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THE GENERAL CHARACTER OF MATHEMATICAL LOGIC

A. Descriptive Approach

B. Analytic Approach

A. Descriptive Approach

1. Traditionally logic has been concerned with terms, propositions, inferences; the investigation of deductive systems has been considered too complex to be attempted.

   The original and valuable contribution of mathematical logic (henceforth ML) has been the study of deductive systems as wholes possessing determinate properties.

   Thus, given any apparently suitable set of definitions, postulates, rules of derivation, one may ask what will result; in particular, one may consider:

   The problem of coherence: can both sides of a contradiction be deduced?

   The problem of completeness: is there some relevant proposition that can be neither proved nor disproved?

   The decision problem: is there some automatic procedure for solving all problems that arise within the proposed system?

2. This investigation has been closely linked with mathematics.

   Advantage of concrete and easily controlled question, How much mathematics can be deduced from what basis?

   Again, since mathematics is deductive, the question of possible deductive systems includes the question of possible mathematical systems; hence, ML becomes mathematics at the most general level.

   Finally, at the beginning of this century mathematicians were concerned with legitimacy of an axiom of choice in set theory and with paradoxes involved in the notion of the set of all sets; the Bourbaki group illustrate the waning of this concern which, however, is only one aspect of the link between ML and mathematics.

3. The investigation has been symbolic and technical.

It has been symbolic because, inasmuch as a symbolism possesses determinate characteristics, it becomes possible to envisage the totality of possible deductive chains resting on a given basis.

It has been technical in the sense that the only permissible symbols, operations, changes of order, omissions, substitutions, additions, occur in accord with explicitly stated rules.

The significance of this combination of the symbolic and the technical is an objectification of mind and a consequent independence of any particular mind.

Illustrate by Euclid's lack of rigor and of necessity.
Illustrate by process of taking square root.
Recall great strides of XIXth century mathematics through elimination of geometrical intuition in calculus, through Dedekind's definition of real number, Cantor's synthesis in theory of abstract sets.

Value: unless all insights explicitly formulated in initial axioms and rules, then

you will be unable to exploit all their virtualities,
you will operate with concealed presuppositions,
you may be engaged in self-contradictory enterprise.

Note, however, that when ML is named 'formal' it is this symbolic and technical character that is meant; hence Umdeutung of 'formal' in name, Formal Logic.

Again, note pre-established harmony between this approach to logical issues and an empiricist, pragmatic mentality recall B. Malinowski).

4. The symbolic-technical character of ML gives rise to problems unknown in scholastic (classical) tradition.

When a scholastic comes upon a difficulty in his position, he draws a distinction.

A symbolic technique cannot draw distinctions; distinctions habe to be included in initial definitions; they are equivalent to the introduction of a casual insight, to the transition from one logical formalization (henceforth LF) to another.
Unless distinctions excluded once deductive process has started, one is dealing not with a single well-defined LF but with an undefined series of LF's.

Scholastics accept the principle of excluded middle, but their acceptance does not mean that they waive the right to refuse disjunctions, to draw distinctions, to reformulate issues.
ML either accepts or rejects excluded middle, and on that basis distinguishes different logical systems.

Note principle of excluded middle takes on quite different meaning according as casual distinctions are admitted or excluded.

Besides distinctions of content, scholastics have a retinue of structural distinctions, e.g. re, ratione; exercite, signate; first and second intention; etc.

Structural distinctions appear in ML as stratified series of languages: 1. object language
2. meta-language for syntax, for semantics,
3. meta-meta-languages......

Scholastics employ analogous terms.
In ML there emerge indefinitely large stratifications; e.g. Church's series of meanings for implication and quantification,
Curry's for canonicity,
Wang's for set.

5. Paradoxes (Cretan, reflective relation) have played a basic role in development of ML.

I think it important not to view the paradoxes as mere fallacies that a few apposite distinctions would clear away.

Aristotelian logic was a defence of mind against pretence of sophistry, distinction of prooof from persuasion.

The techniques of scholastic tradition correspond to natural dynamism of human mind.

But it is well to distinguish sharply between this natural dynamism and the exact idea of rigorously deductive system: else one will operate in accord with spontaneous dynamism and simultaneously entertain the illusion that one is rigorously deductive.

Again, it is impossible to investigate the properties of rigorously deductive system and admit distinctions to be introduced as need arises, for then one really is investigating series of LF's with no clear idea of the series.

B. Analytic Approach

1. General Character of Technique

Pretechinical: grasp how to do it [practical insight] and do it yourself (sequence of movements).

Movement towards technical: increasing accumulation of # insights (applied science, engineering, technicians, skilled workers) and increasingly detailed instructions for execution, both organizational (workers, foremen, supervisors, superintendents,....) and mechanical (tools, machines, power-driven machines, automation).

Fundamental characteristic of technique.
A highly complex process controlled throughout by practical intelligence,
but only fragmentary intelligence of total process in any of the individuals engaged,
hence division of intellectual labor.

2. Symbolism as technique.

Pretechnical: mental arithmetic.

Technical: plan of operations guided by rules.
One can learn and execute rules without understanding why they work; hence take square root of 1784 but not of MDCCLXIV. Hence,

Economy of intelligence: one can do it equally well though one does not understand; and even if one understands, one can do it without any effort of intelligence as mere effortless routine.

Economy of reason: one does it not only without understanding what one is doing but also without raising any theoretical question about correctness of procedure; all that is needed is practical checking (have rules been observed? does it work?)

Objectification of mind: liberation from individual variations and aberrations; complete irrelevance of 'I see it this way, I am inclined to think, etc.' The technique works independently of any particular mind; as such, it provides completely transparent communication.

3. Symbolism as model

N.B. We are not using name, 'model' in sense employed by mathematicians or in ML. See A. Church, Introduction to Mathematical Logic, p. 325, and note 451; or J. Ladriere, Les limitations internes des formalismes, introductory chapter; also P. Suppes, Introduction to Logic, p. 253 .

Mathematician not only produces symbols but also perceives symbols he produces; at once writer and reader, speaker and hearer.

Moreover, while anyone can look at symbols, mathematician looks intelligently; he is intelligent percipient, reader, hearer; cf. Plato, Meno, anamnesis; Aristotle, De anima, forms grasped by mind in images.

Symbolism as model is symbolism as potential object of intelligence.

Distinguish: model and object.
Model: sensible mark, sequence of such marks.
Object: what is conceived as result of insight into model.

Compare Euclidean diagram and defined points, lines.
In analytic geometry, double model: 1. diagram,
2. algebraic symbols; single object, e.g. conic.
In physics, double model: 1. diagram
2. symbols; object, e.g. free fall.

Explicit and virtual elements in model.
Explicit: what is represented in diagram, expressed overtly in symbols.
Virtual: what is added to explicit model from suggestiveness of symbols, from familiarity with technique.

4. Symbolism as both technique and model

The mathematician as agent and percipient, as speaker and hearer, as writer and reader, effects transformation of symbols.

Translates problem into explicit model; virtualities of model suggest possible sequence of changes; end result is solution of problem. E.g., prove De Moivre's theorem.

Note difference between this process (in which many premises suppressed) and what traditionally has been meant by logical analysis.

5. Isomorphism.

Consider: analogy of proportion; prolonged analogy of proportion; note that it consists in similarity of relations and involves independence of what are related.

Consider that the same symbolic technique can provide model for objects inseries of isomorphic fields.

E.g., for algebraic, geometrical, physical relations.

Now algebra, geometry, physics include deductive processes; therefore, the same symbolic technique will provide a model for these logical relations.

Generally, there can be constructed symbolic techniques that serve as models for total range of logical relations.

6. Mathematical logic (symbolic logic, logistics) is the investigation of the field of logical relations through the development of suitable symbolic techniques.

a. Symbols: Russell-Whitehead, Hilbert, Polish

b. Systems: see table of 13 systems in appendix to A.N. Prior, Formal Logic, Clarendon Press, 1955.

Add combinatory logic, outlined by R. Feys, La technique de la logique combinatoire, RPL 44(1946) 74-103; 237-270

c. Note plurality of systems (very summarily)
Classical Propositional Calculus worked out differently by Russell, Hilbert, Lukasiewicz, et ali.
Lewis modal systems: add strict implication
Intuitionistic: omit excluded middle
Three-valued: admit third alternative to false and true.

d. In general: one can have any type of logic one wishes, and one can construct any symbolism one pleases to attain more readily a specific objective, e.g. all rules of substitution of form $P_1 a \rightarrow aQ_1$

e. The obvious question.
"To sum this all up. The choice of languages is itself a problem that cannot be solved linguistically by only by some non-linguistic method." L. O. Katsoff, "Ontology and the Choice of Languages," Proceedings XI Internat. Cong. Phil., Brussels 1953, XIV, 32.