r_{i1} r_{i2} r_{i3} r_{i4} ...... $r_{i(n-1)}$ r_{in}

dr_{i2} dr_{i3} dr_{i4} ------ dr_{in}

S_{i1} S_{i2} S_{i3} S_{i4} ---- $S_{i(n-1)}$ S_{in} carrying costs

ds_{i2} ds_{i3} ds_{i4} ------- ds_{in}

$$\text{Circ. Cap.} = r_{i1} + S_{i1} + \sum_{j=2}^{j=n} (dr_{ij} + ds_{ij})$$

dr_{ij} varies ā increases in initial payments per turnover

~~does not vary ...~~

varies ā turnover frequency i.e. fixed costs that are not "piece-costs" ways per day

ds_{ij} varies ā increases in

S_{1j} S_{2j} S_{3j} S_{4j} {transitional prices / transitional quantities (depend upon ~~frequency~~ size)}

← number of instances varies ā frequencies

DM' r_{i1} r_{i2} r_{i3} → a same contribution

DM''

Quantity is a function of frequency

Quantity × frequency = rate of payment per interval

1. Secstions of a Monetary Circulation.

Though the main body of any analysis consists in relations between terms that themselves are defined by relations, initially it is necessary to assume or describe some objects or points of reference. Here this preliminary work of assumption and description will consist in marking off sections of a monetary circulation, but we would note at once

10. Price and Process Indices.

To indicate correlations between accelerations of the monetary circuits and accelerations of the productive process, it is necessary to introduce price and process indices. The present section is a summary presentation of some fundamental definitions.

When any quantity, q_i, of any object, i, is exchanged for any quantity, q_j, of any object, j, then it is always possible to determine a ratio, p_{ij}, such that

$$q_i = q_j p_{ij} \tag{19}$$

In any such case, p_{ij} is the price per unit of the object, j, and this price is measured in units of the object, i.

Transformations from prices measured in one unit to prices measured in another unit are ruled by the equations,

$$p_{ij} p_{ji} = 1 \tag{20}$$

$$p_{ij} p_{jk} = p_{ik} \tag{21}$$

as may be shewn readily by writing down the definitions of p_{ji}, p_{jk}, p_{ik}, on the analogy of equation (19), and then eliminating in two ways the terms, q_i, q_j, q_k.

Money is a medium of exchange and a common measure of prices. A medium of exchange is any object, from Homeric hides through pieces of eight to digits in a banker's ledger, which under given historical conditions regularly appears as at least one term of the vast majority of exchanges. A common measure of prices is had when all prices are expressed in units of the same object.

With respect to equations (20) and (21), if the object, i, is money and if the objects, j and k, are different and not money, then by equation (20) the price of the object, j, in monetary units is the reciprocal of the price of money in units of the object, j; further by equation (21) the price of the object, k, in units of the object, j, is equal to the monetary price of the object, k, divided by the monetary price of the object, j.

Prices measured in non-monetary units will be termed real prices; they are to be understood whenever a price is denoted by using a double suffix. On the other hand, it is superfluous to have a suffix to denote money in monetary prices, which henceforth will be defined by the equation,

$$m_i = p_i q_i \tag{22}$$

where q_i units of any object, i, are exchanged for m_i units of money, so that the monetary price of the object, i, is p_i. Then, the price of money in units of the object, i, is the reciprocal, $1/p_i$; and the price of the unit of the object, j, in units of the object, i, is the quotient of the two monetary prices, p_j/p_i, so that equation (21) becomes

$$p_i p_{ij} = p_j \tag{23}$$

where p_{ij} is a real price and both p_i and p_j are monetary prices.

As appears from equation (23), the real price, p_{ij}, determines no more than a ratio between the monetary prices, p_i and p_j. Hence monetary prices may all change while real prices remain unchanged; further a change of monetary prices may have two components, a change of real prices and a further change of monetary prices. In this case the second component, and in the former case the total change, is a change in the price of money. Inflation is a fall, deflation is a rise in the price of money.

The consistency of prices is the consistency of equations representing simultaneous exchanges in the same economy. If prices are consistent, it is impossible to obtain directly or indirectly from simultaneous exchange equations two different prices, say p_i and p_i', for the same object, i. Now the consistency of prices may be postulated by taking it as a definition of the object, i, so that any apparent inconsistency is attributed to the presence in the exchange of some further object, j. Then, in reality p_i' becomes a mistaken expression for two prices, namely, $(p_i \pm p_j)$. The price, p_j, of the latent object, j, may be the price of special services, of more agreeable circumstances, of pride of purse; it may be the price of information, of the use of agents, of transportation; it may be the price of exceptional initiative, of foresight, of knowledge, of great acumen; or negatively it may be the price of lack of foresight, lack of initiative, of ignorance, of gullibility. Generically one may say that p_j is the price of a lack of consistency, but it is always possible in any particular case to give a specific content to this generic interpretation; otherwise it would be impossible to account for the difference in prices, and it would be meaningless to assert that prices tend to be consistent.

The latter view, that prices tend to be uniform throughout the economy though actually they never attain uniformity, is the more common one. It offers the advantage that the analysis of exchange is not loaded with latent objects and so in practical discussions it is a preferable mode of statement. The foregoing statement, however, has the theoretical advantage of greater generality and simplicity; and it is presupposed by the affirmation of a tendency to uniformity which is no more than a tendency to reduce latent prices to a minimum.

~~departures from consistent prices are to be accounted for. Not a little of the hedonism of the old school of economists was derived from efforts to explain what has to be taken for granted.~~

If prices are consistent, the price of money is consistent. It follows that there is one price of money for the whole economy, and that inflation and deflation are variations in this one price and so affect all ~~prices~~ monetary prices in like manner.

To define price and process indices, let DZ be any definite rate of payment, which in a first interval is DZ_i and in a second interval is DZ_j, where D^2Z_j is the excess of the latter over the former. Let **z** be any object sold in either of these two intervals and paid for in the rate, DZ. Let p_z be the price, q_z the quantity sold in the first interval; and let $(p_z + dp_z)$ be the price and $(q_z + dq_z)$ be the quantity sold in the second interval. Thus, summing with respect to all instances of **z**, one has

$$DZ_i = \sum p_z q_z \qquad (24)$$

$$DZ_j = \sum (p_z + dp_z)(q_z + dq_z) \qquad (25)$$

$$D^2Z_j = DZ_j - DZ_i = \sum (p_z dq_z + q_z dp_z + dp_z dq_z) \qquad (26)$$

Let the price indices and process indices of the two intervals be P_i, P_j, DQ_i, DQ_j, and let the definitions of these indices be the solution of the equations

$$P_i DQ_i = DZ_i \qquad (27)$$

$$P_j DQ_j = DZ_j \qquad (28)$$

$$P_j = P_i + DP_j \qquad (29)$$

$$DQ_j = DQ_i + D^2Q_j \qquad (30)$$

$$DQ_i DP_j = \sum q_z dp_z \qquad (31)$$

$$P_i D^2Q_j = \sum p_z dq_z \qquad (32)$$

where, in general, the last two equations must be mere approximations.

It follows that the definitions of the indices are in themselves no more than approximations. The reason is not difficulty of obtaining information about p_z, dp_z, q_z, dq_z, nor the fact that the prices may have to be averaged, though both those difficulties also exist. None the less even with perfect information and no averaging, the definitions of the indices remain approximate, unless the four sums of $p_z q_z$, $p_z dq_z$, $q_z dp_z$, $dp_z dq_z$ happen to be a fourfold proportion; for in fact these four sums are by the defining equations made equal to $P_i DQ_i$, $P_i D^2Q_j$, $DQ_i DP_j$, $DP_j D^2Q_j$, which are a fourfold proportion; but there is no ground to expect that the four summations will also be in proportion. Hence equations (31) and (32) are only approximate and so the definitions are approximate.

Though for six unknowns there are the six equations (27) to (32) still the fourfold proportion introduces a condition of consistency and so leaves an infinity of solutions possible. Hence solution is by introducing any arbitrary base, say 100, as the value of P_i. This determines DQ_i by (27); thence follow approximate determinations of DP_j and D^2Q_j by (31) and (32), and of P_j and DQ_j by (29) and (30); a check and adjustment of P_j and DQ_j follows by (28).

The significance of the indices is that it provides a differentiation between increments in prices and increments in quantities sold as the rate of payment moves from DZ_i in one interval to DZ_j in the next. The corrected value of DP_j indicates the price increment, and that of D^2Q_j indicates the quantity increment.

Thus, the condition that an increment in the rate of payment is accompanied by a proportionate increment in quantities sold is that DP is zero; on the other hand, the greater is DP, the greater is the inertia of the process of goods and services against circuit accelerations.

The process indices disregard all process change that has no aggregate quantitative manifestation. Increments in one quantity may cancel against increments in other quantities when the two sets are of opposite sign. Similarly, changes in quality do not appear in the indices when the emergence of the new is balanced by disappearance of the old. Hence D^2Q is an increment per interval in the aggregate of price-weighted quantities.

11. Systematic Costs and Profits. Price and process indices provide a method of denoting the concomitance or divergence of the productive process and the monetary circulation. It is now necessary to consider the inter-action of the two and, to begin, it will be well to consider certain general phenomena of particular importance in a "capitalist" economy.

Systematic costs and profits are costs and profits in much the same sense as forced savings are savings, that is, they bear an important relation to costs and profits in individual units of enterprise but they are variables not of any individual unit but of the total situation in any exchange economy. Thus by systematic costs and profits are meant neither the costs and profits of the accountant nor again the windfall profits and losses of the equilibrium analyst. For the accountant costs and profits are a division of payments made with a relation to payments received. For the equilibrium analyst windfall profits and losses are accidental variations in the distribution of the receipts of industry and commerce. On the other hand, systematic costs and profits are a division of aggregate initial payments to factors of production, and the division is based upon the tendency of the products to effect an acceleration of the total economy. The significance of this division is that the resultant acceleration tends to ~~ai~~ reduce and in the limit to reduce to zero whatever systematic profits may have existed.

The aggregate of initial payments in any interval is the sum (DO' + DO") which is identical with total income (DI' + DI"). Systematic profits are defined as K.DO" where K has any value in the range 0 to 1 inclusively, and is defined as the fraction of surplus initial payments made with respect to surplus products which are supplied not to replace or maintain worn out or obsolescent equipment but are increasing the capacity and efficiency of old units of enterprise or fitting out new units.

Now systematic profits have a twofold significance. First, with regard to the monetary circuits, it is plain that to maintain rates of payment at their acquired levels total income has to be spent either in itself or in its equivalent: were DE' + DE" to drop permanently behind DO' + DO", there would result a continuous contraction of the pree circulatory process until it was reduced to zero. However, it makes a notable difference whether total income is being spent for basic products and the maintenance and replacement of capital equipment or, on the other hand, there is over and above that expenditure an element, K.DO", which purchases additional capital equipment. To begin, there is the psychological difference: in the former case people are merely making a living; not matter how high their standard of living may be already and no matter how much they are adding to it, still they are adding only to their enjoyment and not to their ownership of industrial and commercial wealth sources of wealth; this runs counter to current ideas on the "successful man" who can emerge only when there is in income an element, K.DO", which ministers to the increase not only of living standards but as well, over and above all increase of living standards, to the increase of ownership of means of production. Besides this psychological difference, there is a monetary difference: K.DO" yields an equal K.DI"; now there is no necessity that the recipients of K.DI" should also be the spenders of K.DI"; the continuity of the circuits at their acquired levels is assured as long as DD" remains zero, so that the recipients of K.DI" may devote this part of their income to the purchase of redistributive goods, to real estate or stocks and securities, to augment their ~~financial~~ prestige or to increase their financial power, as long as this subtraction from the circuits is balanced by an equal and opposite movement from the investment market to the purchase of surplus goods and services.

Besides the psychological and the monetary significance of systematic profits, there is also a real significance. The products generating systematic profits fit out new firms and expand old firms. In so far as this constitutes a net increment of capital equipment, by over-balancing the effects of the current rate of liquidation of units of enterprise, the productive process tends to accelerate in long-term style. In so far as the net increment occurs in surplus units, the surplus stage is due for acceleration immediately and the basic stage is due for a still greater acceleration ultimately. In so far as the net increment occurs in basic units, the basic stage is due immediately for an acceleration.

However, these real effects of systematic profits have a repercussion on systematic profits themselves. In the long run, as has been shown, the acceleration of the productive process involves a decrease of K as the portion of surplus activity devoted to maintenance and replacement increases with increasing capital equipment. ~~But besides this long-term effect, there is an immediate effect upon the ratio of systematic profits to total income. If this ratio is denoted by H, then~~

equipment. However the movement towards this ultimate position is full of interest. Let H be the ratio of systematic profits to total income, so that

$$H = K.DO''/(DO' + DO'')$$

or

$$H/K = DO''/(DO' + DO'') \qquad (33)$$

If now we follow through the emergence and development of a long-term acceleration, we find a first period in which H is increasing, a second period in which H is decreasing, and a third period in which H is zero. As the long-term acceleration begins, DO' is constant but both K and DO" are increasing; H increases as does the product of K and DO" in the numerator with a slight drag because of the presence of DO" in the denominator. Further, this first period lasts as long as DO" is increasing more rapidly than DO', that is, as long as the efforts of the surplus stage are more on equipping the surplus than the basic stage of the process; for then H is still increasing though less and less rapidly as DO' approaches the rate of acceleration of DO". However, every increment in the surplus stage stands at least point-to-line to increments in the basic stage and unless the expansion of the surplus stage is mere blundering, sooner or later DO' will begin to increase more rapidly than DO" while at best K is constant. Then H begins to decrease, and the more successful the expansion of the basic stage, the more rapid the rate of decrease. Finally, K begins to decrease, as the surplus stage has to devote more of its efforts to mere maintenance and replacement; and if the long-term acceleration works itself out K returns to zero and H has to reach zero ahead of K.

The single condition to this movement (if we abstract from the favourable balance of foreign trade and from deficit government spending, which will be discussed later) is that there does not supervene a rate of liquidation of old or new firms to eliminate from systematic profits their tendency to accelerate the productive process. Thus there is possible a dynamic situation in which the surplus stage of the process is yielding capital equipment that generates systematic profits but not yielding an aggregate increment of consumer goods that reduces the ratio of systematic profits to total income. This situation is most easily verified in an industrial revolution that is the work of "new" men: because an industrial revolution is in process, the new capital equipment is simply displacing old equipment; because the industrial revolution is the work of "new" men, the displacement of old equipment occurs not as a cost of obsolescence on existing firms but as fresh investment constituting the emergence of new firms. On the other hand, the more industrial and commercial enterprise is in the hands of vast corporations which by their command of talent and resources stand in a virtually impregnable position, the less would seem the possibility of evading the effects of variations in systematic profits by a concomitant rate of liquidations.

Further, it may be noted that it would be absurd for the great corporations to attempt to plan an elimination in the variations of the ratio of systematic profits. For while the planning itself would be possible, perhaps, the objective of the planning would be manifest stupidity: what would be planned would be a steady flow of surplus products that did not yield their increment in basic products; it would be a matter of devising better machinery and more efficient organization, of effecting both, and then of using them as though they were not better than what already existed. It would be a planned economy in which the idea of the plan was to effect a maximum change in the surplus stage while keeping the basic stage in a relative status quo.

10. The Cross-overs. The distinction between the two circuits involves a cross-over from basic outlay to surplus income and from surplus outlay to basic income. It has been argued that unless these two cancel (DG = 0) the process is apt to be submitted to an expansion of one circuit and an a contraction of the other. We have now to investigate the conditions of their cancelling.

Let all recipients of income, whether basic or surplus, be divided into groups of n_i members each. Let each member of the same group receive approximately the same income per interval, o_i, and devote the same proportion of income, g_i, to expenditure at the basic final market. In any later interval let the situation have changed to the extent that in group, i, numbers have increased by dn_i, income per interval by do_i, and the proportion of income devoted to basic expenditure by dg_i. Then basic income per interval, DI', and the increment of basic income per interval, D²I', may be expressed as summations with respect to all instances of the groups, i.

$$DI' = \sum g_i o_i n_i \qquad (16)$$

$$D^2I' = \sum(g_i o_i dn_i + o_i n_i dg_i + n_i g_i do_i + g_i do_i dn_i$$
$$+ o_i dn_i dg_i + n_i dg_i do_i + dg_i do_i dn_i) \qquad (17)$$

If DI' is visualized as a rectangular solid, then D²I' may be visualized as the elements added to expand the rectangualar solid, three plates added to three faces, three bars added along three edges, and a little cube added to the corner.

To estimate the relative importance of the various components of D^2I' one may note the following. As one passes from group to group one finds that as o_i increases, n_i decreases and also g_i decreases: the greater the income, the fewer that receive it and the smaller the proportion of it spent on consumer goods and services. With regard to the increments, dg_i is always quite small since it is a change in a proper fraction, dn_i may be quite large as employment increases of or decreases, and do_i may be extremely large amounting in the aggregate to billions of dellars as an economy moves from the peak of a boom to a slump or from a slump to an all-out war effort. Further, dg_i is usually opposite in sign to do_i: savings increase somewhat more rapidly than aggregate income; however in the highest income brackets changes in income are apt to be cancelled by the opposite changes in dg_i, while as income decreases this tendency is more and more reduced until in the lowest brackets the effect of ~~sax~~ increased rates of savings may be small. Hence, for notable changes in basic income, one has to llok look to the factors $g_i o_i dn_i$ and $n_i g_i do_i$ and, with respect to these two, one may discount/positive increments to income in groups which already are spending all they intend/or can manage to spend on consumer goods and services.

Now the condition of cross-over equilibrium (DG = 0) may be written from equation (11) in the form

$$DO'/DO'' = (1 - G')/G' \qquad (16)$$

where

$$DI' = (1 - G')(DO' + DO'') \qquad (9)$$

Hence when the ratio, DO'/DO'' is decreasing, G' has to increase, and when DO'/DO'' is increasing, G' hasto decrease to satisfy the

11. Trends. Trends are determinate relations between successive intervals. Two types of trend are considered: process trends and circulation trends. Both have the same general form, namely, on certain suppositions with regard to intervals 1, 2, 3,... i, j,... n, the quantitative process of goods and services or the circulatory process of payments and transfers behaves in such and such a fashion. Nine classes are distinguished in each type of trend; names of the classes and their definitions are to be had in the following table.

Process Trend:	DQ'	DQ"
Circulation Trend:	DM'	DM"
Level:	0	0
Basic Expansion:	+	0
Surplus Expansion:	0	+
Compound Expansion:	+	+
Basic Contraction:	-	0
Surplus Contraction:	0	-
Compound Contraction:	-	-
Basic Disequilibrium:	-	+
Surplus Disequilibrium:	+	-

Thus the nine process trends are defined according as process indices DQ' and DQ" are zero, positive, or negative. Similarly, the nine circulation trends are defined according as circuit increments in the quantity of money, DM' and DM", are zero, positive, or negative. In each case the nine classes are complete enumerations with respect to all possible values of two variables. If over a series of intervals, DQ' is zero and DQ" is positive, the process trend is a surplus expansion. If over a series of intervals, DM' is negative and DM" is positive, the circulation trend is a basic disequilibrium.

To complete the definitions of the process trends, it is necessary to specify the meaning of DQ' and DQ". Let then DQ' be defined by the rates of basic expenditure, DE'_i and DE'_j, in any two successive intervals, i and j. Let DQ" be defined by the rates of surplus expenditure, DE''_i and DE''_j, in the same two successive intervals. Wti With DE'_i, DE''_i, DE'_j, DE''_j determinate with respect to prices and quantities, calculation of DQ' and DQ" proceeds as outlined in equations (83) to (91) in the preceding section.

It is to be observed that process trends are merely functions of aggregate weighted quantities sold at the basic and the surplus final markets. Differences that do not appear in the process indices do not affect the trend. Hence if one quantity increases and another decreases proportionately to the respective weights, indices which regard aggregates are unaffected. Accordingly any amount of qualitative change may be going on without any change of trend. It follows that the process level differs enormously from the neo-classical stationary state. The latter is a pattern of unchanging routines. But the process level, at least in theory, is compatible with an industrial revolution moving along in a strait jacket.

The circulation trends are functions of changes in the quantity of money available in the basic and the surplus circuits. Their general character may be deduced from the circuit equations.

However, it will facilitate the course of such a deduction to set down at once a few general theorems: a) there is a concomitance of variations in initial, transitional, and final payments of uniformly specified types; b) the rates of absorption, DA' and DA'', are no more than incidental adjustments; c) with D^2S' at zero over a series of intervals, DS' may be positive, zero, or negative over the same series of intervals; and the same holds for D^2S'' and DS''; d) the difference between turnover differences of transitional payments made and turnover differences of transitional payments received is an adjustment factor; e) turnover frequencies are resultant, and not initiating, factors of acceleration.

First, there is a concomitance of variations in initial, transitional, and final payments of uniformly specified types. The ground of the concomitance lies in the definitions of the terms: transitional payments are initial payments in a process of double summation; final payments are initial payments at the end of the double summation. Any one of the three may begin to move out of step, but unless the others follow, then it has to revert to its original level. There will be lags proportionate to the production-and-sales period between different rates. Transitional movements may change their route so as to involve more or fewer transitional payments, and so give greater or less aggregates of transitional payments. But apart from incidental xx differences of such a nature, variations are concomitant. There can be no systematic divergence over a series of intervals with one rate increasing and another xxxx constant or decreasing.

Secondly, the rates of absorption, DA' and DA", represent no more than incidental adjustments. They were defined by the equations,

$$DA' = DI' + DD' - DE' \tag{67}$$

$$DA'' = DI'' + DD'' - DE'' \tag{68}$$

Now DI' and DI" represent money earned per interval; DE' and DE" represent money spent per interval; if DD' and DD" were defined as the excess of money spent and not earned over money earned and not earned per interval, then DA' and DA" would always be zero. It is desirable, however, to define DD' and DD" in terms ~~as~~ of savings, and not to count as savings the money earned at the end of one interval and spent at the beginning of the next. Thus, DA' and DA" represent the excess of the carry-over from this interval to the next over the carry-over from the previous to the present interval. As such, they are not systematic factors in a trend but incidental adjustments.

The third theorem follows immediately from definitions. DS' is a turnover sum, D^2S' is a turnover difference. As long as DS' is the same interval after interval, D^2S' will be zero. The same holds for DS" and D^2S''.

The fourth theorem regards the significance of T' and T" which are defined by the equations,

$$T' = D^2t' - D^2T' \tag{93}$$

$$T'' = D^2t'' - D^2T'' \tag{94}$$

~~Evidently, on the supposition of synchronized turnovers, T' and T" are always zero: transitional payments made are then identical with transitional payments received. Without synchronized turnovers, T' and T" tend to zero as to a statistical average. Hence within intervals they represent factors of adjustment.~~

Now D^2T and D^2t do not refer to the same turnovers. This may be seen by inspecting the following equations and checking by equations (24) and (36).

$$D^2T = \sum_i \sum_1^n dT_{ij} = \sum_i \sum_1^n (T_{ij} - T_{ij'}) \qquad (53 \,\&\, 22)$$

$$= \sum_i (T_{in} - T_{i0}) \qquad (95)$$

where the constant, K, has been omitted and the limits result from the fact that

$$dT_{i1} = T_{i1} - T_{i0} \qquad (22)$$

On the other hand, since

$$dt_{i1} = t_{i2} - t_{i1} \qquad (32)$$

it follows that

$$D^2t = \sum (t_{in'} - t_{i1}) \qquad (96)$$

where n' is written for (n + 1) and K again is omitted. Hence with respect to three successive intervals, D^2R and D^2T refer to the last turnovers of the first and second, while D^2t, D^2O, and D^2S refer to the first turnovers of the second and third. For turnovers, 0 and n, are the last turnovers of the first and second intervals, while turnovers, 1 and (n + 1), are the first turnovers of the second and third intervals.

With regard to T' and T", then, on the supposition of synchronized turnovers and a constant acceleration, the difference between turnover differences of transitional payments, D^2T and D^2t, will be always zero. Without synchronized turnovers but with constant acceleration, T' and T" tend to zero as a statistical average. Finally, in so far as the acceleration is changing, T' and T" tend to some positive or negative quantity as statistical averages. However, changes of acceleration are incidental adjustments; they are not trends but intensifications or reversals of trends.

The fifth theorem is that changes in turnover frequency are resultant rather than initiating factors of acceleration. Granted an acceleration is in progress, one may expect a greater efficiency of production and sales to supervene and intensify the acceleration. For with the acceleration in progress, opportunities to introduce improvements multiply and selling is strong. On the other hand, without an acceleration in progress, opportunities to introduce improvements are restricted while the weakness of sales ~~discourages expensive modifications~~ prevents the greater efficiency in selling necessary to convert shorter production periods into shorter turnover periods. Finally, in a deceleration of the process one may expect turnover frequencies to diminish; sales are falling; and reduced rates of production prevent the most efficient use of means of production.

To turn now to the circulation trends. It has been shewn already that

$$DM' - DA' = D^2R' + D^2T' \tag{71}$$

$$DM'' - DA'' = D^2R'' + D^2T'' \tag{72}$$

Since the two equations are similar, one may discuss their implications without distinguishing between basic (') and surplus (") rates. On the supposition that DM is positive over a series of intervals, then, since DA is an incidental factor and since D^2R and D^2T keep pace, it follows that both D^2R and D^2T will be positive over the series of intervals; again, if DM is negative, then D^2R and D^2T will be both negative; and if DM is zero, then D^2R and D^2T will average zero.

The conclusions hold no matter what the reason for DM being positive, zero, or negative. It may be any solution of the equations defining DM' and DM", namely,

$$DM' = DS' + DD' + DG$$

$$DM'' = DS'' + DD'' - DG \qquad (14)$$

and so an acceleration of the circulation may be due to excess transfers to supply, DS, to excess transfers to demand, DD, or to a cross-over difference, DG. The relative importance of the three in effecting accelerations of the circuit may be estimated as follows. With DD', DD", DG each at zero, entrepreneurs are receiving back their aggregate outlays (including the payment of profits to themselves). In such a situation, demand is neutral and prices may be termed normal. On the other hand, with DD', DD", DG above or below zero, there is a strengthening or weakening of aggregate demand independently of supply; demand is not neutral but asks for more or for less; and since this asking is independent of supply, it can effect nothing but an upward or downward movement of prices (unless it is very slight and met out of inventories). Now the upward or downward movement of prices will stimulate at once the whole series of speculative producers to increased or decreased scales of operation and, in the main, such changes in the scale of operations involve excess transfers from or to the redistributive function. In current practice changes in the quantity of money in the circuits are changes in the volume of short-term loans; and short-term loans are made not to purchasers but to producers; they affect not DD' and DD" but DS' and DS". It would seem that in general DM' and DM" depend on DS' and DS", while the role of DD', DD", DG is to act as stimulants to the scale of operations of agents of supply.

By combining equations (65) and (93), (66) and (94), one obtains the correlations of turnover differences in the form,

$$D^2R' + D^2S' = D^2O' + T' \qquad (97)$$

$$D^2R'' + D^2S'' = D^2O'' + T'' \qquad (98)$$

where T' and T" are adjustment factors ~~tending to zero as a statistical average~~ discussed above. Since the equations are similar, the argument may disregard the distinction between basic (') and surplus ("). With D^2S at zero over a series of intervals, D^2R and D^2O tend to be equal. But with D^2S at zero, DS is constant interval by interval; however this constant may be positive, zero, or negative; ~~hence~~ so that D^2R and D^2O tend to be equal ~~wither~~ whether increasing, averaging zero or decreasing. Hence with D^2S at zero, according as DS is positive, zero, or negative, one has D^2R, D^2T, D^2t, and D^2O similarly positive, averaging zero, or negative. The circulation is expanding, level, or contracting interval after interval. Entrepreneurs are receiving back in final payments (and transitional payments) all that they are spending in initial payments (and transitional payments); no matter how great the aggregate income they are paying to themselves, it keeps coming back. To intensify or reverse any such trend, DD', DD", DG may provide stimulation, but, in the main, the work will be done by a positive or negative D^2S which effects a change in DS.

~~So far the argument has dealt with turnover differences. Now to be considered are the ~~tt~~ turnover sums, DO', DO", DR', DR". The first point to be considered is their relation to the turnover differences, D^2R', D^2R'', D^2O', D^2O''. The latter, then, determine the difference between the aggregates of last or first turnovers in two successive intervals. This may be seen by inspecting the following.~~

So far the argument has dealt with turnover differences. One now has to ask how do DO', DR', DO", DR" behave when with DM positive, zero, or negative interval by interval over a series of intervals, D^2R, D^2T, D^2t, D^2O are similarly positive, averaging zero, or negative. ~~New~~ Immediately one notes that DO and DR are turnover sums and so vary either from variation in turnover magnitude or from variation in turnover frequency; thus because

$$DO' = \sum_{1} \sum_{1}^{n} i'_{ij} \tag{46}$$

DO' may be greater either because instances of i'_{ij} are greater or because there are more instances; and there may be more instances of "j" from increasing turnover frequency, or more instances of "i" because additional new units of enterprise exceed liquidated units; and in the latter case the turnover frequency of the additional units may raise or lower the existing average frequency per unit of turnover magnitude.

Now with respect to change of turnover magnitudes, which includes the excess, positive or negative, of new units of enterprise over liquidated units, the ~~utn~~ turnover differences D^2R, D^2O, etc., give information only on the first and last turnovers of each interval. However, we have a source of information with ~~g~~ regard to intermediate turnovers ~~from~~ in the conclusion that, in the main, DM' resulted from DS' and DM" from DS". For DS' and DS" are also turnover sums, double summations of m_{ij}, which is the difference between payments made in the later turnover and payments received in the earlier turnover with respect to all turnovers. For DS to be positive, zero, or negative, interval by interval over a series of intervals, means that the aggregate of turnover magnitudes during an interval are above, or at, or below the average of the previous interval. Hence DO' and DR', DO" and DR", as far as turnover magnitudes are concerned, have their trend determined as increasing, averaging zero, or decreasing according as DM' and DM" respectively are positive, zero, or negative.

As to the change in turnover frequencies, the change in the general average from the replacement of new firms by old firms by new firms and the emergence of additional new firms cannot be determined generally. On the other hand, as argued previously, there should seem to be a tendency for turnover frequency to increase during an expansion but be hampered and decrease during a contraction. However, the expansions have to be both process and circulation expansions and, similarly, the contractions have to hold in both orders; for turnover frequency is a matter of the efficiency of both production and sales. In situations that are neither double expansions nor double contractions the probabilities of change in turnover frequency are less determinate: a circulation level is less incompatible with change

Granted the existence of both demand and supply functions valid with respect to concrete conditions, there remains the question of solutions. Now no solution exists, there is no exchange, no price, and the quantity sold is zero, if with respect to any determinate quantity the most offered by the most eager buyer is less than the least required by the most eager seller. On the other hand, solutions exist if the most eager buyer offers more than or at least as much as the [redacted] most eager seller requires. However, the postulate of consistent prices admits only one solution; further, the same postulate requires the elimination of the prices of all objects distinct from the objects of the class, i, so that the one solution required by the postulate is a minimum price. But a minimum price is had

71 + 72

$DM - DA = D^2R + D^2T$

1 D^2R and D^2T same sign

2 if quantity added is greater than positive rate of absorption then D^2R increasing until ... internal

if quantity removed is greater than negative rate of absorption then D^2R decreasing until ... internal

3 [illegible] true whether addition be DS DO $\div DG$ but DS action _ DO DG stimulation – or supplies $+DO$" covering ^+DG

4 D^2S depends on changes in DS in succession

intervals may be 0 or DS + 0 –

hence

$D^2S + D^2R + D^2T = D^2t + D^2O$

$T = D^2t - D^2T$ adjustment factor

$D^2S = 0$

$D^2R = D^2O + T$ until ... internal apart from adjustments

4: 82 · 85 · 90
5: 297
10: 5 9 18.19 8: 19-21
22: 90
18: 19-20
27: 94 '11 24 687

5 hence – provided added money is not counterbalanced by absorption [decreasing income velocities] – removed money is not replaced virtually by absorption (increasing income velocities) – both limited – then D^2R D^2T D^2O move up or down according to DS

intensification "changes with stimulation of trend D^2S / DO DG stimulation eager buyer is greater than the least that the most eager seller is ready to take. Further, on the postulate of consistent prices, there will be only one solution or one common point to the two functions. And as prices of ignorance, gullibility, etc., are eliminated, this solution lies in the region in which the least eager buyer and the least eager seller come to terms and so in the region in which the maximum quantity that can be sold is sold.

e) The Nature of Prices. Let the term, valuation, denote any appreciation on any grounds of any object. Let a comparative valuation denote a decision with respect to alternatives: of two events, A and B, only one is possible; the comparative valuation is the decision, A is preferred to B. Let a marginal comparative valuation denote a decision with respect to alternative quantities of alternative objects. The question is, How many units of A and how many of B is preferable to any other combination when the possible numbers, say x and y, are defined by the equation

$$Ax + By = C \tag{22}$$

As appears from the foregoing account of supply and demand, prices result from the marginal comparative valuations of the community. The demand and supply schedules are solutions of equations (18) and (20) which correspond to equation (22): M is the constant C, Ax is $p_i q_i$ on each of a series of hypotheses regarding p_i, and k is By

6 DO follows D^2O
DR follows D^2R

7 frequency acceleration
[positive = expansion
[negative = contraction

5	+2	2	7
7	+2	0	9
9	−2	−4	7
7	−2	0	5
		−2	

8 price curves DP' DP''
positive = expansion – decrease = frequency acceleration
negative = contraction – ~~[illegible]~~ opposed by frequency deceleration

9 DQ' DQ''
follow DR' DR''
apart from DP' DP''