Hence by taking successive positive integers as numerators and avoiding denominators with ~~common~~ factors common to the numerator, one can set up an infinite series of infinite series of proper fractions; e.g., 3/4, 3/5, 3/7,.... 4/5, 4/7, 4/9,....

Each of the proper fractions can be taken as an exponent, so that the fraction, 1/2, means a square root, the fraction, 2/3, means the square of the cube root, etc., etc.

Hence there are an infinite series of infinite series of types of root; and each can be applied to each term in the infinite series of infinite series of proper fractions.

This yields an infinite series of infinite series of rational proper fractions and, further, an infinite series of infinite series of infinite series of infinite series of surd proper fractions. Each fraction corresponds to a cut in any part of a continuum no matter how small.

There is something baffling about this conclusion. ~~Nor is this to be thought surprising. The list of series of series of series of fractions is an operation of intelligent formulation. The formulations assign rules for operating ad indefinitely, and once the rules are reached, the formulation ceases. What is guided by the rules is indefinite recurrence, and that appeals to the empirical residue. The contin~~
Nor is this to be thought surprising. The continuum illustrates the essential divergence of the empirical residue from the field of intelligent formulation. The list of series of series is an operation of intelligence that ceases as soon as a rule is reached. The continuum, as it were, mocks the rules. It possesses cuts to correspond to all definable fractions, rational or surd, both in itself as a whole and, further, in any of its parts no matter how small. ~~Obviously~~, the cuts must be more dense when taken in the part. ~~Inversely, it follows~~ that the class, "definable fractions," must be far smaller than the class, "cuts of a continuum." In fact, one begins to grasp the nature of the continuum when one grasps that it lies beyond the definable, that it represents the divergence of the empirical residue from the terms of formulating intelligence.

It would seem that

Similarly, it would seem

The principle for the division of kinds of continua has already been assigned. There are imaginary continua, known by appealing to the empirical residue of imagination. There are immediately concrete continua; they are given to sense here and now; they are known to be given, not by verifying the infinity of infinity of cuts, but by verifying the properties of mathematical continuous functions. Finally, there are remotely concrete continua, given to oneself at other places and times or given to other people at any other place or time than the present.