'ON THE SCHOLASTIC PHILOSOPHY OF GEOMETRICAL KNOWLEDGE'

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Our lectures, with respect to the matter treated, can be considered as an evolution of the books of Aristotle called the Posterior Analytics.

In this treatise the philosopher treats of science strictly so called which results from apodictic demonstration. The books of the Prior Analytics consider the syllogism purely according to its form, where the syllogism can be formed from propositions which are neither certain nor true. The Posterior Analytics, on the other hand, have as their object demonstration by the syllogism, in which the propositions used as premises are true, certain and necessary, and which produces science strictly so called. Thus the matter of the syllogism is also considered.

S. Thomas thus describes the character of each treatise (Anal. Post. I, lect. 1, n.6).:

"The part of the Logic which serves the first process [i.e. 'the process of reason which produces necessity'] is called the judicative part, because the judgement resulting has the certitude of science. And because a certain judgement concerning effects cannot be had except by resolution into first principles, hence this part is called Analytic, that is resolutory. But the certitude of a judgement, which is attained by resolution is, either from the mere form of the syllogism alone, and this is the object of the book of the Prior Analytics, which deals with the syllogism simply; or in addition to the form, from the matter, because propositions per se and necessary are envisaged , and this is the object of the book of the Posterior Analytics, which treats of the demonstrative syllogism."

Aristotle proves in his investigation that in demonstration - that is in science as opposed to the understanding of principles - the search for premises for conclusions cannot be by way of a never-ending resolution, nor is a strictly circular demonstration legitimate; hence in the analysis one must reach some first propositions, first principles, axioms., postulates, 'hypotheses'. So much with regard to propositions in so far as they are the elements of syllogisms.

Something similar must be said of the notions which are the elements of propositions: Not all our notions (ideas) can be defined by other notions; here also one must arrive at primary notions.

It is in this way that Aristotle examines the general structure of the sciences, which consists in the connexions which are discovered between the notions and the first principles and between the notions and the propositions (truths) which are derived from them by way of the syllogism and definition. He does not examine this structure in the same way as the special science itself (e.g. geometry) does, namely in order to construct its own (geometric) system; the object of the Analytics is not the object of a special science; its object is science itself, its origin and its structure. The Analytics, therefore, consider the operations of the human mind themselves; not indeed under their psychological aspect, inasmuch as they are acts of the mind, but with respect to the manner in which these acts attain their objects. The Analytics are therefore a theory of knowledge, even in this first part which examines the general structure of the sciences.

But they also have another function by which they equally constitute a theory of knowledge.

For in this analysis of science we arrive at first principles, which if the conclusions are to be apodictic, must themselves be certain and necessary. Hence the process of the 'logic of judgement' leads of itself to the problem: from where does the certain knowledge of principles arise in the human mind itself, whence is derived the certain knowledge of their necessity. And this is the second part of the enquiry of the

<u>Posterior Analytics</u>.

Thus a twofold problem is considered in these books: 1. the problem of the structure of science, which arises from the connexions between the principles and what is derived from them, 2. the problem of the first principles.

It seems to be clear that these problems are not considered in the abstract, i.e. in such a way that no <u>special</u> science is examined and this is particularly true in the second problem, inasmuch as this is primarily concerned with the <u>matter</u> of the propositions, "necessary and per se propositions are taken as subject matter".

Thus it is that Aristotle in these investigations almost always has before his eyes as the type (or model) of science, geometry; both where he treats in general of demonstrative science, and, obviously, in the numerous cases where he takes his examples from geometry. St. Thomas already noted this in the beginning of his commentary; he remarks (Anal. Poster I, lect. 1, n.10):

"Aristotle makes clear the proposition used as a premise in induction. And in the first place, in the demonstrations in which science is acquired. Among these the mathematical sciences are pre-eminent, because of their most certain mode of demonstration."

The reason is clear. In the time of Aristotle geometry was the only demonstrative science which constituted a systematic body of doctrine. Less than a quarter of a century after the death of the philosopher, Euclid wrote his <u>Elements</u>, which from that time on for almost twenty centuries was used by the human race as an almost perfect manual. But the contents of this work had already in the time of Aristotle been discovered and to some extent reduced to a system; the final and greatest elaborations were discovered in the school of Plato, especially by Eudoxus and Theatetus. It is no wonder that in his examination of the nature of apodictic science Aristotle always keeps geometry before his eyes

The geometers themselves therefore, in the constitution of their science, made the resolution to the first principles, which they did not demonstrate. These principles, the terms of resolution of the geometers, were considered as 'hypotheses' in the school of Plato (Rep. VI 510c.) and were simply accepted (admitted) by the geometers; for they "starting from these principles as being evident to anyone, do not consider them to require any justification (explanation)". But the philosophers did demand and seek an explanation of these principles; and in Plato it was precisely the function of the 'dialectical method' to undertake this enquiry; and so "dialectic" does away with (Rep. VII 532 c) these hypotheses.

This doctrine, so it seems, reappears in Aristotle more fully developed. He distinguishes, (Anal. Post. I, cap. 2) in the propositions which are the principles of the sciences, between those which are axioms (the common conceptions of the mind, 'dignities') from those which are either hypotheses (suppositions) or postulates ('petitions'). The difference can be briefly described thus: an axiom is evident to anyone; this is not the case with those other propositions. If a principle of a special science agrees with the opinion of the learner, it will be a supposition, if it does not, it will be a postulate. Plato also had already spoken of the agreement that there is between the teacher and the learner (Rep. VII 533 c). But all these principles must be received by the geometer from another and a higher science, whether they are there proved by a strict demonstration, or by a consideration of the terms. This other science, which takes the place of the Platonic dialectic and from which the special science receives its principles, is according to S. Thomas either Metaphysics (Anal. Post. I, lect. 5,n.7) or 'physics' (ibid. lect. 5.n.7); but it is always philosophy.

It is clear that in peripatetic philosophy it is the function of the philosopher to settle the question of the first principles of geometry and that will become clearer in the following lectures; whether you attribute that function to Metaphysics, or to natural philosophy, or to a part of philosophy distinct from these, which you could call cognitional theory.

Thus the resolution of geometrical propositions leads to certain first propositions and in the execution of this resolution (analysis) almost everything must be left to the mathematicians themselves; only the general questions which envisage the general structure of science will have to be considered by the philosopher also; for these, without any doubt, concern the epistemologist. But in examining the validity of the principles, the principal task belohgs to us philosophers. This we can discharge for a twofold end. The first is this: in order that all the foundations of a complete geometrical science may be shown to have their justification, so that it may be clearly seen that an integral science of geometry can be safely constructed on these foundations; we will not aim at this total treatment in our lectures, though we will treat of the principal questions.

But there is also a second aim in this philosophical enquiry: we shall discover conclusions (and perhaps methods) which will be of great value for a <u>general</u> theory of knowledge. We shall present one immediately, leaving the others to emerge in the course of the lectures. Certainly this theory cannot be constructed unless the judgement of the human mind is examined with respect to its meaning, its origin, its value. But in order to pass judgement on the judgement itself, it is not sufficient to start with a kind of general definition of the judgement and then proceed to analyse (resolve) it; rather we must elicit some definite (determined) judgements- they must therefore arise in our mind - and these judgements must be examined with respect to their origin and their value. As with our mind's eye, we look at these necessary and universal judgements themselves in concrete instances -

numerous examples will occur in the course of our exposition -, so a universal theory of the judgement itself will come to light from aptly chosen specific judgements, which will then be considered not only as judgements of such and such a special science, but as specimens of the very judgement as such, which reveal the nature of this operation of the human mind to the mind itself. The examination of the principles of mathematics will provide such ideal specimens, for the remark we have heard from S. Thomas is true, namely that this science has the most certain mode of demonstration. For demonstration does not only imply correctness of form, but also and especially the known necessity of the matter, hence the necessity of the principles. Hence for a philosopher who is studying the theory of cognition to neglect these principles is the height of folly.

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These views need to be stressed in our time more than formerly. To understand this it will be useful to recount briefly the history of the problem of mathematical knowledge. Up to about the middle of the last century the common opinion of both philosophers and mathematicians - and of the whole human race - was the following: mathematical knowledge is ultimately derived from the data of sensibility, whether from the external senses or from the imagination. S. Thomas makes this point when he says (In Boeth. de Trin. q.6, a.2): "For in mathematical science knowledge expressed in the judgement must terminate in the imagination not in sense-apprehension". In the further explanation there was indeed a difference of positions. For Plato, on account of the imperfection of sense-knowledge, the data of the senses were only an occasion or preparation required in order that the mind should remember what it had formerly seen (See Meno 82b - 85b and elsewhere). And the mathematical forms themselves, it seems, could only be imperfectly participated by sensible realities. Aristotle held that mathematical concepts were acquired by abstraction from sense-data, both with respect to the notions and with respect to the first principles, which themselves are said to be known by a kind of induction. Not indeed in such a way that the judgement of the mind only registered the data afforded by the senses, but, as in the abstraction of ideas from sense-data, the operation of the agent intellect comes into play, so also in the origin of those judgements which are the first principles there is a similar operation which transcends the sense and attains to the nature; in the data of the imagination the mind by a true mental vision sees the natures (Anal. Post. I, 12,77b,30): "Mathematics is, as it were, a seeing with the understanding".

To explain the necessary character of geometrical knowledge Kant conceived the subjective form of external sensibility. We will have much to say of this opinion later, now it will be sufficient to note that in this opinion also there is recognized an intuition (not an intellectual one) in the phantasm, which should explain the origin of geometrical knowledge.

These schools therefore agree in admitting the necessity and exactitude of mathematical knowledge. In the preceding century empiricism, especially as it is presented by Stuart Mill, readily admits the origin of mathematical knowledge from the data of the senses but for this reason it equiparates it wholly with the knowledge of nature, with the result that it also denies the apodictic necessity and total exactitude which the human race had always attributed to mathematical knowledge.

We will be able later on to dispose of this theory of the empiricists as wholly unconvincing; it does however indicate two problems, the solution to which must be found by the epistemologist if he wishes to vindicate the necessity and exactitude of mathematical knowledge. This is very necessary today, not on account of that theory of empiricism, but on account of the modern theories which have their origin in these difficulties. For these theories entirely reject the classical theories and by that fact they totally undermine the age-old conviction which humanity used to have concerning the value of geometrical science. Without doubt there is a pressing need for the examination of these problems.

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Here are the two problems. The first is this: The data of sensation, i.e. the objects that are known by the senses and inasmuch as they are known only by the senses, are of themselves contingent facts; but the judgements of mathematics present themselves as absolutely necessary, both in arithmetic and in geometry. If they owe their origin to these sense-data, from where does that necessity arise?

This problem is found, as we have said, both in geometry and in arithmetic. But geometry alone involves another problem: the data of the senses in relation to a continuum intuited by the imagination, and in relation to the perceptions of the external senses - even with the aid of perfect instruments - are not exact. The senses do not perceive points without extension, but only very diminutive bodies which are not points; similarly they cannot perceive lines but only long bodies which have some breadth. The senses cannot distinguish between a line which is straight and another which is only slightly curved; they cannot decide whether three lines meet almost, or exactly in one point; not can the imagination distinguish all these things. For all sense-knowledge there is a "threshold of exactitude" which the senses cannot surmount. And nevertheless geometry claims to make absolutely exact judgements on such matters. It teaches that the three median lines in a triangle meet in one point with total - "infinite" - exactitude. Whence does this exactitude arise? This problem does not arise in arithmetic, the senses can perfectly perceive and decide whether three or five objects are there and what is the result of the addition of these numbers; there exactitude is not lacking in the sense-data.

Not all authors give a good description of this second problem; Kant seems to have given no attention to it at all; it is given paramount importance in the modern problematic, although as shall see all were not able to give clear expression to the diverse aspects of the problem.

Thus the modern theories have their origin primarily in the difficulties of the second problem; with the exception of one case which will be considered later, they scarcely speak of the problem of necessity; they seem to consider this problem solved by the fact that they describe 'pure' mathematics as a pure 'creation' of the human mind; in this view the problem of exactitude also seems to disappear. But then, clearly, other questions arise: the question of the real meaning of such a doctrine created by the human mind, the question of its applicability to the real world, to which they nonetheless apply this doctrine. Nor is it surprising that in this question the crude doctrine of empiricism sometimes reappears. Thus Einstein in 'Geometry and Experience': "How can it be that mathematics, in spite of the fact that it is produced by the human mind independantly of all experience can be so perfectly applied to reality?. Can the human intellect without experience, by its pure thought, attain to the properties of real things? In my opinion the brief answer to this question must be: insofar as mathematical theses refer to reality, they are not certain; inasmuch as they are certain, they do not envisage reality."

That such theories, even allowing for their rhetorical exaggeration, demand a new examination of the foundations of mathematics, is evident. They presuppose the further evolution of the doctrines which reject to a greater or lesser degree the imaginative intuition of the continuum from which, nevertheless, geometrical knowledge takes its origin; they even characterize it as a source of errors.

This modern evolution has followed a twofold path; and each of these courses involves the problem of the passage from the inexactitude of sensation to the exactitude of mathematical understanding. The first way leads to the 'arithmetization' of the continuum, as it is called, the second to the doctrines which are derived from the criticism of the fifth postulate of Euclid. Let us briefly consider this evolution.

A. On the arithmetization of the continuum.

The Greeks deduced numbers from the consideration of the intuitive continuum. Aristotle, as is well known, distinguished continuous and discrete quantity, the latter esulting from the division of the former. Multiplicity arises from division and multiplicity measured by one is number, integral number, natural number.

It is not necessary to explain here how the notion of fractional number arises from the same division. The Greeks, it seems, did not want to consider fractions as numbers; where the Egyptians (and the moderns) considered a fraction also as a number, the Greeks recognized the proportions between continuous quantities, lines, which behave like numbers, thus: if lines of different length can be reduced to unity by multiplication by means of different numbers, then their proportion is inverse to the proportion of these numbers.

But they had also already discovered that there were lines (v.g. the side and the diagonal of a square) which did not admit of any numerical proportion, which were incommensurable, irrational. The difficulties which arose from these irrational proportions (but real) were brilliantly solved by Eudoxus with his theory of proportions (which is described in the fifth book of Euclid's Elements)

The Moderns, certainly from Descartes on, considered all these propositions as numbers and, as the Greeks had a 'continuum of proportions' which is the corpus of all the proportions in a continuum, so the Moderns have a continuum of numbers, the corpus 'of real numbers', which contains not only integers but also fractions, and irrational numbers (both algebraic and transcendant). The modern 'analysis' could be established on the basis of these numbers and by means of the infinitesimal calculus this analysis can effect the calculation (computation) of the continuum, even continuous changes. But these numbers, this analysis were deduced from the intuitive continuum; these numbers are only another name for the proportions of the Greeks.

Difficulties were raised on this point in the course of the last century; because of the lack of exactitude in sense perception, the intuitive continuum did not seem to be an apt medium from which the corpus of real numbers could be deduced; hence they tried to effect this deduction by another means, by the pure arithmetical intuition of the series of integral numbers (1,2,3....to infinity) which is exempt from that lack of exactitude.

Few authors describe this difficulty exactly; here are two very well-known authors who speak with the greatest clarity on this point.

F. Klein in his book 'Anwendung der Differential-und Integral-rechnung. Eine Revision der Prinzipien (which, as the second part of the title indicates, is entirely devoted to this problem) is expressly concerned with this difficulty. He explains well that in measurement by means of sense perception (and also in the imagination) there is a threshold of exactitude beyond which our perception cannot go:

"In all these practical areas there is a threshold value of exactitude" But in the arithmetical definition of real number (v.g. by means of the decimal fraction) there is no such threshold:

"In the ideal area of Arithmetic there is not a finite threshold value as there is in the empirical area, but the exactitude with which the numbers are defined or at least are considered to be defined, is unlimited".

We will not consider a certain difficulty which we have, nor the way in which he proceeds in the construction of the analysis (and of abstract, 'precise' geometry, "Praezisionsmathematik").

Let us hear also the words of H. Poincare:

"Intuition (of the continuum) cannot give us exactitude, nor even certainty; this has always been progressively recognized".

"We have, therefore, different kinds of intuition; in the first place the appeal to sense and imagination; then generalization by means of induction. . . finally we have the intuition of pure number . . . The first two cannot give us certitude; I have shown that above by examples; but who would entertain serious doubts about the third, who would have doubts about Arithmetic?

But in contemporary analysis, if one wants to maintain strict rigour, this can only be by the syllogism and the appeal to this intuition of pure number, which alone cannot deceive us. It can be said that absolute exactitude is attained today."

Hence Analysis, because it is a purely arithmetical construction starting from the series of integral numbers, no longer depends as formerly on the intuition of the continuum, but vice versa is used for the construction of an abstract geometry. The reason for this, as we have heard, is because these authors will not trust the intuition of the continuum owing to the inexactitude of sense-data. They even think that they have certain examples (we will examine them later) which are deduced from analysis and are said to positively contradict imaginative intuition.

If these assertions are true they present us with a very serious problem: how can it be that the human mind was for centuries so firmly convinced of the exact value and the supreme certitude of geometry, deduced from such an intuition? There are some who, on this ground, assert that this classical method of constructing geometry, in spite of the real genius of many of those who employed it, remains only a dream, "eine Utopie", Thus the celebrated 'realist' mathematician E. Study, says in his book 'Die realistiche Weltansicht und die Lehre vom Raume':

"This circumstance seems to us decisive in passing judgment on the present issue, that a geometry really independant of analysis, which was precisely what the ancient ideal demanded, is clearly seen to be a dream".

But the difficulty becomes even greater if further examination shows that this classical geometry cannot even be constructed from analysis and in dependance on it. For in that case the most brilliant mathematicians up to the middle of the last century were wrong not only in their method but also in what they thought they had discovered.

And indeed, in the case we are considering, this seems to be so. True, if it is a question of approximative application (Approximations Mathematik in F. Klein's expression) there is no great difficulty; for this only deals with experimental extension, as it is known by the senses by means of inexact sense-perception. But if it is a question of the properties of abstract extension, of the extended as extended, about which classical geometry argued, these properties cannot, in our view, be deduced from analysis, without all the same difficulties appearing once more in this very application. And so our problem remains.

If that is true must we not assert that neither does the human intellect perfectly know this extension, something men have always believed up to this? What are we to say of Aristotle's theory of knowledge, which held firmly that this science (geometry) had its origin in sense-data?

There is another point. Shortly after the beginning of this century difficulties arose in this very confident arithmetical analysis, which according to some authorities (so Weyl, Brouwer and others) lead to what may truly be called a 'vrisis' in Mathematics. And there are some (v.g. O.Hölder) who consider that the difficulties cannot be solved except by a return to the consideration of the intuitive continuum; an opinion which seems true to us. This makes it all the more urgent to enquire how the inexactitude of sense-data can be overcome; and this is what we will try to do in our lectures.

## B. On the Criticism of the Fifth Postulate of Euclid.

Another way in which the modern evolution of the philosophy of mathematics made progress was the criticism of the Fifth Postulate of Euclid, or the postulate of the single parallel. The principle which is expressed in this postulate is indeed not immediately evident, it cannot be immediately abstracted from the sense-data; and this again on account of the lack of exactitude in these data, as we shall see subsequently. (It is remarkable that very many mathematicians who call this principle 'less evident' than other axioms, do not seem to be able to indicate the root of this defective evidence). Thus it belongs to those principles which, according to Aristotle, are not '<u>axioms</u>' but '<u>postulates</u>'; and which in consequence are admitted in geometry as '<u>hypotheses</u>', but must be explained and justified by another, higher science, namely by philosophy.

Already in antiquity, as shown by Proclus, difficulties were raised concerning the postulate and others were proposed in its place, but they in turn were equally defective. The critique of the last century led to this conclusion, which is now finally generally accepted by mathematicians: besides classical, Euclidian geometry, another is possible which rejects the fifth postulate and replaces it with another contradictory postulate. Many, but not all, of the theorems deduced from this postulate contradict the classical theses; but there is no internal contradiction in any one of the systems of geometry nor will such a contradiction be found in the future. From this they conclude: for many centuries Euclidean geometry seemed to the human mind to be the only possible and necessary one; this now seems to be false; the modern view holds that along with Euclidean geometry there are other systems which are equally possible. We are confronted with a problem which must be solved by a theory of knowledge.

At first sight the necessity of geometry seems to be denied; and indeed there are some authorities who in this sense reckon geometry with the purely physical sciences, which, in the same way, lack this evident necessity. This is certainly false, as can easily be shown; each individual system is intrinsically necessary and they are known by us as such. But the comparison between the geometrical and physical sciences deserves our attention.

However there remains another very serious problem: the whole human race and all the mathematicians, even those who were unhappy about the Fifth Postulate, recognized Euclidean geometry as the <u>one</u> necessary system; if they are in error, how is this to be explained? if they are not wrong, what are we to say about the severe critique of the moderns? This much is clear: again there is question of a serious problem, which is very relevant in a general theory of knowledge. Did not Aristotle and S. Thomas select theorems from Euclidean geometry when they wanted to give an example of a position (thesis) which was absolutely certain?

C. <u>On 'Axiomatics'</u>

From this critique there evolved a complete science, part of 'inductive logic' which is called <u>Axiomatics</u>. We have spoken above of the intrinsic necessity and intrinsic immunity from contradiction which is proper to every system of geometry, including the non-Euclidean system.

To establish these properties the first principles of such a science must be enuntiated - each and every one of them. If this is done one system, e.g. Euclidean geometry can be enuntiated as a massive conditional proposition. Let us indicate the principles by the letters A, B....and F, then the conclusions by the letters, P, Q, R....The whole geometry can now be enuntiated thus: If A and B....and F, then P and Q and R and . . . This proposition, as a true conditional, enuntiates the intrinsic necessity. And this science will be: hypothetico-deductive. Many 'axiomatics' hold that pure geometry is no more than such a hypothetico-deductive system. The fundamental hypothesis, i.e. the series of axioms A, B...F, is no longer examined in relation to the real value of the propositions; indeed according to the extreme 'axiomatics', not even in relation to their meaning. The words with which Hilbert begins his exposition in the first chapter of his book '<u>Die Grundlagen der Geometric</u>' are well known:

"We conceive three different systems of things: we call the things (entities) of the first system <u>points</u> and we indicate them by the letters A, B. C...; the things of the second system we call <u>straight lines</u> and we indicate them by the letters a, b, c...; the things of the third system we call <u>planes</u> (surfaces) and we indicate them by the letters a, B, y."

Thus neither the (real) value nor the meaning matters, but it is only required that from the hypothetical premiss the deduction be made by means of valid syllogisms according to pure formal logic.

At first sight this position may seem quite Aristotelian; for according to the philosopher science strictly so called - as opposed to the understanding of first principles - envisages solely conclusions logically deduced. But in due time we shall see that there is a vast difference.

For if account is not taken of the meaning of the axioms, it is clear that there can be no question of the evidence of the axioms. Hence we cannot be certain a priori that no contradiction will follow from such a system of axioms. Thus it is that 'axiomatics' set about finding ways of proving that no contradiction will follow.

This position leads both to problems and to results which are valuable. The first problem is the same one that always recurs if we abandon the natural procedure of the human mind ("le cheminement de la pensée). The human mind thinks that it knows objective extension perfectly and that it can construct a perfectly certain science of that object. Such an axiomatic, non-contradictory system only attains this object within limits; for it to be applied to reality, the whole classical problem of the transition from inexact sense-data (and possibly contingent facts) to exact and necessary knowledge seems to return. For classical geometry was rediscovered not <u>only</u> by the hypothetico-deductive system. But 'axiomatics' also discovered valuable results. Thus: as we have said, it begins with a system of 'axioms' concerning the evidence of which it makes no suppositions and consequently without any doubt a system could be constructed from which a contradiction would follow very soon or after a more lengthy deduction. To insure that the deduction is valid, provision must be made against this. And they sought and found methods of demonstrating that a determined system is immune from contradiction even in its future conclusions. Such methods, sometimes very brilliantly conceived, are without doubt a valuable evolution of logic.

Further these methods allow investigations into the particulars of the structure of science, just as Aristotle did in the Posterior Analytics with regard to its general structure. Not all the conclusions require an integral series of axioms for their deduction; and it could be determined in different cases how the system of axioms could be distinguished into different parts which have their influence on different parts of science. Thus the structure of science can be more distinctly known. We have a further evolution of the doctrine of Aristotle. To this question and to the problem of non-contradiction are linked the methods of enquiring into the true mutual <u>independance</u> of the axioms which are proposed; and these also make a contribution to 'the logic of judgment'.

And this entire method of proving non-contradiction seems to lead to a problem which is shared by ontology, namely, to the theory of possibles. Possibles are described as: objects whose notes do not imply any mutual contradiction. Can this theory be applied to systems which according to 'axiomatics' are immune from contradiction, e.g. to non-Euclidean geometry? And in general to the problem of 'Mathematical existence' about which Aristotle and S. Thomas also have a theory open to further development?

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From this brief survey we can conclude: in this part of philosophy we recognize, with considerable regret, an absence of enquiry on the part of the scholastics. Without any doubt this part of 'the logic of judgment,' the science which was so auspiciously begun by Aristotle, must receive renewed study, especially in view of the modern critique of these questions; in question are the most serious problems in cognitional theory, not only the special but also the general theory, the problems of logic and the epistemological problems regarding the structure of science. It is a question also of fully establishing the foundations of geometry; but we will not give special attention to this; the first problems will demand out attention because these especially involve the first principles, the purely philosophical matter. We will not need to enter into questions of higher mathematics; the elements of mathematics will suffice for our purpose.

We will conclude with one observation: in the lectures which follow we will not be treating of a science (certainly philosophical) already <u>constituted</u> but rather of a philosophical science as yet <u>to be constituted</u>: of the scholastic philosophy of geometrical knowledge.

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