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Paul Lorenzen: Einführung in die operative Logik und Mathematik
Berlin - Göttingen - Heidelberg 1955

——— " ———: Metamathematik. BI-Taschenbuch Bd 25
Mannheim 1962

——— " ———: Formale Logik. Sammlung Göschen Bd 1176/1176a
Berlin [2]1962

W. Kamlah - P. Lorenzen: Logische Propädeutik BI-Taschenbuch Bd 227/227a
Mannheim 1967

Vladimir Richter (S.J.): Untersuchungen zur operativen Logik
Alber, Freiburg i. Breisgau 1965

A short account of Paul Lorenzen's operative approach to Logic.

Philosophical relevancy: The operative approache to logic is an example that sowes, in which sense it may be said that some valid statements are the objective explicitation of reasonable operationes, the reasonableness of which is expressed by generallz admissible rules.
This procedure is useful for an account for first sentences.

Basic idea of the operative approach: From a syntactical viewpoint, logical systems are usually viewed as based on conventions. For different systems and also for expressions of different type in a system there are often analogous laws valid. For this systematic ambiguity and also for the logical laws an account may be given from a semantical viewpoint: logicallt valid are statements, which hold for ant state of affairs. But this semantical account has to use a metalanguage, which presupposes again similar logical laws.
In order to avoid a regressus in infinitum or a circle-argument, Lorenzen develops an operative approach (Paul Lorenzen: Protologik. Ein Beitrag zum Begruendungs-problem der Logik. Kant Studien 47 (1955/56) 350-358). This is based on an introduction rather than a presupposition of the required languages and metalanguages. Such an introduction is possible, because it is not alwazs necessarz to speak a language, if one wants to leanr a pattern governed activity, e.g. laying bricks. Then a language (language game) may be introduced in such a way that one can speak about this operation and make statements about it. The validity of these statements may be proved by performing the pattern goverend activity,which one has learned and about which one is speaking, and finding out if the result, which has been asserted in the statemant, can be reached by this activity - i.e. by checking the statement about the activity whith this activity.
This language may be itself viewed as an pattern governed acitivity and a metalanguage of this language developped, and so on. Its statements are to be checked by the language referred to.
This procedure may be applied to the special case of a pattern governed behaviour, which are operations of a calculus, which operations are determined by the rules of the calculus. Presupposing such a calculus, a rule is called an "admissible rule", if its application does not yield results, which cannot be yielded without its application. The proof for the admissibility of a rule consists in indicating a procedure by which the use of this rule can be eliminated, i.e. replaced by only apllying basic rules of the calculus. The investigation of patterns of such procedures for elimination (Eliminationsverfahren) is the field of "Protologik".
These considerations may be generalized for different calculi with their basic rules (basic formulas are interpreted as rules permitting the introduction of this formula as an expression of the calculus):
"Htpothetically admissible" is a rule, if some other rules - the hypothesis - are admissible or basic.
"Generallz admissible" is a rule, if the hypothesis is empty.

The logical laws are then the concequences of introducing the logical symbols by introduction rules. The logical laws are then yielded by application of rules, which are admissible after introduction of the logical symbols into any calculus. This hows some similarity with the calculus of natural deduction. The dissimilarity lies in the fact that only some of the rules, on which natural deduction is based, are used conventionally, namely for introduction of the logical symbol, and other basic rules of natural deduction are shown to be admissible under the hypothesis of these introduction rules.

Another way for giving an account for logic, used by Lorenzen, is presented in the framework of a dialogue game. The logical symbols are introduced by means of special rules for their use by the partners of the game. Logically true is a sentence, if the proponent of the statement is in possession of a strategy to win the game.

Examples:

1) pattern governed activity (schematisches Operieren):
a) laying bricks
b) calculus C:

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                                   &
                  &o                                            &&&
          &oo           &&o&                 &&&O                          &&&&&
    &ooo   &&oo&  &&o&o  &&&o&&     &&&oo    &&&&o&              &&&&&o       &&&&&&&
 &oooo &&OOO&    ...........
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2) Language about this activity (exmpel 1b: calculus C):

Rules of calculus C: R1: $\to$ &
R2: a $\to$ ao
R3: a $\to$ &a&
Admissible rules for C: R4: a $\to$ &ao&
R5: a $\to$ &&a
R6: o $\to$ o&

Proof by elimination prcedure:

ad R4: application of R4 eliminated by application of R2 and R3.

ad R5: To show the elimination of R5 one has to consider the way, how R2 and R3 have been used to yield "a". In order to yield the result of an application of R5 on "a" without using R5 one has to add in the way of apllying R2 and R3 in order to reach "a" starting from & (R1) an application of R3 before the first application of R2 in this way.
R5 is an example of an admissible rule in C, which can not be "deduced" from the basic rules.

ad R6: This case is a trivial one, because R6 can never be applyied in C, as according to R2 "o" can never occure ~~on~~ isolated. Therefore the admission of this rule in this calculus does not yield expressions, which can not be constructed only by means of R1, R2 and R3.
For any expression, which does not belong to a calculus, Lorenzen writes a Lambda (Λ).
The general form for these trivially admissible rules is therefore: $\Lambda \to a$
The negation ~~is~~ "-a" is later introduced by: $a \supset \Lambda \to -a$

3) Statements about rules and their admissibility:

$R1, R2, R3 \vdash R4$ $\qquad R1, R2, R3 \vdash R5$ $\qquad R1, R2, R3 \vdash R5$

The validity of these statements has been shown by procedures of elimination.

Trivial cases of such metarules, which are admissibbe in any calculus, would be (in the following R_1, R_2, ... are not rules of C but any rules):
$R_1, R_2, \ldots, R_n \vdash R_i \quad (i = 1, 2, \ldots, n)$ Proceeding to rules about metarules this could be written as: $\Vdash R1, R_2, \ldots, R_n \vdash R_i \quad (i = 1, 2, \ldots, n)$

Among these rules would also hold :

$R_1, \ldots, R_n \vdash R;\ R_1, \ldots, R_n \vdash S;\ R, S \vdash T \ \Vdash R_1, \ldots, R_n \vdash T$

4) Introduction of logical symbols:

Rules for introduction of new signs: Rules admissible after introduction:

$R_{\supset}$ $\qquad A \to B \vdash \ \to A \supset B$ $\qquad\qquad \to A \supset B \vdash A \to B$

Proof: The only way to arrive at $\to A \supset B$ was an application of $R_{\supset}$, which presupposes that $A \to B$ is admissible, q.e.d.

$R_{\&}$ $\qquad A, B \to A\&B$ $\qquad\qquad A\&B \to A \qquad A\&B \to B$

Proof in a similar way by elimination.

$A \nrightarrow A \vee B$ $\qquad A \rightarrow B \vee A$ $\qquad A \vee B,\ A \supset C,\ B \supset C \vdash C$

$(A \vee B) \& (A \supset C) \& (B \supset C) \rightarrow C$

$A \supset \wedge \rightarrow -A$ $\qquad -A \rightarrow A \supset \wedge$

$\cancel{A \& A}$ $\quad A \& -A \rightarrow \wedge$

$\rightarrow -(A \& -A)$

By means of this appraoch one arrives at the intuitionistic propositional calculus. The propositional calculus is reached, if one adds $\rightarrow A \vee -A$.

5) Dialogue game:

Partners: P(roponent), O(pponent)

Gëneral rules: P Starts, then everyone has alternatively one move. This may be an attack or a defence.
P may attack any formula O has stated, but defnde only against the last attack beought forth by O.
O can take action only against the last move of P.
O is allowed to make primitive statements, P only, if O had made this statement before.
P wins, if O connot move. Otherwise O wins.

Special rules:

sign	attack	defence
$A \& B$	L?	A
	R?	B
$(x)fx$	a?	fa
$A \vee B$	?	A / B
$(Ex)fx$	?	fa
$A \supset B$	A?	B
$-A$	A?	

	O	P
1		$p \supset p$
2	$p!$	p ✓

	O	P
1		$p \& q \supset p$
2	$p \& q!$	2L?
3	p	d2 p ✓

defence against last attack in 2.

	O	P
1		$p \supset p \vee q$
2	$p!$	$p \vee q$
3	?	p ✓

	O	P
1		$p \vee \sim p$
2	?	$\sim p$
3	$p!$ ✓	

	O	P
1		$p \supset \sim\sim p$
2	$p!$	$\sim\sim p$
3	$\sim p!$	$p!$ ✓

	O	P
1		$\sim\sim p \supset p$
2	$\sim\sim p!$	$\sim p$
3	$p!$ ✓	

	O	P
1		$(p \vee \sim p) \supset (\sim\sim p \supset p)$
2	$p \vee \sim p!$	$\sim\sim p \supset p$
3	$\sim\sim p!$	
		2?
4	p ¦ $\sim p$	d3 p ¦ 3! $\sim p$
5	¦ p	✓ ¦ 4! p ✓

It can be shown that the dialogue-methode yields the same logic as the first approach, i.e. a logic which is rather an intuitionistic logic(without pv-p).
The classical logic can be interpreted in one of the following two ways:
a) making "classical assumptions" of the kind pv-p;
b) viewing it as a "fiction", as an easier way for expressing formulas, which contain instead of p the double negation --p. The reason is that if one transforms the classical formulas F into formulas, which result by double negation (F'), it can be shown that F → F' and F' are logically valid in the intuitionistic logic.

Advantages of the operative approach:
1) It does not view logic as merely conventional and gives a better account for that, what is often referred to as the evidence of logic. Nevertheless there is not made an uncritical appeal to evidence in order to justify logic.This is accomplished by the procedures of elimination. At the same time the necessary minimum of conventional element in logical language is made clear.
2) Logic is not justified by presupposing a logic in the metalanguage. The required metalanguages are not presupposed, but introduced, in a constructive way.
3) The minimum conventional element of logical language consists in the rules for introducing logical signs. These signs are not introduced by a strict definition in the usual sense, but by rules, which specify their basic use. The essential meaning of these expressions is therefore given by their use. Other rules, which determine the use, are to be shown as admissible rules after introducing this sign.
4) As the pattern for introducing a sign and the pattern of the rules admissible after this introduction, are independent of a special language, the isomorphism of analogous logical laws in diefferent languages or in different types of expressions of the same language can be easily accounted for. The operative approach gives therefore a better understanding for the systematic ambiguity.

Philosophical relevancy:
1) To view the meaning of an expression in terms of its use is in good accord with the development of Analytical Philosophy after late Wittgenstein.
2) Though formalized languages are according to Goedel and Tarski essentially restricted, the patterns of operationes (indicated in introduction rules and in admissible rules) are not. If therefore the operational account for logic is valid, it shows, how the justification of logic opens at the same time a way to deal with problems, which are not restricted to a special language.
3) An essential feature of this way is that terms, which express in a language a meaning, that by itself is not restricted to this language, are not introduced into this language by a (explicite or implicite) definition within this language (Objective definition), but by an operative definition, by a pattern of operation.
4) Thus a correlation is established between operation and expression. Though the expression is tied into the restrictions of the language, the pattern of operation is not. Statements which result by application of the introduction rules for an expression and of the connected admissible rules upon a special language an objective explication of this operational pattern.
5) This may be used to give an account for the logical structure of the argument Aristotele uses for justifying the principle of contradiction.
   a) Positively it can be shown that after introduction of negation and conjunction the principle of contradiction in its categorical form (-(p&-p)) is the result of admissible rules and holds therefore in any language into which negation is introduced in this way.
   b) Negatively this can be interpreted such that anyone who uses the word negation in the meaning in which it has been introduced and doubts the validity of the principle of contradiction is liable to a contradictio exercita, i.e. between what he is saying and what he is doing in saying it. In order to have been able to use this argument it was necessary to show the relation between operation ("doing") and linguistic expression. This was done be "operative definition" and the whole operative approach.

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