3.4 Geometry.

Space and Time, as described, have both a material and a formal aspect. Materially, they are totalities of concrete extensions and durations. Formally, they are such totalities as ordered, for without the order either the concreteness or the totality vanishes. But there are as many distinct ways of ordering concrete extensions and durations as there are possible references' frames; and there are as many distinct, possible reference frames, as there are possible concrete positions, directions, and instants that may be taken as the origin and orientation of the frames.

Hence it is that, while Space and Time considered materially are single identities, for there are only single sets of concrete extensions and durations to be ordered, none the less Space and Time considered formally are potentially infinite groups, for there an infinity of possible reference frames for ordering them.

However, by the principle of independence, the explanatory correlations and system of Space and Time will be independent of reference frames. For the multiplicity of reference frames arises from the multiplicity of particular positions, directions, and instants that may be taken as origin and orientation of the frames; and such particular positions, directions, and instants must be correlative to human experiences, otherwise the reference frame would not be the order of concrete extensions or concrete durations. Since then the explanatory correlation or system is independent of human experiences, the explanatory correlation or system of Space and/or Time will be independent of reference frames.

Similarly, by the derived principle of invariance, the expression of the correlations or system of Space and/or Time will be independent of reference frames.

Now by this general property of invariance it is possible to define a series of metrical geometries. This we proceed to do, and we begin by recalling the more general mathematical notions that are needed to make use of the property of invariance.

Consider any mathematical expression involving a number of variables, say, the function,

$$F(x_1, x_j, x_k, \ldots) = 0 \tag{1}$$

in which there are n variables. Consider, further, any n equations that relate these variables, x, to other variables, x', say,

$$x_i = X_i(x'_1, x'_j, \ldots)$$
$$x_j = X_j(x'_1, x'_j, \ldots) \tag{2}$$

and so on to n equations. Now employ these equations as substitutions to eliminate the variables, x, from the function (1) and so obtain the function,

$$f(x'_1, x'_j, x'_k, \ldots) = 0 \tag{3}$$

in which there are n different variables. Finally, let it be possible to solve the set of equations (2) and so obtain the inverse set that makes possible the substitutions in (3) that will restore the initial function (1).

2.5   The Meaning of Covariance.

Physics and natural geometry differ from other explanatory sciences.

Consider the two expressions: Twice two is four; John is here now. The first has the same meaning when uttered in any place or at any time. The second has as many different meanings as there/places in which it is uttered (and times at which it is uttered.

are/

By the principle of independence explanatory relations and their expression do not vary with changes of place and time, where place and time are understood concretely as relative to human experiences. Hence the first statement is an explanatory relation and the second is not.

Now other sciences have no difficulty whatever in securing the proper independence for their explanatory statements. There is no question of transforming the multiplication table, the binomial theorem, the formulae for chemical compounds, or the principles of the theory of evolution. "Twice two is four" cannot vary from change of place or time because it makes no allusion to either place or time.

But in physics and natural geometry, things are not so simple. Their principles and laws have to be applicable to any concrete extension or duration within their domain. And any concrete extension means any part of Space, as any concrete duration means any part of Time. Only by introducing reference frames can we refer to the totality of concrete extensions or to the totality of concrete durations; for without the introduction of some order or structure either the concreteness or else the totality vanishes, and then we are left either with a merely imaginary totality or else with the limited extensions and durations actually experienced. But any order or structure or reference frame has its origin and orientation in some particular position, direction, and instant; all the specifications of the reference frame are relative to the origin and orientation; the origin and orientation are relative to some concrete position, direction, instant; ~~and an positions, directions, instants cannot be concrete~~ and we cannot designate a concrete position, direction, or instant without ~~appleas~~ appealing to our concrete experience.

In brief, to designate "any concrete extension" or "any concrete duration" there are needed both a reference frame and concrete human experience of some concrete extension or some concrete duration. And then 1) the designated concrete extension, 2) the designated concrete duration, 3) the reference frame, and 4) the experienced extension or duration, all are related; a change in any one involves changes in some other; some of the related terms refer directly to human experience and some do not. It follows that explanatory relations and systems must be independent of variations in any of the four just listed, for all four fall under the definition of descriptive elements.

This brings us to the meaning of covariance. When principles or laws in physics or geometry are expressed in covariant form, automatically they possess invariance under a group of transformations. Covariance secures for the mathematical expression of geometrical or physical laws 1) the independence that is due to them as explanatory correlations, 2) the independence automatically enjoyed by the laws of mathematicals, chemistry, biology, 3) the independence they otherwise do not enjoy because of their close connection with Space and Time.