments; a group of middlemen, $E_j$, whose transitional payments form the total receipts of the initial group; and a final group, $E_k$, whose total receipts are basic expenditure, DE', and whose transitional payments are the total receipts of the middlemen. There are two conventional elements in this structure of basic supply. The first conventional element lies in the manner of the description: we speak of groups of entrepreneurs when really we have no interest in entrepreneurs; we are studying not entrepreneurs but payments in their circular relationships, and, in fact, the entrepreneurs in the three groups are mere figure-heads; what they stand for are sets of payments and receipts, and really what is under discussion are such sets. The second conventional element lies not in the nomenclature but in the structure itself. No real structure of basic supply admits the elegant simplicity of the above description; there are endless complications, and these complications are not constant but shifting. But whatever the complications and their changes, there is one constant feature, namely, the balancing of ledgers, the equality of receipts and payments. The complexities of interdependence represented by the balancing of ledgers are not the object of our study, but that balancing itself. Hence we are content to study such balancing under the simple conditions of three groups of entrepreneurial figure-heads.

Let us say that the volume of payments per interval of the initial group, $E_i$, is $\sum r_i n_i$, of the group of middlemen, $E_j$, is $\sum (r_j n_j + s_j n_j)$, and of the final group, $E_k$, is $\sum (r_k n_k + s_k n_k)$, where "r" denotes initial payments, "s" transitional payments, "n" turnover frequencies, and the three summations are taken with respect to all instances of "i", "j", and "k" respectively. Then since initial payments are identical with outlay we have

$$DO' = \sum (r_i n_i + r_j n_j + r_k n_k) \qquad (39)$$

On the further assumption that in each case payments of the interval equal receipts of the interval, we have

$$DE' = \sum (r_k n_k + s_k n_k) \qquad (40)$$

$$\sum s_k n_k = \sum (r_j n_j + s_j n_j) \qquad (41)$$

$$\sum s_j n_j = \sum r_i n_i \qquad (42)$$

Equation (40) states that the receipts of the final group are equal to their initial and transitional payments. Equation (41) states that the receipts of the group of middlemen are equal to their initial and transitional payments. Equation (42) states that the receipts of the initial group are equal to their initial payments, which, ex hypothesi, are their sole payments. But the transitional payments of the final group are identical with the receipts of the middlemen, and the transitional payments of the middlemen are identical with the receipts of the initial group. Hence the summation of $s_k n_k$ appears in both (40) and (41), and the summation of $s_j n_j$ appears in both (41) and (42). On the elimination of these summations, it appears that DO' and DE' are equal, for both equal the summation of the rates of initial payments per interval. Let us further suppose cross-over equilibrium, so that DG equals zero, and DO' equals DI'. It then appears that the condition and, inversely, the consequent of entrepreneurial receipts equaling entrepreneurial payments is that the expenditure of basic demand, DE', equals the income of basic demand, DI'.

Now it is important to distinguish two different aspects of equations (39) to (42). Under a certain aspect these equations express a truism: if entrepreneurial receipts and payments equate, then they equate not only among entrepreneurs but also between entrepreneurs and the third party, demand. But under another aspect the same equations, so far from expressing a necessary truth, express an almost unattainable ideal, namely, a dynamic equilibrium to which any actual process continually attempts to approximate by varying prices and changing quantities of supply. To study the truism is to study book-keeping, to study the art of double entry, and to learn the magic of the variable items, profit and loss, which perforce make the books balance. To study the ideal is to study equilibrium analysis. The book-keepers are wise after the event. But if the entrepreneurs are to be wise, they have to be wise before the event, for their payments precede their receipts, and the receipts may equal the payments but they may also be greater or less, to give the entrepreneur a windfall profit or loss. Such justification or condemnation of payments by receipts the book-keeper records but the entrepreneur has to anticipate, and the grounds of his anticipations, their effects upon his decisions, and the interaction of all decisions, form the staple topic of equilibrium analysis. Now the view-point of the present discussion is neither that of the book-keeper nor that of the equilibrium analyst. Equations (39) to (42) are regarded not as a set of facts recorded by book-keepers, nor as an ideal which entrepreneurs strive yet ever fail to attain, but as a first approximation to the law of the circulation in the basic circuit. The first approximation to the law of projectiles is the parabola: one might, if one chose, consider projectiles as aiming at/or tending towards the ideal of the parabola yet ever being frustrated by wind-resistance; one might elaborately describe the trajectory of the projectile as an indefinite series of parabolas, each one in succession the goal of its tendency only to be deserted because adverse circumstance set it on another track. In such a description of trajectories there is to be found at least a superficial resemblance with the statement that an economy is tending towards equilibrium at ever instant, ~~thus~~ though towards a different equilibrium at every ~~different~~ successive instant. But whatever the resemblance, and however deep and significant the difference, we here propose to ~~study~~ take a circuit in equilibrium as a first approximation to the law of the circuit and examine first the implications of this law and then the second approximations that are relevant to our inquiry.

If we suppose that equations (39) to (42) represent any first interval and that in a second interval, in which cross-over equilibrium is assumed, the increments in the terms are $D^2O'$, $D^2I'$, $D^2E'$, $D^2R'$, $dr_i$, $dr_j$, $dr_k$, $dn_i$, $dn_j$, $dn_k$, $ds_j$, $ds_k$, then the conditions that the acceleration is a success, i.e., that the acceleration has extended round the circuit in accordance with the first approximation to the law of the circuit, are to found in the following equations.

$$D^2O' = D^2I' = \sum (dr_i n_i + dr_j n_j + dr_k n_k + r_i dn_i + r_j dn_j + r_k dn_k) \quad (43)$$

$$D^2E' = D^2R' = \sum (dr_k n_k + r_k dn_k + ds_k n_k + s_k dn_k) \quad (44)$$

$$\sum (ds_k n_k + s_k dn_k) = \sum (dr_j n_j + r_j dn_j + ds_j n_j + s_j dn_j) \quad (45)$$

$$\sum (ds_j n_j + s_j dn_j) = \sum (dr_i n_i + r_i dn_i) \quad (46)$$

These equations are derived by substituting in equations (39) to (42) a $(r_i + dr_i)$ for an $r_i$, etc., etc., multiplying out the expressions, neglecting the products of two increments, such as $dr_i dn_i$, and eliminating through (39) to (42) the products containing no increments, such as $r_i n_i$. The initial substitution implies that the increments are such as to satisfy the conditions defined by the initial equations. The final elimination separates the donditions of an equilibrium circulation from the conditions of a~~n equilibrium or~~ successful acceleration of a circulation. The neglect of the products of two increments makes equations (43) to (46) approximate except when one is considering pure frequency accelerations (when all instances of dr and ds are zero) or pure quantity accelerations (when all instances of dn are zero).

The significance of equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circuit, a meaning for the already indicated distinction between a pure quantity acceleration and a pure frequency acceleration, a basis of discussion for abortive accel rations, and finally a means of contrasting such circulatory success or failure with the success or failure of acceleration of the productive t rhythms.

Our first task is to complete our inspection of the circuit. There remains the question of quantities of money and velocities of money in basic demand. Now if we define DD' by the equation

$$DD' = DE' - DI' \qquad (47)$$

there seems no lack of plausibility. The equation means that savings are in the redistribution function, so that when people in basic demand are spending more than they earn, they are transfering savings to basic demand to make up the difference; again when they are spending less than they earn, they are creating savings and so transfering money from basic demand to the redistribution function. However, this equation has a further implication, namely, that income velocities, in the aggregate, cease to be an added variable in the system. By giving DD' the above precise meaning, one eliminates such suppositions as that DD' adds to the quantity of money in basic demand merely to decrease the velocity of money there or, inversely, that it subtracts from the quantity of money there merely to increase its velocity. DD' has been tied down to an exclusive role of quantity acceleration, and income velocities in the aggregate are determinate when DI' and DE' are determinate. To put the same point differently, the concept of income velocities has been eliminated; there is a rate of income, DI'; there is a rate of expenditure, DE'; but between the two there is no rate, but only a quantity of money, which DD' has the function of increasing or decreasing. This last statement assimilates the analysis of the basic demand function to that of the basic supply function; for in supply we consider only rates of payment, turnover sizes and frequencies, and between such rates we do not posit virtual monetary velocities but only quantities of money in reserve, quantities which DS' augments or diminishes.

If now we revert ot to equations (43) to (46), we observe that the successful acceleration of the circuit involves two types of increment, quantity increments, $dr_i$, $dr_j$, $dr_k$, $ds_j$, $ds_k$, and frequency increments, $dn_i$, $dn_j$, $dn_k$; reference to $D^2O'$, $D^2I'$ $D^2E'$, $D^2R'$, is omitted since they are but other names for the same realities. Now with respect to the quantity increments, the question arises, To what extent are they due to DS' and to what extent are they due to DD'? In other words, the quantity increments show that there is more money in circulation, say, DM', where DM' is defined by the equation,

$$DM' = \sum (dr_i + dr_j + dr_k + ds_j + ds_k) \qquad (48)$$

or on the suppositions of equation (37)

$$DM' = \sum [(1 + v_i)dr_i + (1 + v_j)dr_j + dr_k] \qquad (49)$$

the summations being taken with respect to all instances of "i," "j," and "k," that is, with respect to all members of the three groups of entrepreneurs. Now when we were engaged in the study of basic supply, it was natural to consider this total quantity increment as the work of the net transfer, DS'; but now evidently such a total increment in the whole circuit may to some extent be the work of DD', so that

$$DM' = DS' + DD' \qquad (50)$$

Which, then, of the two net transfers has the preponderant role in increasing xhx or decreasing the quantity of money in the circuit?

First, with reggrd to increases, it should seem that the role of DD' can be little more than initial stimulation. People may spend more than they earn by drawing on savings, but they cannot do so to any great extent. To give DD' a preponderant role would make quantity accelerations of the basic circuit both small and short-living short-lived. On the other hand, our society has developed vast mechanisms to provide entrepreneurs with the means of making large and sustained additions to the quantity of money in the circuit. It practised mercantilism to obtain more gold when money was gold. It developed banking and bills of exchange. It replaced a gold currency, first, with a gold standard fiduciary issue and, later, with what is to all practical purposes a new kind of money, money-of-account. These developments did not take place to enable consumers to spend more than they earn, but to enable entrepreneurs to pure increase the size of their turnovers. Consumers cannot pay interest on their consumption or even on the increments in their consumption. But entrepreneurs can pay interest on the size of their turnovers. There is a second argument. It is that in an economy in which supply is responsive to demand, any positive action of DD' would be immediately stimulate entrepreneurial expansion. The middlemen and the initial group would begin increased turnovers before they began to receive increased receipts. The increased initial payments in the turnovers would mean an increased rate of income, so that to maintain a positive DD' a further immediate increase in the rate of expenditure would be necessary. There is a third argument which follows out of the second, namely, that at the beginning of an expansion outlays are increasing more rapidly than goods at the final market, so that the increase in income is larger than the immediately available increase of objects on which income may be spent; hence unless DD' is negative and people are spending less than they are earning, prices will rise to give a positive DP' and make excessive expenditure equal to insufficient goods. But as soon as a negative DD' is needed to prevent price inflation, a positive DD' would only accentuate the price inflation; and while this positive action on the part of DD' would reduce to some extent the need of positive action on the part of DS', it also would increase that need especially if the acceleration of the supply of goods and services continues; for the rising price level would be communicated back from the final market through the transitional markets and at the same time a demand for increased wages would arise, so that the whole basis of calculation of initial and transitional payments is raised.

The three arguments tend to show that quantity acceleration of the circuit through the action of DD' is ~~unnecess~~ impossible, inasmuch as people cannot to any great extent spend more than they earn, unnecessary, inasmuch as entrepreneurs will effect the quantity acceleration in response to any real stimulus, inadvisable, inasmuch as such throughout industry remove

activity ~~does more to raise prices than anything else.~~ is apt to raise price levels and so multiply the need for more money No one of the three arguments is peremptory, but the three combined tend to limit positive action by DD' to a stimulation of the quantity acceleration and to leave to DS' the main part of the work of providing the increased quantities of money for the increased turnovers.

~~Stimulating action on the part of DD' is not, of course, necessary. Its origin~~ The immediate stimulation of basic supply we shall find later in cross-over disequilibrium. Meanwhile it will be interesting to note a correlation between the rise in prices, DP', and the rate of acceleration, DQ', when DD' is zero. For any rate rate of acceleration of goods and services, DQ', there is an excess of goods in production over goods at the final market; the former is increasing ~~incomes~~ outlays and so also incomes while the increase in goods for expenditure lags; corresponding to this lag

Mutatis mutandis, the same arguments that hold at the beginning of an acceleration hold throughout it during.
For any rate of acceleration of goods & services, DQ', ~~there is an~~ ~~lag~~ goods in production are in advance of goods on the final market, and ~~income~~ increasing income is in advance of the possible basic expenditure at initial prices. This lag is a function of DQ', increasing with its increase, & decreasing with its decrease; and corresponding to the size of this lag there is some single advance in prices, DP', that clears the basic market with DD' at zero. When the lag changes ~~DP~~ another change in prices is necessary if DD' is still zero. But when the acceleration DQ' eventually becomes zero, a positive DD' is for the first time desirable, namely, to prevent the negative DP' implied in the disappearing of

8. Prices. Any exchange involves a ratio between the quantities of the objects exchanged. When $x_i$ units of an object "i" are traded for $x_j$ units of an object "j", then there is always some ratio, say $p_{ij}$, such that

$$x_i = p_{ij}x_j \tag{8}$$

The ratio, $p_{ij}$, is named a price: it is a ratio between quantities of objects exchanged; and it is defined by equation (8) as ~~mm~~ a multiple of the number of units of the second object, "j". When prices are expressed as multiples of the units of some standard object, that object is named a money; the subscripts "j" of equation (8) may then be understood and one may write

$$x_i = p_i x \tag{9}$$

so that $p_i$ is the price of the object "i" in monetary units.

The price of the same object at the same time is the same in all exchanges of that object. The statement is methodological. It is a definition of "same object" and "same time." Thus, if what seems to be the same object is traded at different prices, the larger price is larger because it includes, besides the price of ~~something-else~~ the object, the price of something else: for instance, between different places, this price of something else would be the price of information, transportation, the use of agents, and the like; again, in the same locality, this price would be the price of special services, more agreeable circumstances, ignorance, gullibility, pride of purse, and the like. In other cases the apparent divergence is a divergence between clock time and economic time: the new price has not yet become operative at a more remote market.

The significance of prices is not limitted to formal exchanges. This appears at once if equation (8) is differentiated on the supposition that the price, $p_{ij}$, is a constant. Then,

$$dx_i = p_{ij} dx_j \qquad (10)$$

which the is the same equation and admits the same varieties as equations (7), namely, the marginal comparative valuations that make the quantities of the productive process determinate. Hence constant prices and determinate quantities in process are concomitant: if the one, then also the other. Which is cause and which is effect, is a further question. But one has at once that if either exists, the other also exists. Now the possibility of determinate ~~exchanges~~ quantities is not limited to exchange economies. Hence, the ratios that emerge as prices in exchanges are a more universal reality than exchanges.

In an exchange, then, it is necessary to distinguish between two aspects: there is the aspect of free consent; there is also a pragmatic equivalence between different quantities of different objects. The aspect of free consent consists in the fact that an exchange occurs when there is a coincidence of two opposite decisions; one party prefers $x_i$ of "i" to $x_j$ of "j"; the other party prefers the opposite; and the two come to terms. On the other hand, the aspect of pragmatic equivalence lies entirely in the coming to terms, in the objective effect of the exchange. This objective effect may exist regardless of preferences or free consents. There is a pragmatic equivalence between the work and the upkeep of slaves; it exists independertly of their

The significance of the equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circulation, a meaning for a distinction between pure quantity and pure frequency accelerations, a basis of discussion for abortive or non-successful accelerations, and finally a means of contrasting circulatory success, so defined, and the success of accelerations of the productive process.

The first step is a consideration of the general conditions of such an acceleration. In so far as there are increments in the quantity of initial or transitional payments per turnover, the net transfers, DS' and DD', have been active. For variations in income velocities can be eliminated from the discussion by defining DD' by the equation

$$DD' = DE' - DI' \qquad (47)$$

This is not arbitrary

in terms of known, i.e. already assigned variables.

This procedure is not arbitrary even though the argument used involves an arbitrary control over the precise meaning to be attributed to "money in the demand function" and "net transfers during an interval." The absence of pure arbitrariness may be shown by establishing the same result in another way. The assigned variables determine the quantities and rates of initial payments;per turnover; they also determine the quantities and rates of final sales;per turnover; but the initial payments are income and the final sales are expenditure. The quantities and velocities in basic demand begin from income and end at expenditure; changes in the quantities are determinate by when DD' is determinate; but if we know the rate of flow at either end and the quantity in between, we also know the velocity in between.

The significance of the equations (43) to (46) is conceptual. They provide a definition of a successful acceleration of the basic circuit, a meaning for a distinction between pure frequency acceleration and pure quantity acceleration, as indicated above, a basis of discussion for abortive accelerations, and finally a means of constrasting circulatory success, so defined, with the success of an acceleration of productive rhythms.

The first step is a consideration of the general conditions of the successful acceleration and, to begin, there is the question whether income velocities are a further variable or whether they are determinate when the already assigned variables are determinate. Two considerations favour the second alternative. The first is that income velocities are periods of time between initial payments and final sales; but the assigned variables determine the quantity of initial payments per turnover and the number of turnovers per interval; they also define the quantities of money involved and the rapidity of turnovers of final sales. ~~Itxremainsxtrue~~ It follows that the aggregate of income velocities have some determinate index. Again, if a circular flow is successful, then the elements of the flow are not piling up at one point to leave a vacuum at another

Surplus Circuit

1) Repeat all said about basic

2) Case of DD" different

Investment
1) Redistributional
2) Increase of financial loans
3) New capital equipment DD"
4) New monetary circulating capital, DS' DS"

3) Difference & Once expansion in process
then DI" = DE"
purchase of new capital equipment at DD" = 0

& different DD" can be positive over long periods
[illegible]
& this positive gives cross-over disequilibrium
a new complication of basic circuit

## Abortive Accelerations

year profits & losses

correlated 1 index of expansion or contraction

1° Varying inventories [ 2 acceleration starts of funds without going round.

uncorrelated

2° Windfall profits & losses

X makes outlay & Y gets it in addition to his own outlays

Y gets it & makes a − DD'

X' spends + DD' — Y uses it to decrease c. cap. on short term loans

## Successful acceleration & success of process

wrong to put DE = DD'

− DD' at beginning of expansion = a saving needed [illegible]

~~±DD~~

successful process says a good deal more

DP' DD' at beginning and

DS' & no repayment of principal

Negative acceleration

$DM' = -K$

General squeeze – falling prices

if at once & universal, shutdown de facto, falling prices & reducing production which augments $-DM'$ – unless it systematic – & does not provide solution

Return from positive acceleration to zero acceleration on acquired higher level

I [ Acceleration – proportionate lag
– unless $-DS'$, automatically then $+DP'$

End of acceleration begins – unless $+DP'$, then $-DP'$

can't take it, because unorganized for general price drop

and cause can't be avoided, because unorganized for $-DD'$

] 

II No $-DS'$ – if expansion on increasing volume of loans

then remaining at peak of expansion means remaining at peak of volume of loans

to pay the loans off & not renew them – reduce volume – prices $-DS'$, gives reduced turnover size.

~~But more prices~~

Prices : Mechanism or Norm ↗ perhaps "the merely empirical"

Prices & Acceleration

Competition – Cost Prices
→ actual & potential consistency
→ monopoly vs. potential competition

1) The ~~rigidity~~ inertia of real prices

~~$q_i = q_j \, dp_{ij}$~~

$$q_i = q_j \, p_{ij}$$

$$dq_i = q_j \, dp_{ij} + p_{ij} \, dq_j + dp_{ij} \, dq_j$$

suppose $dq_i = 0$

then $dp_{ij}$ and $dq_j$ have to be opposite in sign

more means a lower real price

less means a higher real price

And everything cannot accelerate at once

2) The problem of "more money"

1) Mercantilism

2) Bills of exchange, banking

3) Gold standard

4) No gold standard

3) Booms an artificial acceleration

Slumps cutting out the artifice — propensity to consume max (or zero acceleration)

One may reflect that all this has been said before. But one hundred years practice followed in the wake by theory, has moved away from the view that fluid prices and competition as the panacea for all economic ills and the guarantee of ever greater benefits. No doubt that view provided an effective means for enforcing the consistency of prices: it worked the whole world into a single market place. Again it gave a free hand to the entrepreneur - - -

But it eliminated itself. ~~Competition is no more~~ ~~a very valuable type of [illegible] an embargo upon~~

~~Too harsh~~ Competition an embargo

Too harsh

An age of corporations

Neither fluid prices nor competition

But not panacea of fluid prices & competition. Restore statements =

Not a norm but a mechanism. Definition -

But the idea of mechanism: handle the

Result - -

merely empirical - Postulate & effect enlightenment