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Thought is systematic when all its terms(and relations) are defined or clarified.

Terms are primitive or derived. Derived are defined by using primitive terms. Primitive terms are clarified by

- (1) by taking an instance in which the term is illustrated
- (2) by asking why this instance is such and such
- (3) by grasping the necessary and sufficient conditions for its being such and such

more precisely

there is a set of nominally defined terms
 there is a set of basic insights
 there is a set of explanatorily defined terms
 there are postulates and rules of procedure

Cf. p. 333

Clue, concepts, image, question, insight

Genesis: bring them together re "circle"

Extend to systematic thought

Illustrate by arithmetic, higher viewpoints, etc., etc.

End up with closed system of terms and relations x ; x

where the terms fix the relations and the relations fix the terms;
 as answer to the question, What are we doing when we are knowing?

2.8 Implicit Definition

Aristotle Met VII cc. 10 & 11: pparts of matter, of form

D Hilbert; implicit definition; skip the matter entirely;

a straight line is a line fixed by two and only two points;

eg (a, c) and (b, d) are ppoints and $y - c = (x - a)(c - d)/(a - b)$
 is what is meant by a straight line.

Complete generality, perhaps more than you want...

Higher Viewpoints

In considering primitive terms we introduced "system"

Now have to consider transition from less general to more general system; transition to "higher viewpoint"

Geometry: from Euclidean to Riemannian geometry;
n-dimensional curved space

More manageable: from arithmetic to algebra

3.1 Positive integers

Multitude of instances of one

Suppose as familiar: one, plus, equals

Infinite series of definitions: last plus one equals next;
a single insight grounds infinity of definitions

3.2 Addition tables

3.3 Homogeneous expansion

Multiplication: adding several instances of same number

Powers: multiplying several instances of same number

Subtraction, division, roots, going back to starting point

3.4 The Need for a Higher Viewpoint

Allow full generality to inverse operations:

subtraction yields negative numbers: $2 - 3$

division yields fractions: $28/9 = 3 \frac{1}{3}$

recurrent decimals: $28/9 = 3.\bar{3}$

roots yields surds $\sqrt{2}$, non-recurrent decimals, imaginary $\sqrt{-3}$

Further there is needed clarification of one, equals, minus

$|a - b| < e$, where e is as small as one cares to choose,
then a and b are equivalent; 'one' is equivalent to $.9$

3.5 Formulation of Higher Viewpoint

Rules for fractions, for signs, for equations, for indices

Hence redefinition of notions of addition, multiplication,
powers, subtraction, division, roots

Hence redefinition of numbers: generated not merely by
addition but by any operation on any number.

3.6 Successive Higher Viewpoints

The process from the virtual image, doing arithmetic, to a set of rules defining or ~~a~~ redefining operations to yield a new field of "numers"^b or mathematical entities, is RECURRENT

One can begin from maximum generality (at least currently known maximum) of set theory ~~xxx~~ and descend to ever less generality (good for teachers; whether good for students, we don't know yet)

But one becomes a competent mathematical technician only by gradually mounting from the more particular to the less, from the less general to the more.

3.7 The Significance of Symbolism

In the recurrent process, the symbols are(part of)the image in which one discerns the new more general rules governing the new operations

Not symbols are equally significant: /MDCCLXIV

the greatest discovery in arithmetic: zero, decimal, binary notation (position takes over major component of meaning: M becomes 1,000; C becomes 100, X becomes 10
Newton \bar{y} , Leibniz dy/dx

.. the symbolism is apt inasmuch as its immanent patterns as well as the dynamic latterns of its manipulation run parallel to the rules and operations that ~~xxxxx~~ generate the numbers, of other mathematical entities

symbolism takes over notable part of solutions: the symbols plus habits that have become automatic dictate what is to be done

symbolism constitutes heuristic technique ('eurisko); not only seek unknowns, but name them, formulate what is known about them (clues), put formulatins together into equations, solve equations

symbolism eliminates element of chance: Euclidean Cartesian symbolic pattern expresses meaning: acceleration, spin, wave, torque

symbolism provides virtual image in which one discerns rules for next higher level

Inverse Insight

Direct insight grasps the point

Inverse insight grasps that there is not point, nothing to be grasped -- the only point is that there is no point

1) Incommensurable magnitudes, irrational numbers, surds
Why is $\sqrt{2}$ a surd? What is the x such that $x^2 = 2$?

It is some number between 1 ($1^2 = 1$) and 2 ($2^2 = 4$)

Let us say that it is some (improper fraction, m/n ,
and let us suppose that all common factors have been removed
ie if $m/n = abcdef/aceghi$, then replace it by def/ghi

$x = m/n$

$x^2 = m^2/n^2 = 2$ ex hypothesis

but if m is prime to n , then m^2 must be prime to n^2

there is no fraction, m/n , such that $m/n = \sqrt{2}$

2) Non-countable multitude (first next next etc)

A multitude is countable if it can be ordered/and its
components set in one-to-one relationship with positive integers
eg 2, 4, 8, 16... is countable

Two parallel lines, drop perpendicular, select pivot, O ,
swing about O

One-to-one relationship between points in perpendicular and
points in infinite line

But there is no first or next: if interval, then infinity of
pts.

Diagonal theorem of ^{infinite} ~~XXXXXXXXXXXX~~ decimals

3) First law of motion keeping
What is the cause of a body moving with constant velocity
in a straight line as long as no external force intervenes

Is it being pushed along by its "inertia"? Occult cause

What requires a cause is not velocity but acceleration:
inverse insight.

4) If you were travelling with the velocity of light,
what would you see?

Empirical Residue

Inverse insight: particular problems in arithmetic, mechanics, electromagnetics

Empirical residue: general problem, how is collaboration possible (different investigators pooling results of experiments performed at different places and times) how is generalization possible (properties of this hydrogen atom to properties of 55% of the universe)

Heuristic structures: fuller determination of qq.

ER consists in positive empirical data

is to be denied any immanent intelligibility of its own

is connected with some compensating methodological intelligibility

Positive data: therefore not just vacuum

Denial of immanent intelligibility, not a denial of experience or of description: pointed out conceived named considered discussed affirmed denied

though no less given than color sound heat, conceived no less accurately, talked about no less fluently

still not to be explained in terms of transverse longitudinal waves, molecular motion, or any other theoretical construction

Inverse insight also compensated, but within a determinate range and contrary to one's anticipation of intelligibility

ER of general scientific relevance, and no one expects he is going to understand it.

Particular places and particular times

differ as a matter of fact

differ only as a matter of fact: no immanent intelligibility

Consider three equidistant positions, A B C, why do they differ?

Because of distances AB BC CA; why do the distances differ?

Because of the different positions? Accordingly the different positions differ because they are different!!

They ~~it~~ can be unified, organized, highly intelligibly by reference frames.

Yes but there are a vast ~~a~~ variety of possible reference frames; they all organize different places and times; they all presuppose differences in space and time; none explain these differences

Hence particular places timesas, they have no immanent intelligibility of their own, so they do not modify the immanent intelligibility of anything else

If the same experimnt is repeated at different places with different results, the different results are not due to difference in the place but difference in something at the place

If repeated at different times with different results, the difference is due not to difference in time but to something at one time and not at the other

Hence possibility of collaboration between all laboratories over all times

Again, similars are similarly understood

Every chemical element and every chemical compound differs from every other element and compound; they differ intelligibly, and the difference has to be explained (composition structure)

Every hydrogen atom differs from every other hydrogen atom; the difference is as real as the atoms; but the difference does not have to be explained; it is an aspect of ER, individuality. For there to be a difference in explanation, there has to be a difference in the data, not just more data similar in all respects

One explanation does for all hydrogen atoms, 55% of universe.

Abstraction

- 1) from place time individuality
- 2) from further aspects that are regarded as incidental irrelevant negligible

- 1) presupposes ER
- 2) presupposes an insight that selects what is not incidental but essential not irrelevant but significant not negligible but important

circle: circumference centre radii plane surface
color ~~xxxxx~~ size material (chalk rubber steel)

definition selects from the data what is essential significant important to the insight into the data

definition neglects what ~~ix~~ in the data is superfluous and so incidental irrelevant negligible as far as the occurrence of the insight goes

Judgement: Do circles exist? Approximate, yes. Beyond 'n' decimal places, we can hardly determine!!