II

The Development of Mathematical Logic

1. The development of ML has been the pursuit of an ideal, viz., a rigorous hypothetico-deductive system that from minimal suppositions would embrace the whole of known mathematics.

2. The ideal has been formulated as an axiomatic system or logical formalization.

It distinguishes terms and propositions, divides both into derived and not-derived, conceives not-derived as relative to system, and names them primitive.

Derived terms are defined by primitive.

Derived propositions are deduced from primitive.

Rules of derivation must be stated explicitly, and no derivations are admitted except in accord with stated rules.

Let P and Q denote two LF's, and let p be any proposition that can be constructed in P.

Then P and Q are equivalent, if primitive of P are derived in Q and primitive of Q are derived in P.

P is complete, if one can derive either p or Np.

P is coherent, if one cannot derive both p and Np.

The primitive propositions of P are independent if no one can be derived from the others.

The primitive propositions of P are elegant if they offer the simplest basis for deriving in the simplest manner all the propositions of P.

3. Principal lines of endeavor.

a Axiomatic set theory: Zermelo - Fraenkel - von Neumann

b Whitehead-Russell, Principia mathematica; aims to base whole of mathematics on logical axioms; a magnificent unitary view that remains one of the principal directions.

However in both first and second editions there is a non-logical axiom of infinity.

In first addition there is also a "theory of types" (to avoid paradox of class of classes that do not contain themselves) and an axiom of reducibility (to make possible Dedekind's definition of real number, excluded by theory of types).

In second edition the axiom of reducibility is eliminated and there is employed a weakened theory of types that eliminates syntactical but not semantical paradoxes.

c Hilbert proposed a two-level approach.

First, a formalized deduction of the whole of mathematics from mathematical axioms; on this level there were to be admitted infinities of objects and of operations.

Secondly, a metamathematics that on a strictly finite basis would investigate logical properties (especially consistency) of the first mathematical level.

Results: short term, geometry worked out with axioms verified intuitively in model that supposes validity of counting numbers; arithmetic could be shown to be consistent only if some axioms were omitted or all weakened. However, as will appear, this has proved most fruitful line of inquiry.

d Intuitionistic school: Brouwer, Heyting

Insists that LF is only tool, that mathematics is essentially constructive, that excluded middle cannot be invoked indiscriminatly. Program involves lopping off more of classical mathematics than mathematicians are ready to sacrifice.

e Gonseth; review Dialectica

Tends to conceive axiomatic ideal just an outdated Euclidean avatar; insists on development, interaction between maths and cultural movements; relativist in tone.

f Bourbaki group: Hilbert's first level; metamathematics is a separate department of no particular interest to mathematician; weak point that rigid axiomatic structure neither accounts for past development of maths nor opens way to developments of future.

4. Gödelian limitations.

Jean Ladriere, Les limitations internes des formalismes, Louvain (Nauwelaerts) and Paris (Gauthier-Villars) 1957. Pp. 702

There have been demonstrated a series of theorems setting limitations to the possibility of reaching the ideal of the rigorously deductive mathematical system. The general form of the argument in such cases is approximately as follows:

a An LF is a symbolic technique capable of representing a manifold of deductive sequences.

Consider an LFL and an LFM, which symbolically are identical or sufficiently parallel, but differ inasmuch as LFL is interpreted logically while LFM is interpreted mathematically.

b. Now in mathematics there exist non-enumerable sets, i.e., aggregates that do not admit a one-to-one correspondence with the positive integers, and so cannot be enumerated (counted).

Hence, to suppose that such a set is enumerable (e.g. the set of infinite decimals) results in a contradiction, and this contradiction can be demonstrated.

c With sufficent ingenuity it is possible to make the LFL sufficiently parallel to the LFM so that the proof of non-enumerability in LFM is

is matched by a proof of logical impossibility in LFL.

In other words, the proof that the proposition "K" has been enumerated" is contradictory is paralleled by a proof that the proposition "K" is a theorem, or "K" is a soluble problem, or "K" is true, or "K is definable" is contradictory.

d Such theorems are extremely complex; they have been worked out in a variety of manners; they arise when the LFL is sufficiently powerful to represent the theory of division, resolution into prime factors, and the unicity of such resolution.

e Their proximate significance was the refutation of Hilbert's proposal to settle the logical validity of arithmetic on a finitist basis.

Godel's demonstration was followed by a demonstration by Gentzen that arithmetic was non-contradictory, where however the LFL had to employ transfinite induction.

f The ultimate significance, however, of such Godelian limitations seems to be the same as of inverse insight; cf. irrationals, transcendental numbers, Galois on fifth degree equations, Newton's first law.

5. The Transcendence of Godelian Limitations.

a Mere avoidance: J. S. Myhill (JSL 15(1950) 185-196) avoids such consequences by employing a logic without quantification and without negation.

b Use of indefinitely large stratifications (analogy)

Church: "implication" and "quantification" take on different meanings on different strata

Curry: similar procedure re his basic notion of canonicity.

c Skolem paradox shows that by different modes of stating one-to-one correspondence, "enumerable" takes on different meanings.

d L. Henkin's study of relations between LFL and models showing that LFL lacks absolutely definite meaning.

e Hao Wang: indefinite series of sub-systems; at each level new resources of construction and new meaning for enumerable; the consistency and the theory of truth for any level, m, demonstrable at level (m + 2).

f Significance: ideal of IL moving from static and closed to analogous and open.

III

The Truth of an ML System

1. The truth of what?

a not the truth of what is seen or written that may be or may not be wf, but only wf expressions can be under consideration, since not-wf is just jumble of symbols

moreover, what is seen or written is usually only part of the expressions that are possible or necessary in the system; the larger range of what might be seen or written is also relevant and may be decisive

finally, strictly expressions are not true or false but merely adequate or inadequate; what is true or false is w hat is meant, intended.

b the truth of a MLS is the truth of what is conceived, considered, meant, intended by one w ho understands the MLS, who not only can judge whether any given expression is a wff in the MLS, but also can manipulate the MLS and so bring out all its virtualities of expression

c hence the truth of an MLS is the truth of a virtual totality of propositions

where the same proposition can be expressed in any of several languages, and the same expression in any given language can be uttered any number of times

where the virtual totality consists in all the propositions that can be formed within the system and can be derived in accord with explicitly stated rules from the axioms.

2. What is the general character of such truth?

a Aristotle, Met. H 10 1087a 15-20, distinguished science (episteme) in potency and science in act, affirmed that science in potency is itself indeterminate and of the universal and indeterminate, while science in act is determinate and of the particular and determinate.

b Clearly then, the truth of an MLS is the truth of universal and indeterminate and determinable knowledge of the universal, indeterminate and determinable.

c Again, Aquinas distinguished abstraction of universal from particular, and abstraction of form from matter

The type of abstraction of an MLS is of form from matter, of a forma artificialis, of a net-work of relations linking unspecified propositions, arguments, predicates, classes, relations.

That MLS is such a form, appears from fact that an MLS is a logical interpretation of a range of symbolic expressions that admit other isomorphic interpretations

3. What is meant by "truth"?

a Distinguish definition and criterion of truth.

Truth is defined as "adaequatio intellectus ad rem."

However, the criterion of truth is the precise element in knowledge by which one knows that such an "adaequatio" has been attained; commonly, this criterion is conceived by Scholastics as "perspicientia evidentiae sufficientis qua sufficientis."

b The criterion of truth is analyzed more exactly in "Insight," Chap. X as a grasp of the virtually unconditioned, i.e., as a grasp of 1) a conditioned, 2) a link between the conditioned and its conditions, and 3) of the fulfillment of the conditions.

c One of the modes of the virtually unconditioned is that of the analytic proposition, where

1) the conditioned is the proposition in question

2) the fulfillment of the conditions is the set of definitions of the terms contained in the proposition

3) the link between conditions and conditioned are the syntactical structures in accord with which single terms in their defined sense coalesce to form a proposition.

d Another of the modes of the virtually unconditioned is the analytic principle, where

An analytic principle is an analytic proposition whose terms, in their defined sense, occur in true judgements of fact.

In other words, an analytic principle adds an existential reference to an analytic proposition, where existence is defined by its connection with factual truth.

On factual truth and on the three main modes of analytic principles, see Insight, chap. X.

4. An MLS is by postulation a virtually unconditioned.

For an MLS is a virtual totality of propositions that result through explicitly stated rules of derivation from primitive terms and propostions.

Hence the MLS is a conditioned: it results from something inadequately distinct from itself.

The MLS has conditions, namely, the primitive terms and propositions.

These conditions are fulfilled by postulation, just as the definitions of the analytic proposition are fulfilled by postulation.

The MLS is a conditioned linked to its conditions, for the rules of derivation determine what is the totality of propositions that pertain to the MLS; and the rules of derivation are posited by postulation, just as the syntax of the analytic proposition is posited by postulation.

Accordingly, every MLS (satisfying the definition of an MLS) has the verbal type of truth that pertains to the analytic proposition.

In other words, if you were to speak or think in certain defined manners, you would eo ipso be committed to accepting such and such an MLS.

5. Various types of MLS contain fragments of factual truth.

The present assertion stands to the preceding as the analytic proposition to the analytic principle.

However, analytic principles may be absolute (as in metaphysics), provisional (as commonly is the case in empirical science), or serial (as in mathematics).

In our next lecture, we shall raise the question of the foundations of logic; this will be equivalent to the problem of putting logic on foundations of the same absolute type of metaphysics. However, the authors of the various MLS entertain no such intention.

Again, while MLS is closely related to mathematics and may be conceived as a generalization of mathematics, still there is no universal agreement among mathematicians upon the exact nature of mathematics and so there is no possibility at the present time of a universal agreement upon the generalization of mathematics.

Cf. P. Bernays (Zurich), Zur Beurteilung der Situation in der beweistheoretischen Forschung, Rev. Intern. de Phil., VIII (Brussels 1954 7-13, considered that while the choice of method and of deductive framework for investigation of mathematical foundations may soon be settled, the determination of the notion of mathematics remains very remote.

On the basis, then, of our triple division of types of analytic principle (absolute, provisional, serial), it would seem that the various MLS possess factual truth of no more than the provisional type. In other words, the various MLS are just a series of hypotheses on the nature of deductive system.

This view is confirmed by our preceding lectures on the general character and on the development of ML; in other words, the history of ML is the history of the gradual and as yet incomplete discovery of what an MLS really is; a broad and a priori ideal has been gradually trimmed down into accord with the facts of logical possibility.

This view is also confirmed by the truth of the assertion that heads this section (5). For a very brief analysis serves to show that while many of the various MLS are incompatible, still the basic assertions contained in each are true as far as they go. Thus,

a There is no doubt about the existence, the factual truth of the occurrence, of propositions, negative propositions, compound propositions in the truth-functional sense. There follows the factual truth of the various elaborations of the classical propositional calculus, in sensu aienti.

in global judgment

b There is no doubt about the factual truth of strict implication. Thus, there is a strict implication of an MLS in its axioms and rules of derivation.

There follows, in sensu aienti, the factual truth of some system of strict implication; but it is to be noted that Lewis's systems, S 4-5, are incompatible with S 6-8.

Further, since CPC makes no provision for strict implication, CPC can be factually true only in sensu aienti.

c There is no doubt about the factual truth that contingent futures are neither true nor false for minds to which they are future.

Many formal implications may be the limit of an operational procedure — i.e. in perfect systems, there is an instance of strict implication If A then B without also If B, then A

Hence, in sensu dicti, a three-valued logic (true, false, neither) is factually true; and this reveals the fragmentary character of two-valued logics.

d There seems to be no doubt that, as long as knowledge is still in genesis, in potentia, the principle of excluded middle cannot be built into a logical system and so applied indiscriminately.

Hence, "intuitionistic" logic possesses a measure of factual truth; and this measure involves a restriction on the value of logics that accept excluded middle as an automatically operative principle.

See F.B. Fitch, Symbolic Logic, New York, Ronald Press, 1952, who weakens excluded middle and thereby frees himself from Russell's theory of types which he argues to be self-referentially incoherent.

IV

The Foundations of Logic

1. Originally logic was a clear-headed assertion of human rationality against sophistry (abuse of language to deceive mind) and apart from rhetoric (concerned with legitimate persuasion with regard to contingent).

However, it was not a pure assertion of rationality but an assertion implemented in technique of figures and moods of syllogism, &c. None the less, this technical aspect was considered of minor importance, and no one felt it a matter of any moment what one thought, say, of the fourth figure.

Moreover, interest in epistemological issues led to a distinction between major and minor logic, with major logic a field for enormous differences of opinion, while minor logic remained the unquestioned object of acceptance for all sane people. One may perhaps add that such unquestioning acceptance played no small part in moving mathematicians to solve all their ultimate difficulties by basing mathematics on logic.

2. The development of MLS has changed the situation.

First, it s hould be conceded that now there exist techniques that are far superior in precision, in capacity of very complex refinements, in ability to deal with such enormously involved issues as the properties of axiomatic systems.

Secondly, it has to be recognized that the name, logic, as employed in a broad and steady stream of articles and books, denotes not any concern with the immutable laws of mind but rather familiarity with some or all of a set of symbolic techniques.

Thirdly, it would be, I think, a strategic blunder to be concerned over minor divergences between ML and Aristotelian logic. There are types of MLS that involve differences from AL; but it has been shown possible to construct an LF that coincides with the assumptions and implications of Aristotelian technique.

What, finally, is essentially new in the present situation is the invasion of the field of logic by the philosophic differences that formerly were confined to major logic. Accordingly, it no longer is possible to treat logic adequately without going into philosophic issues that previously could be neglected (by common consent) within logic. An instance of the interdependence of the whole of knowledge that should not be unwelcome to the real philosopher.

3. The Ambivalence of the Technical Achievement

There exist logical techniques that virtually are independent of any particular mind (equivalent to computers), that can handle problems too

of Aristotle's Psychological theories of mind, memory—

complex to be considered previously, and that thereby raise the question whether they are simply a tool for mind or rather a substitute for mind.

This issue has been suggested from the first lecture, and now it may be put explicitly as follows:

If one pleases, one may for one's personal satisfaction work out a theory of truth and come to the conclusion that an MLS by definition has the truth of an analytic proposition and that the various MLS represent a series of fragments of factual truth as well. On such grounds one becomes rationally committed to the acceptance of MLS either hypothetically or, under certain restrictions that vary with various systems, absolutely.

But one may, and many do, find it more pleasing to consider the matter of rational commitment to truth as a private and minor matter. What alone counts is the external fact that you employ some language and that any language presumably can be reduced to a logical calculus plus a vocabulary. What really counts, is not anyone's private and internal dedication to truth (which is a greviously abstruse matter0, but the public and external fact that he talks or ventures to write.

The ambivalence of the technical acheivement of ML is that it seems to offer the alternatives of either a rational commitment to truth or of a non-rational pragmatic acquiescence in the fact of talk.

4. Some symptoms

H. Behmann (Proceedings, Second Intern. Cong. of the Intern. Union for the Phil. of Science, held Zurich 154, published Neuchatel 1955, vol. II, pp. 97-108) considered that strict implication was natural only in the sense that the notion of flat space is natural.

At the Colloque de Logique held in connection with 1953 Brussels Intern. cong. Phil., reported by R. Feys, RPL 1953, published Rev. Intern. de Phil. 1954,

F. Gonseth and A. Tarski met head on, when Gonseth put forth his highly nuanced views on the nature of proof, and Tarski stubbornly maintained that he could find nothing rational in such proposals. It would seem that by "rational" Tarski meant "symbolic and technical".

In PL the CPC holds a dominant position, not, I should say because of any merits of a logical character, but solely because it brings about a maximum assimilation of logic to the modes of mathematical technique.

The philosophical movements of Logical Atomism(Russell) and of Logical Positivism (Wittgenstein)

(1) presuppose that a language is equivalent to a logical calculus plus a vocabulary

(2) presuppose that no questions are to be asked about the logical calculus itself

(3) and by their inability to carry out their own programs have brought about their own demise, at least, at Oxford.

See J. O. Urmson, Philosophic Analysis: Its Development Between the Two World Wars. Oxford, Clarendon, 1956.

Also G. J. Warnock, Fellow of Magdalen College , "I am not, nor is any philosppher of my acquaintance, a Logical Positivist." In "The Revolution in Philosophy" by A. J. Ayer et al., London, Macmillan, 1956, p. 124.

5. The Question of Foundations.

a There are many ALS and, while some are equivalent, many are not. This fact posits as a problem the theory of choice of any given system either absolutely or relatively to a particular task.

b When the field to be formalized is as complex as arithmetic, the MLS becomes extremely complicated. Transfinite induction needed for logical theory of ordinary methematical induction. It follows that logic is not a source of greater evidence but rather of bigger problems. What is the basis from which these larger problems are attacked?

c While it is the problem of the infinite that makes an LF of arithmetic so complicated, there seem to me to be parallel problems in the finite domain of ordinary matters of fact. For,

c' ordinary concepts are not the simple smooth regular homogeneous nuggets needed to conform to an MLS, but they are open heuristic structures subject to enormous differentiation and variation; see Insight on Common Sense and on Classical, Statistical, Genetic, Dialectical, Heuristic Structures. Also notion of "person" in my Divin. Pers. Conc. Anal.

c" Hellmut Stoffer (Bonn), Die moderne Ansatze zu einer Logik der Denkformen, Zeit. f. phil. Forschung, 10(1956), 442-466, 601-621, in a very fully documented pair of artiśles, argues for six types of logic needed to classify and deal w ith the expressed forms of thinking, (1) Plane (2) Dialectical, (3) Existential, (4) Magical, (5) Mystical, (6) Hermeneutical.

All MLS would fall under the first category; discussion of the first five would fall under the sixth.

This direction of thought would be confirmed by the English experiment as described by Urmson.

d It would seem that (1) Developing Intelligence and (2) perfectly transparent expression and inference form a dialectical couple; the pursuit of either involves some sacrifice of the other.

6. Samples of Foundations of Logic

a Middle term: "understand means "know cause"
Causa essendi: phases of moon because of sphericity.
Causa cognoscendi: sphericity of moon because of phases
Systematic expansion: textbook of a science vs. history of the same science.

b Subject term: Aristotle, Met., Z, 17, "quid" means "propter quid"

c Predication (basic case): same data understood as individual (notion of thing) and as of a kind (notion of property: descriptive (heavy, hot); explanatory (mass, temperature).

d A Judgement: virtually unconditioned; if A, then B; but A; therefore B, where A and B are propositions, sets of propositions; where A is simply experience; where "if A then B" is merely implicit insight, invariant of thought, invariant of expression.

Significance of syllogism is not Kantian regress to 2^n premisses but manifestation of conclusion as virtually unconditioned.

Even necessary object (God, analytic principle) known by us contingently.

Fallacy of waiting to be necessitated: inevitably heads toward scepticism (XIVth century; rationalism).

Necessity of personal commitment, a personal responsibility; where commitment is intrinsically rational; where commitment is to absolute though in us it occurs contingently.

e Three levels to Aristoteliam sullogismos epistemonikos.

(1) The level of the words, symbols, sensible data to which words or symbols refer.

(2) The level of understanding: grasping subject (b above); grasping predicate and predication (c, above); grasping ground of predicate's being in subject of being known as pertaining to subject (a, above).

(3) The level of judgment and personal commitment (submission) to imanent rational necessity (d, above).

The three levels effectively distinguished only in so far as one moves behind terms, propositions, inferences to their ground in the experiencing, intelligent rational subject.

Again, the subject ek-sists philosophically only in so far as he distinguishes the three levels effectively.

To acknowledge explicitly only the first involves one in materialism, sensism, phenomenalism, positivism, pragmatism.

To acknowledge explicitly only first and second involves one in an idealism, relativism, essentialism, immenentism, Kantian criticism.

Only when all three acknowledged is Thomist realism reached.

Note that what is significant (effective distinction) lies not in subject's formulation of the three levels (for that may be superseded by more accurate formulations) but in the subject's immediate grasp in himself of his preconceptual, prejudicial inability to get around fact of three levels. Cf. Insight, chap. XI.

The subject in this self-knowledge is the foundation of logic; it is a foundation in the reality of the subject himself, and in every experiencing intelligent, reasonable subject; it is a foundation in a reality and so it is beyond the relativism of successively more nuanced and more accurate formulations (statements) of philosophic position.

f This foundation of logic is also a foundation of metaphysics and of the general form of ethics. Insight chap. XIV to XVIII.

g This foundation is dynamic

ML moved from naive ideal of grand deduction from single set of axioms to necessity of set of deductive levels with each level far richer in resources than preceding (Lang)

because it started out with supposition of human knowledge, not as a process of knowing-coming-to-be, but as an aggregate of ready-made univocal terms and ready-made true propositions.

The above exhibits the coming-to-be of the subject-term, the coming-to-be of predication, the coming-to-be of the middle term, the coming-to-be of judgement.

The omission of strict implication in CPC and its derivatives, is the omission of the coming-to-be of judgmentthrough link between conditions and conditioned.

CPC can express only compound not complex sentence.

h This foundation grounds grand-scale analogy.

Experience: understanding: judgement :: potency : form : act

For every stage in the development of understanding, of science

i This foundation grounds sequences of developing concepts of same object.

Experience: the phenomena of fire.

Heuristic structure: what is fire?

Sequence: (a) fire is X manifested by these phenomena; (b) fire is one of four elements; (c) fire is manifestation of "phlogiston"; (d) fire is a chemical activity of a certain specified type.

Because of (a), it is possible for (b), (c) , (d) to be statements about the same object and so to be incompatible.

j This foundation is sufficient not only for traditional and mathematical logic, but also for consideration of dialectical, existential, magical, mystical, hermeneutical logics.

Dialectical: start from what seems obvious; led it have its head; absurd conclusions bring to light the limitations of initial "obvious"; e.g. development of ML; cf. repeated use of this technique in Hegel's Phenomenology of Spirit; Toynbee's "Transfiguration" (problem of transportation becomes problem in ethics of motoring, traffic)

Existential: cf. e above.

Magical: cf. section on "Metaphysics, Mystery, Myth" in Insight, chap. XVII.

Mystical: cf. H. Leisegang on St. Paul (Denkformen, 1951^{2})

Hermeneutical: cf. Truth of Interpretation, Insight, chap. XVII.

Mathematical Logic and Scholasticism.

1. A New Factor in Problem of Method.

While ML is strictly indifferent philosophically and this philosophic indifference is maintained by the better and more intelligent writers, still it very easily is given an empiricist and pragmatic twist and so amounts to an invasion of logic by the enormous problem of philosophic differences.

Perhaps the practical procedure would be to continue to treat logic as philosophically indifferent, to use it as an introduction to philosophy, to add perhaps that it gives rise to questions and differences that at a later stage in the course will be treated more adequately.

From a theoretical viewpoint all seems to depend on the solution that one adopts of the general problem of philosophic difference.

On a view associated with the great name of E. Gilson one should begin philosophy with metaphysics.

Now I have no doubt that the ultimate and decisive factor is "sapientia," and that in St. Thomas "sapientia" is (1) a gift of the Holy Ghost that is connected with mystical experience (patiens divina) and (2) within the natural order, Aristotle's Metaphysics.
Accordingly, I am quite ready to grant that this view has a solid foundation in tradition.

However, in Aquinas, while there is a distinction between natural and supernatural, between reason and faith, there is no separation. The Thomist distinction was followed four centuries later by the Cartesian separation. It is within the context of that separation, within the context of subsequent condemnations of fideism and of traditionalism, within the context of our own Anglo-Saxon cultural traditions, that we have to operate.

For such and many other reasons, I have endeavored in "Insight" to work out a genetic account of sapientia, an "ascensio mentis per intelligibile et verum ad ens."

Hence, in the fourth lecture, I sought the foundations of logic in the subject's personal appropriation of his own empirical, intellectual, and rational consciousness; on this view the foundations of logic are by identity the solution of the epistemological problem, the foundations of metaphysics, the foundations of ethics, and the foundations of natural theology.

2. Is Scholastic thought to be cast in the form of an axiomatic system?

There certainly exists a conception of Scholasticism as a deductive system.

Philosophy is the deduction of necessary conclusions from self-evident principles.

Theology is the deduction of further conclusions from the Word of God with (or without) the help of the self-evident principles of natural reason.

However, this view is more easily substantiated by appealing to Scotus than to Aquinas, and more by appealing to Aquinas' statements (which one interprets in the light of one's own deductivist horizon) than from Aquinas' practice.

I think it useful to distinguish (1) non-empirical, (2) empirical, and (3) comprehensive types of inquiry.

(1) In the non-empirical type there is little appeal to concrete matters of fact. Such is mathematics, and it is markedly a deductive science.

Note, however, that this deductive aspect is coupled with a constructive aspect. Mathematics starts from elements (geometrical entities, numbers, etc.) and out of these simpler and prior objects constructs ever more complex objects. And it is precisely this constructive aspect that makes the deductive aspect possible. Because x, y, z, can each be constructed by beginning from a, b, c, it is possible to deduce the relations between x, y, z.

(2) In the empirical type of inquiry, there is a twofold movement.

First, there is the movement from the priora quoad nos to the priora quoad se: this is represented by any history of the origins and development of physics, chemistry, biology; all along the line the decisive factor is the sensible matter of fact.

Secondly, however, there is the opposite movement from the priora quoad se to the priora quoad nos: this is represented by the textbook of physics or chemistry, in which one begins from laws and systems and proceeds deductively towards the concrete and complex.

Note that neither the non-empirical deduction nor the empirical deduction occurs within a single plane.

ML seems to have shown that a mathematical LF has to be, not a single deduction from a single set of premisses, but rather a series of levels each with more comprehensive premisses and its own deductive expansion; where, observe, the series of levels is open, so that there is no top level.

Again, the history of physics, chemistry, biology has been the history of a succession of higher viewpoints, where on each later viewpoint the premisses of the deduction have been different. Einstein includes Newton as a limiting case, but Einstein's premisses were beyond Newton's ken.

(3) Philosophy and theology are inquiries of the comprehensive type: they are concerned, each in its own way, with everything.

Now if a philosophy is to include a philosophy of science, while the sciences are open, developing, changing, then the possibility of its being fixed is that it have the fixity, not of a monolith, but of a form that admits variable contents.

In other words, such a philosophy has to be an invariant structure in which the structured elements are free to change.

Hence, in "Insight" the structure of the knowing subject (a concrete unity of experiencing, understanding, judging) is shown to be invariant (not subject to revision) and to imply a corresponding invariant structure (potency, form, act) in the proportionate object of our knowledge. Where the philosopher knows that there are forms but the various departments of science investigate what the forms are.

Is philosophy so conceived to be cast in axiomatic form? I do not think it impossible. I do not think it to be very useful, because philosophy is not constructive after the fashion of mathematics, and because the real issues in philosophy do not lie in drawing conclusions but in the subjective intellectual development that is the "ascensio mentis per intelligibile et verum in ens".

Traditionally, Scholasticism is not a deduction from a single, limited, well-defined set of principles: it is a sequence of theses that are based on deductive arguments, with the premisses of the arguments coming from all over the map.

Examine any manual of philosophy and you will find this to be so.

Make a logical analysis of the treatment of "soul" in Aquinas' Contra Gentiles, II, about chap. 48 to 94.

3. Does ML eliminate "existence"?

See for nunaced discussion of "quantifiers", J. Dopp, La notion d'esixtence dans la logique moderne, Proceedings, Amsterdam 1948, pp. 735-739.

For a defence of what amounts to the proposition, "Existential propositions do not exist", see G. Ryle's paper "Systematically Misleading Expressions", originally published in Mind about 1938-39, and reprinted in A.G.N. Flew, Logic and Language, First Series, Oxford, Blackwell, 1952.

The answer to this question is dependent on what one happens to mean by "existence".

I mean what is known through grasping the virtually unconditioned in a concrete judgment of fact; it is what is known by the "yes", the "is", in a concrete judgment.

On the whole, "existence" in this sense is beyond the horizon not only of mathematical logicians, logical positivists, but also of so-called existentialists.

On the other hand, "existence" in this sense can be shown to be admitted implicitly by anyone that claims an ML to possess a reference to anything that happens to be.

4. Does an ML eliminate substance?

An ML makes it possible to do an enormous amount of logic without raising any question of substance, and so for this reason it is welcomed by many.

Again, in Aristotle's day, explanatory science barely existed, and so in Aristotle and in those dependent on him for their notion of science, there is insufficient stress on the significance of relational structures in science.

Thirdly, one can do mathematics without a notion of substance.

Fourthly, one cannot dispense with the notion of substance in any science that is not only descriptive but also explanatory, as is shown in Insight.

Fifthly, "substance" is an ambivalent term: it can mean the reality that is an intelligible unity-identity-whole (in spatio-temporal difference); it can also be used to denote the "already out there now real" of spontaneous extroverted animal consciousness. This basic ambiguity rises from differences in the attitude, orientation, degree of ek-sistence, of the subject; and so it is an extremely difficult problem in philosophy.

5. What is relation of ML to (1) Logical Atomism, and (2) Logical Positivism?

See J.O.Urmson, Philosophical Analysis: Its Development between the two World Wars. Oxford Clarendon 1956 A brief exact illuminating and cogent work.

Logical Atomism was the hope that a complete and satisfactory philosophy could be constructed by proceeding from the MLS of Principa Mathematica, substituting ordinary words for the variables in the MLS, and showing that apart from the connectives supplied by the MLS nothing was needed but atomic experiences of the type, "red here now".

Logical Positivism proceeds from a division of possible propositions based upon MLS.

A proposition may be true independently of the truth or falsity of its variables, e.g., EANpqCpq, and it is named a tautology; it is simply a circular combination of functors.

A proposition may be false independently of the truth or falsity of its variables, e.g., ENpp; it is contradictory.

A proposition may be true or false according to the truth or falsity of one or more of its variables, e.g., Kpq is true if both p and q are true, A pq is true provided not both p and q are false, etc. Such a proposition needs some extra-logical means of verification.

Hence, propositions are either tautologies (mere circular combinations of words) or they are empirically verifiable. Only in the latter case have they much in the way of meaning. The crucial problem of LP was to find a verifiable verification principle.